Partial ovoids, partial spreads of finite classical polar spaces, and related objects

Francesco Pavese

Polytechnic University of Bari (Italy)

Let $\mathcal{P}$ be a finite classical polar space of $\mathrm{PG}(n, q)$. A *(partial) ovoid* $\mathcal{O}$ of $\mathcal{P}$ is a set of points of $\mathcal{P}$ such that every generator of $\mathcal{P}$ has (at most) one point in common with $\mathcal{O}$. A *(partial) spread* $\mathcal{F}$ of $\mathcal{P}$ is a set of generators of $\mathcal{P}$ such that every point of $\mathcal{P}$ lies on (at most) one generator of $\mathcal{F}$. In the case when $\mathcal{P}$ does not admit ovoids or spreads, the question about the size of the largest (maximal) partial ovoid or of the largest (maximal) partial spread naturally arises. Some constructive lower bounds on the sizes of the largest partial ovoids of the symplectic polar space $\mathcal{W}(2r+1, q)$, $r \in \{1, 2\}$, and of the Hermitian polar space $\mathcal{H}(2r, q)$, $r \in \{2, 3, 4\}$, will be discussed. Similarly, an improvement on the lower bound of the size of the largest partial spread of the Hermitian polar space $\mathcal{H}(8r-5, q)$, $r \geqslant 2$, will be outlined.

Subsets of generators of $\mathcal{P}$ are also interesting from a coding theory point of view. Let $\mathcal{M}$ denote either $\mathcal{S}_{r,q}$, or $\mathcal{A}_{r,q}$ or $\mathcal{H}_{r,q}$, i.e., the set of matrices of order r over $\mathbb{F}_q$ that are either symmetric, or alternating or Hermitian. Then $\mathcal{M}$ equipped with the rank distance d is a metric space. There exists a bijection between $\mathcal{S}_{r,q}$, $\mathcal{A}_{r,q}$ or $\mathcal{H}_{r,q}$ and the set of generators of the symplectic polar space $\mathcal{W}(2r-1, q)$, of the hyperbolic polar space $\mathcal{Q}^+(2r-1, q)$ or of the Hermitian polar space $\mathcal{H}(2r-1, q)$ that are disjoint from a fixed generator. Moreover, a t-code in $(\mathcal{M}, d)$ is equivalent to a set $\mathcal{F}$ of generators of the corresponding polar space $\mathcal{P}$ that are disjoint from a fixed generator of $\mathcal{P}$ and such that every $(r-t)$-space of $\mathcal{P}$ lies on at most one element of $\mathcal{F}$. In this context, improved upper and lower bounds on the size of the largest 2-code of $\mathcal{S}_{3,q}$ will be presented. In particular, the constructive improvement on the lower bound provides the first infinite family of 2-codes of symmetric matrices whose size is larger than the largest possible additive 2-code.