# Complementary prisms: their cores, automorphism groups, isoperimetric numbers, hamiltonian properties, etc.

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A finite simple graph is a *core* if all its endomorphisms are automorphisms. In general it is a difficult problem to determine whether a given graph is a core. The most basic examples of cores are complete graphs and odd cycles. A more interesting example is the Petersen graph. There are at least two known families of graphs, which generalize the Petersen graphs, and which are cores: the Kneser graphs and the graphs obtained from invertible hermitian matrices over a finite field $\mathbb{F}_4$, where the edges are formed by pairs of matrices $\{A, B\}$ with $\operatorname{rank}(A - B) = 1$. The Petersen graph can be constructed also from the pentagon $C_5$ and its complement $\overline{C_5}$ (which is also a pentagon) by adding an edge between each pair of identical vertices in $C_5$ and $\overline{C_5}$. Such a construction is known as the *complementary prism* $\Gamma\bar{\Gamma}$ and is defined for each finite simple graph $\Gamma$. The pentagon is a graph with several interesting properties. It is vertex-transitive, strongly regular, self-complementary (i.e. isomorphic to its complement), an example of a Paley graph. Hence, it is natural to ask if $\Gamma\bar{\Gamma}$ is a core whenever $\Gamma$ has some of the mentioned properties. This problem will be the main topic of the talk. Our main focus will be on the cases, where $\Gamma\bar{\Gamma}$ posses either some nice combinatorial properties or some degree of 'symmetry'.

In the talk we will consider also some other aspects of complementary prisms. Strangely, we will see that the isoperimetric number (a.k.a. Cheeger number) of $\Gamma\bar{\Gamma}$ is known for each finite simple graph $\Gamma$. Similarly, the automorphism group $\operatorname{Aut}(\Gamma\bar{\Gamma})$ can be always described by $\operatorname{Aut}(\Gamma)$. In particular, the fraction $|\operatorname{Aut}(\Gamma\bar{\Gamma})|/|\operatorname{Aut}(\Gamma)|$ can attain only values 1, 2, 4, or 12. The automorphism group enables us to determine all the cases, where $\Gamma\bar{\Gamma}$ is vertex-transitive, and to show that it is not a Cayley graph whenever $\Gamma$ has more than one vertex. Hamiltonian properties of the complementary prisms will be addressed as well.

The talk will be based on the recent work available at [1].

## References

[1] M. Orel, *A family of non-Cayley cores based on vertex-transitive or strongly regular self-complementary graphs*. Preprint available at `https://arxiv.org/pdf/2110.10416.pdf` (2021).