Perfect matchings of bicubic graphs and latin $3 \times n$ rectangles

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Let R be an $3 \times n$ latin rectangle. Then there is a cubic bipartite (=bicubic) graph $B(R)$ with vertex set $\{1, \ldots, 2n\}$ such that i is connected to $n + j$ if and only if j occurs in the ith column of R. Each row of R determines a perfect matching of $B(R)$. Conversely, we start with a bicubic graph B on $2n$ vertices, and edge-disjoint perfect matchings π_1, π_2, π_3. By labeling the vertices by $\{1, \ldots, n\}$ and $\{n + 1, \ldots, 2n\}$, we construct a latin rectangle $R(B)$ whose (i, j) entry is k if and only if the edge $\{j, k\}$ is in π_i. Two latin rectangles are similar, if one can be obtained from the other by permuting the rows, the columns and the symbols. Similar latin rectangles give rise to isomorphic bicubic graphs.

Our long-term plan is to study the correspondance $R \mapsto B(R)$; more precisely, to characterize the set of latin rectangles that give rise to a given graph. This is related to the problem on the number of perfect matchings in a given bicubic graph. This number is known for some specific classes of bicubic graphs, and the lower bound $6(4/3)^{n-3}$ has been proved by Voorhoeve [1] in 1978.

I will present computational results on the number of perfect matchings of bicubic graphs up to 30 vertices.

References

[1] M. Voorhoeve, A lower bound for the permanents of certain (0,1)-matrices, Indagationes Mathematicae (Proceedings), Volume 82, Issue 1, 1979, Pages 83-86.