Quest for graphs of Frank number 3

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In an orientation O of the graph G, the edge e is deletable if and only if $O - e$ is strongly connected. For a 3-edge-connected graph G, Hörsch and Szigeti [1] defined the Frank number as the minimum k for which G admits k orientations such that every edge e of G is deletable in at least one of the k orientations. They proved that the Frank number is at most 7 in general, but conjectured that the Frank number is at most 3 for every 3-edge-connected graph G. They proved the Petersen graph has Frank number 3, but this was the only example with this property. In [2], we find an infinite family of 3-edge-connected graphs with Frank number 3. In [3], we improved the best known upper bound to 5 and shortly after Goedgebeur, Mácajová and Renders [4] improved it to 4. Hörsch and Szigeti showed every 3-edge-colorable 3-edge-connected graph has Frank number at most 3. It is tempting to consider non-3-edge-colorable graphs as candidates for having Frank number greater than 2. Snarks are sometimes a good source of finding critical examples or counterexamples. One might suspect various snarks should have Frank number 3. However, we prove several candidate infinite classes of snarks have Frank number 2. As well as the generalized Petersen Graphs $GP(2s + 1, s)$. I would like to popularize this very recent problem at the workshop.

References

[1] F. Hörsch, Z. Szigeti, *Connectivity of orientations of* 3*-edge-connected graphs*, European J. Combin., **94** (2021).

[2] J. Barát, Z.L. Blázsik, *Quest for graphs of Frank number* 3, submitted to Australasian Journal of Combinatorics
`https://arxiv.org/pdf/2209.08804`

[3] J. Barát, Z.L. Blázsik, *Improved upper bound on the Frank number of* 3*-edge-connected graphs*, submitted to European Journal of Combinatorics
`https://arxiv.org/abs/2305.19050`

[4] J. Goedgebeur, E. Mácajová, J. Renders, *On the Frank number and nowhere-zero flows on graphs*, manuscript, `https://arxiv.org/abs/2305.02133`