Applying finite projective geometry to the construction of rare snarks

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The celebrated Berge-Fulkerson conjecture suggests that every bridgeless cubic graph can have its edges covered with at most five perfect matchings. Since three perfect matchings suffice if and only if the graph in question is 3-edge-colourable, uncolourable cubic graphs fall into two classes: those that can be covered with four perfect matchings, and those that require at least five. Cubic graphs that cannot be covered with four perfect matchings are extremely rare. Among the 64326024 snarks (uncolourable cyclically 4-edge-connected cubic graphs with girth at least five) on up to 36 vertices there are only *two* graphs that cannot be covered with four perfect matchings – the Petersen graph and a snark of order 34.

In this talk we show that coverings with four perfect matchings can be described by means of flows whose outflow patterns form a configuration of six lines spanned by four points of the 3-dimensional projective geometry $PG(3, 2)$ in general position. This characterisation provides a convenient tool for investigation of graphs that do not admit such a cover and enables a great variety of constructions of snarks that cannot be covered with four perfect matchings. In particular, with the combined forces of several constructions we can prove that for each even integer $n \geq 42$ there exists at least one snark of order n that has no cover with four perfect matchings.

This is a joint work with Edita Máčajová.