The distance function in Coxeter-like graphs

Marko Orel

University of Primorska (Slovenia)

(Joint work with Draženka Višnjić)

Vector spaces of matrices with coefficients from a finite field equipped with the rank-metric $D(A, B) = \text{rank}(A - B)$ are subjects of study in various mathematical areas. They can be found, for example, in algebraic combinatorics (association schemes, distance-regular graphs), in coding theory (rank-metric codes), and in matrix theory (preserver problems). Consider the graph $\widehat{\Gamma}_n$, which has the set $S_n(\mathbb{F}_2)$ of all $n \times n$ binary symmetric matrices as the vertex set, and where two matrices form an edge $\{A, B\}$ if and only if $\text{rank}(A-B) = 1$. It is well known and easy to see that the graph distance in $\widehat{\Gamma}_n$ equals $d_{\widehat{\Gamma}_n}(A, B) = \text{rank}(A - B)$ unless the diagonal of $A - B$ is zero and $A \neq B$ (i.e. unless $A - B$ is a nonzero *alternate* matrix). In the later case, we have $d_{\widehat{\Gamma}_n}(A, B) = \text{rank}(A - B) + 1$.

In this talk, we will consider the subgraph Γ_n in $\widehat{\Gamma}_n$, which is induced by all invertible matrices. Graph Γ_n was introduced in [1] and generalizes the well-known Coxeter graph Γ_3. Here, we are out of the comfort zone because the vertex set of Γ_n is no longer a vector space. In the talk, the graph distance in Γ_n will be described. We will see that the value $d_{\Gamma_n}(A, B)$ depends on the 'type' of the symmetric rank decomposition of the matrix $A - B$.

References

[1] M. Orel, On generalizations of the Petersen and the Coxeter graph. Electron. J. Combin. 22(4) (2015), Paper #P.4.27.