# Extremal edge-girth-regular graphs

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In extremal graph theory, one often considers a problem with the following type: we fix some graph parameters or graph properties and want to deduce the extremal number of another parameter (in many cases, the number of vertices or edges). The problem considered in our paper is motivated by the well-known cage problem. Jajcay, Kiss, and Miklavič ([2]) defined a new type of regularity. An edge-girth-regular graph $egr(n, k, g, \lambda)$ is a $k-$regular graph of order $n$, girth $g$ and with the property that each of its edges is contained in exactly $\lambda$ distinct $g-$cycles.
An $egr(n, k, g, \lambda)$ is called extremal for the triple $(k, g, \lambda)$ if $n$ is the smallest order of any $egr(n, k, g, \lambda)$. Drglin, Filipovski, Jajcay, and Raiman ([1]) gave lower bounds for the order of edge-girth-regular graphs. Their proof is purely combinatorial. We improve the lower bounds for the order of extremal edge-girth-regular graphs using properties of the eigenvalues of the adjacency matrix of a graph ([3]).

## References

[1] A. Z. Drglin, S. Filipovski, R. Jajcay, and T. Raiman, Extremal Edge-Girth-Regular Graphs, Graphs and Combin. **37** (2021), 2139–2154.

[2] R. Jajcay, Gy. Kiss, Š. Miklavič, *Edge-girth-regular graphs*, European Journal of Combinatorics **72** (2018), 70–82.

[3] I. Porupsánszki, *On edge-girth-regular graphs: lower bounds and new families*, arXiv:2305.17014 (2023)