Geometric constructions of small regular graphs with girth 5 and 7

György Kiss

ELTE, Budapest (Hungary) & University of Primorska, Koper (Slovenia)

(Joint work with Gabriela Araujo-Pardo and István Porupsánszki)

The cage problem is a classical problem in extremal graph theory. A (k, g)-graph is a k-regular graph with girth g. A (k, g)-*cage* is a (k, g)-graph of minimum order. A general lower bound on $n(k, g)$, known as the Moore bound, is obtained by counting the vertices whose distance from a vertex (if g is odd), or an edge (if g is even) is at most $\lfloor (g-1)/2 \rfloor$. Graphs attaining this bound are called Moore graphs. For $g = 3, 4$ Moore graphs are the complete, and complete bipartite graphs, respectively. Moore graphs are rare for $g > 4$. When g is even, then there exists a k-regular Moore graph widt girth $2r > 4$ if and only if there exists a finite generalized r-gon of order $(k-1, k-1)$, and the graphs are the incidence graphs of the generalized polygons. When $g > 4$ odd, then there exist Moore graphs only for $g = 5$ and $k = 3, 7$ and possibly 57.

When $g = 5$, then several graphs were constructed by complicated, careful manipulations of the incidence graphs of finite projective planes. Some vertices are removed from these $(q + 1, 6)$-cages, after that matchings or cycles are added to the neighbours of the removed vertices to get back regularity.

In this talk simple geometric constructions are presented for $g = 5$ and 7 using incidence graphs of projective planes and generalized quadrangles, respectively.

References

[1] G. Araujo-Pardo, Gy. Kiss, I. Porupsánszki, *Notes on edge-girth-regular graphs arising from $t-$good structures and biaffine planes*, manuscript.

[2] Gy. Kiss, *Geometric constructions of small regular graphs with girth 7*, manuscript.