ON REGULAR TWO-GRAPHS ON 38 AND 42 VERTICES

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A two-graph is a pair $(\mathcal{V}, \Delta)$, where Δ is a collection of unordered triples chosen from a finite set of vertices $\mathcal{V}$, such that every 4-subset of $\mathcal{V}$ contains an even number of triples of Δ. The triples from Δ are called coherent. A regular two-graph has the property that every pair of vertices lies in the same number of triples of the two-graph.

From a two-graph $\Phi = (\mathcal{V}, \Delta)$ and any fixed $x \in \mathcal{V}$ we construct the graph Γ which has a vertex set $\mathcal{V}$ by setting the vertex x as isolated vertex and letting any two other vertices y, z to be adjacent in Γ if $\{z, x, y\}$ is coherent in Φ. Deleting the isolated vertex x yields a graph on $|\mathcal{V}| - 1$ vertices, which is called the descendant of Φ. The two-graph $(\mathcal{V}, \Delta)$ is regular if and only if each descendant is strongly regular with parameters $(v-1, k, \lambda, \mu)$, where $\mu = k/2$.

All regular two-graphs having up to 36 vertices are known, and the first open case is the enumeration of two-graphs on 38 vertices. It is known that there are at least 191 regular two-graphs on 38 vertices and at least 18 regular two-graphs on 42 vertices. The number of descendants of these two-graphs is 6760 and 120, respectively.

In [1], we classify strongly regular graphs with parameters $(41, 20, 9, 10)$ having nontrivial automorphisms and show that there are exactly 7152 such graphs. We enumerate all regular two-graphs on 38 and 42 vertices with at least one descendant whose full automorphism group is nontrivial and establish that there are at least 194 regular two-graphs on 38 vertices and at least 752 regular two-graphs on 42 vertices. Furthermore, we construct descendants with trivial automorphism group of the newly constructed two-graphs and increase the number of known strongly regular graphs with parameters $(37, 18, 8, 9)$ and $(41, 20, 9, 10)$ to 6802 and 18439 respectively. This significantly increases the number of known strongly regular graphs with parameters $(41, 20, 9, 10)$.

References

[1] M. Maksimović, S. Rukavina, *New regular two-graphs on* 38 *and* 42 *vertices*, Math. Commun. 27 (2022), 151–161