Towards inductive proofs in algebraic graph theory with applications

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One of the main obstacles to solving problems concerning automorphism groups of vertex-transitive graphs and digraphs is that the natural factors of the automorphism group do not necessarily give groups that are automorphism groups of vertex-transitive graphs or digraphs (or even 2-closed groups). Thus using induction can be difficult. We find a class of transitive permutation groups, which properly contains the automorphism groups of vertex-transitive graphs and digraphs, and call them 5/2-closed, as, using Wielandt's hierarchy of permutation groups, the class is larger than 2-closed but contain groups which are not 3-closed. We give a sufficient condition for the natural quotients of transitive groups in this class to remain in this class, in effect giving a sufficient condition for natural inductive proofs to work. We will then give examples of results that have been proven for this new class, solving several problems concerning isomorphisms between combinatorial objects, automorphism groups of combinatorial objects, and more.