Avoiding given size intersections in finite affine spaces $AG(n, 2)$

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We study the set of intersection sizes of a k-dimensional affine subspace and a point set of size $m \in [0, 2^n]$ of the n-dimensional binary affine space $\mathrm{AG}(n, 2)$. Following the theme of Erdős, Füredi, Rothschild and T. Sós [2], we partially determine which local densities in k-dimensional affine subspaces are unavoidable in all m-element point sets in the n-dimensional affine space. We say $[n, m] \to [k, t]$ if the intersection size t is unavoidable, and denote by $\rho(n, k, t)$ the density of values $m \in [0, 2^n]$ for which $[n, m] \to [k, t]$.

Generally we conjecture that $\rho(n, k, t) \to 1$ as $n \to \infty$ for all pairs (k, t) with $0 \leqslant t \leqslant 2^k$. The talk focuses on the case when t is a power of 2, or the sum of two consecutive powers of 2. Building on the ideas of the Szemerédi Cube Lemma [3, Corollary 2.1], and the work of Bonin and Qin [1], we first prove that $\rho(n, k, 2^{k-\ell}) \geqslant \frac{1-\varepsilon}{2^{\ell-1}}$ for all $1 \leqslant \ell \leqslant k - 1$, $\varepsilon > 0$ and sufficiently large n.

For $t = 3 \cdot 2^{k-\ell}$, the spectrum is scattered (has values m missing from it), but still we can show that it has positive density. To prove this, we use the same framework but with added tools including the additive energy of sets, and estimates on the sizes of hypercube cuts.

References

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