

m -SETS OF AFFINE AND PROJECTIVE SPACES AVOIDING CERTAIN CONFIGURATIONS

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The starting point of this talk is the so-called cap-set problem which is the determination of the largest subset of $AG(n, 3)$ that does not contain a complete line, or in other terms, contains no arithmetic progression of length 3 (3-AP). Quite recently Ellenberg and Gijswijt proved a breakthrough result regarding caps in $AG(n, 3)$ [3] building on the ideas of Croot, Lev and Pach [2].

We discuss the generalisation of this problem in the following directions:

Problem 1.

- *What is the maximum size of a point set M in $AG(n, q)$ which does not contain a complete affine k -dimensional subspace, (i.e., k -flat)?*
- *Given integers k, t , what is the spectra of $|M|$ for point sets M in $AG(n, q)$ for which the intersection size of M and a k -flat is different from t , for all k -flats?*

Then we consider the 2-dimensional version of the problem above in the projective setting.

Problem 2. *Given two positive integers $k < q$ and $m < q^2 + q + 1$, are there sets of size m in a finite projective plane $PG(2, q)$ which do not have k -secants?*

Note that for $k = 1$, this problem was initiated by Blokhuis, Seress and Wilbrink [1] who called sets $S \subseteq PG(2, q)$ without tangents *untouchable sets*.

Finally, we study the size of point sets of $AG(n, q)$ that do not contain arithmetic progression of length 3 (3-AP). A 3-AP free set is complete if it is not contained in a larger 3-AP free set. We show contructions of small complete 3-AP free sets for which the size almost attains the trivial lower bound.

The results are joint with **Benedek Kovács** (for Problem 1), **Tamás Héger** (for Problem 2) and **Bence Csajbók** (for the AP-3 problem).

References

- [1] Blokhuis, A., Seress, A., Wilbrink, H. A. (1991). On sets of points in $PG(2, q)$ without tangents. Mitt. Math. Sem. Giessen, 201, 39-44.
- [2] Croot, E., Lev, V. F., Pach, P. P. (2017). Progression-free sets in are exponentially small. Annals of Mathematics, 331-337.
- [3] Ellenberg, J. S., Gijswijt, D. (2017). On large subsets of with no three-term arithmetic progression. Annals of Mathematics, 339-343.