On the number of directions determined by a set of size p

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Rédei proved that a set in $\mathbb{F}_p^2$ of cardinality of p is either a line or determines at least $\frac{p+3}{2}$ directions.

We present a new proof for Rédei's result [2] that relies on a lemma proved in a joint paper with Gergely Kiss. In the mean time we prove the following theorem.

Theorem 1. *Assume f is a polynomial of degree at least 1 with $\sum_{x\in\mathbb{F}_p} f(x) = p$. Then the degree of f is at least $\frac{p-1}{2}$.*

As a strengthening of the result we proved the following.

Theorem 2. *For large enough p, up to affine transformations, there are exactly 2 polynomials f of degree $\frac{p-1}{2}$ with $\sum_{x\in\mathbb{F}_p} f(x) = p$.*

This result leads to a new proof of a result of Lovász and Schrijver [1].

Joint work with Gergely Kiss and Ádḿ Markó and Zoltán Nagy.

References

[1] L. Lovász, A. Schrijver: Remarks on a theorem of Rédei, Studia Scient. Math. Hungar. **16** (1981), 449–454.

[2] L. Rédei: Lükenhafte Polynome über endlichen Körpern, Birkhäuser Verlag, Basel (1970) (English translation: Lacunary polynomials over finite fields, North Holland, Amsterdam (1973)).