Strongly regular graphs with 2-transitive two-graphs*

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joint work with R. Bailey and V. Smaldore

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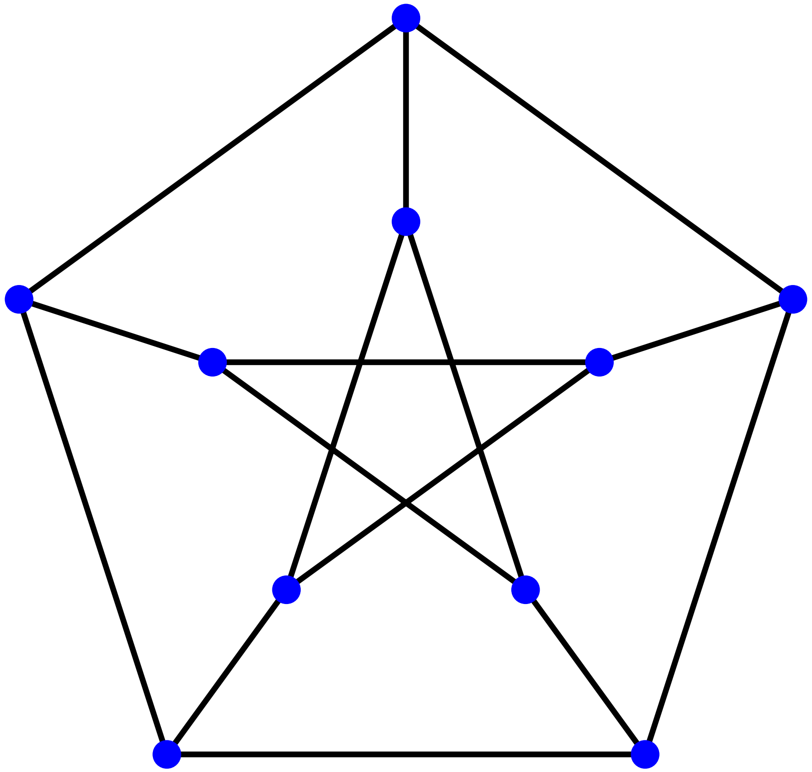
Outline

1. SRGs, two-graphs, Seidel switchings
2. Two-graphs with doubly transitive automorphism groups
3. SRGs in the switching class of linear type two-graphs

Everybody knows:



The Petersen graph srg$(10, 3, 0, 1)$

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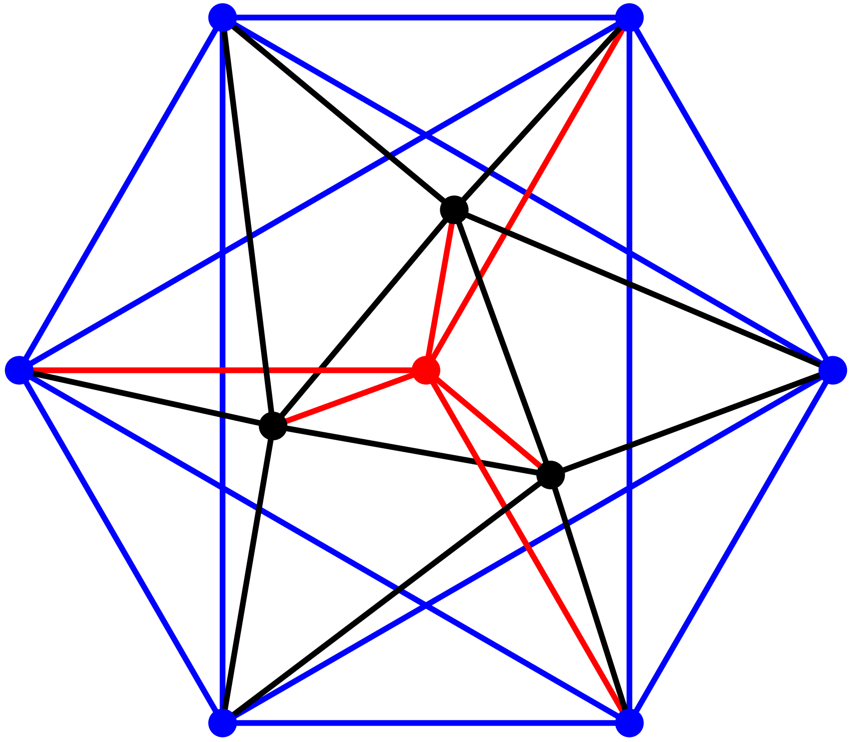

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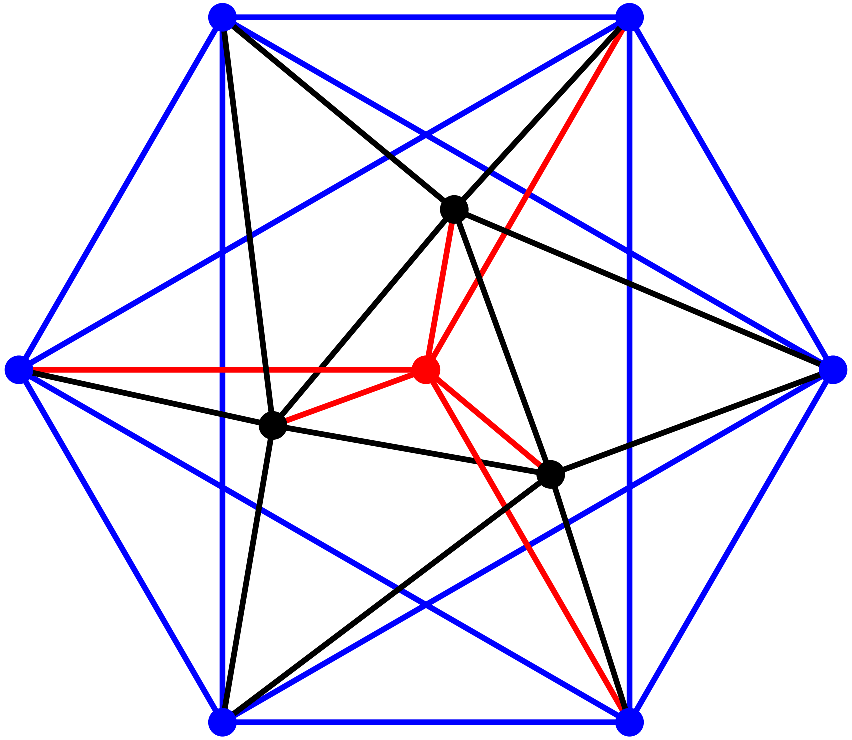

srg(10, 6, 3, 4)

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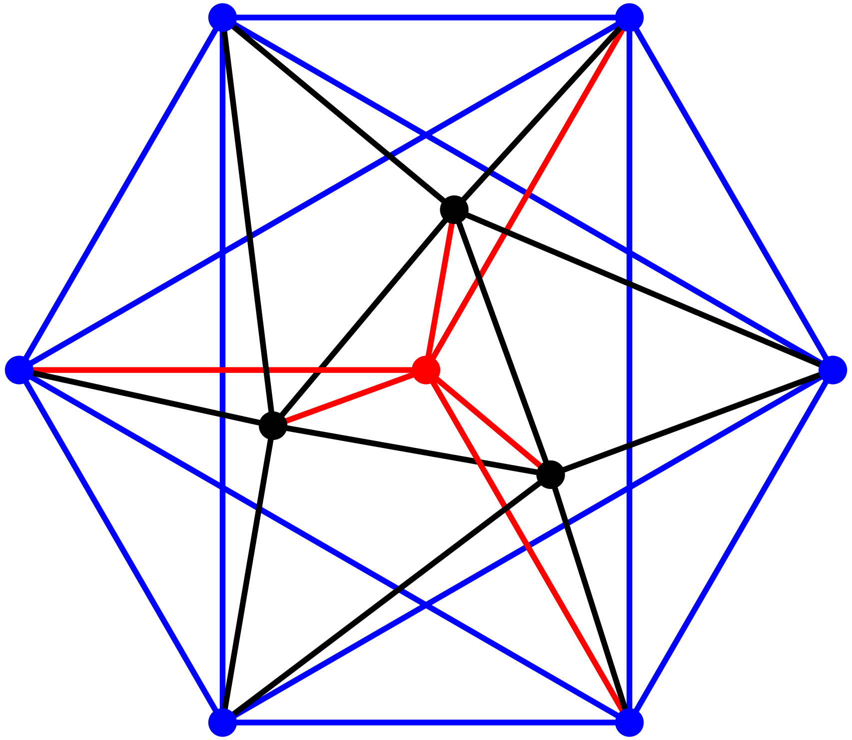

$\mathrm{srg}(10, 6, 3, 4)$
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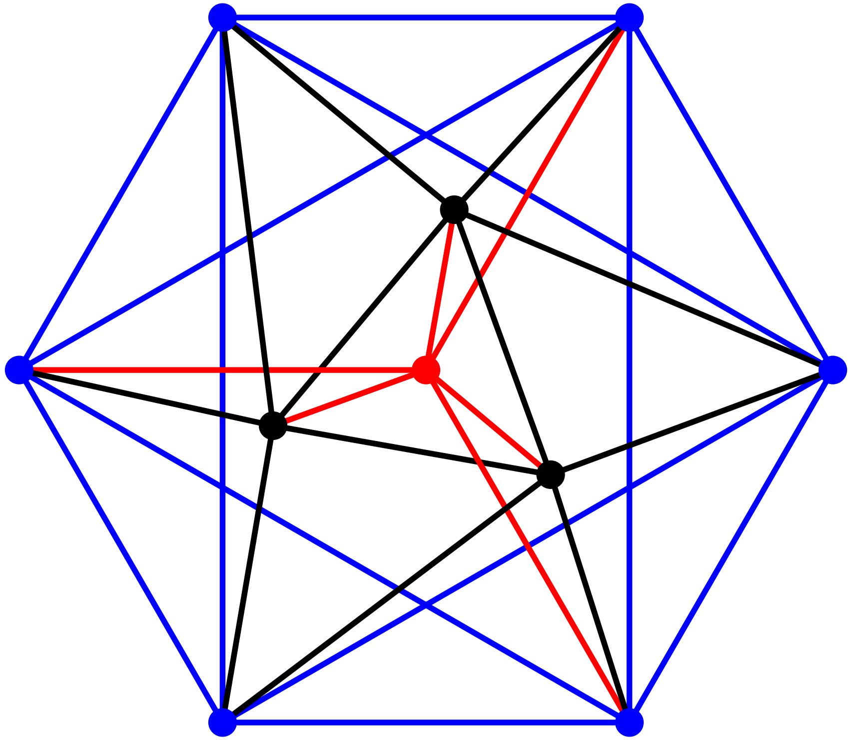

srg$(10, 6, 3, 4)$
complement of the Petersen graph
Johnson graph J$(5, 2)$

Seidel switching

Definition: Seidel switching of a simple graph $G = (V, E)$

Given a subset $Y \subseteq V$, the operation of switching of G with respect to Y consists of replacing

- all edges from Y to its complement by nonedges,
- and all nonedges by edges,
- while leaving the edges within Y or outside Y unchanged.

Fact

The Johnson graph $J(5, 2)$ is obtained from the Petersen graph by Seidel switching w.r.t. the blue/green subsets.

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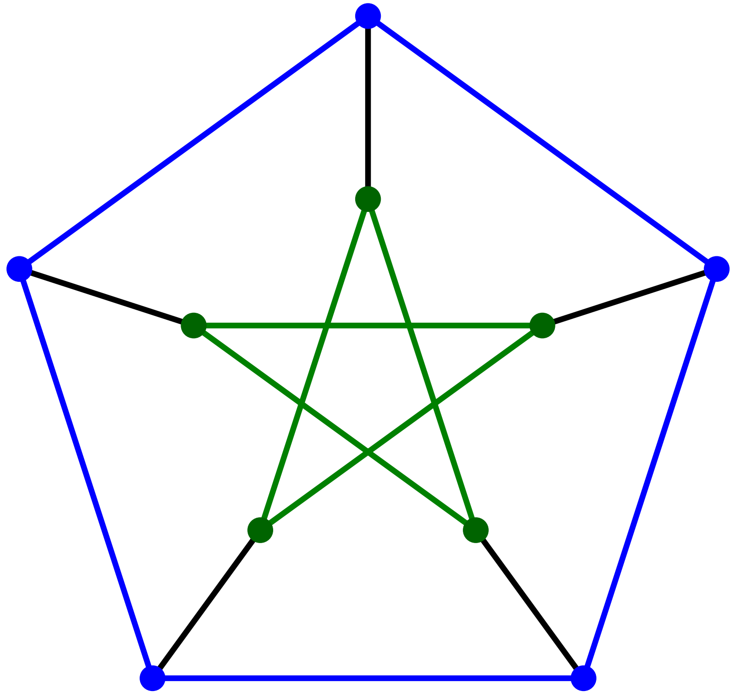

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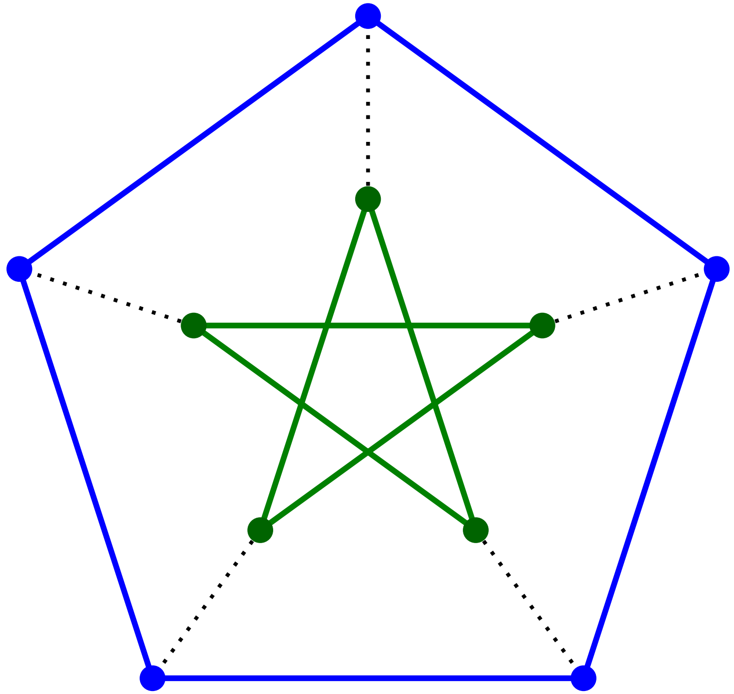

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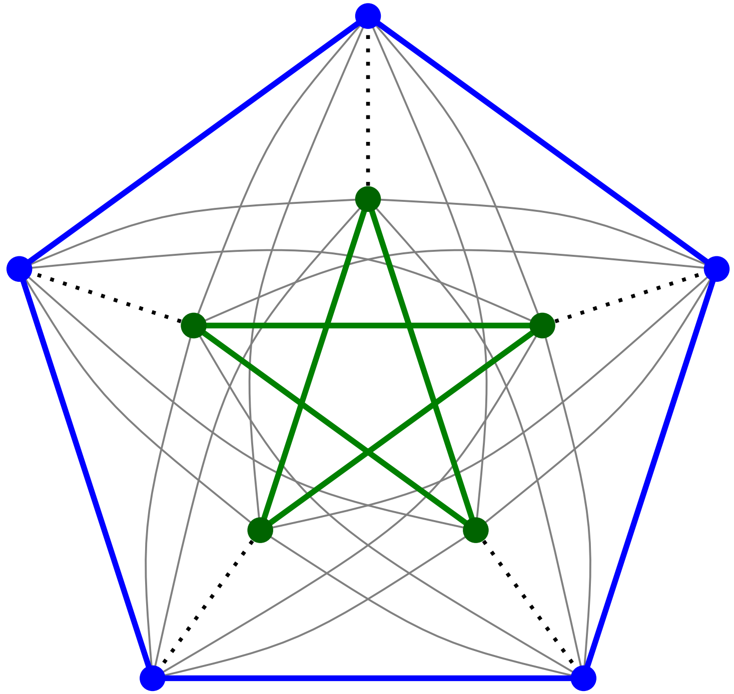

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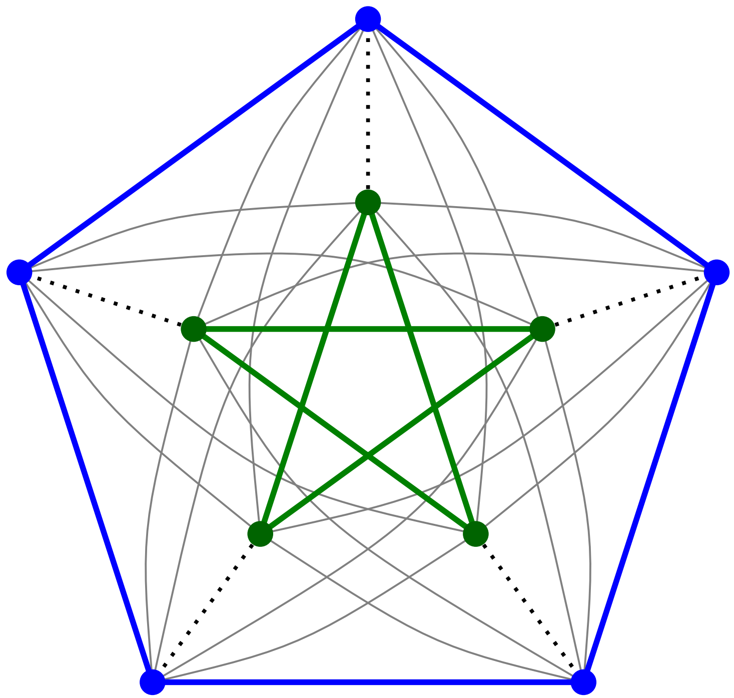

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Two-graphs are not graphs

Definition: Two-graph (Higman)

A two-graph is a pair (X, T), where T is a set of unordered **triples** of a vertex set X, such that every (unordered) quadruple from X contains an even number of triples from T.

In a regular two-graph, each pair of vertices is in a constant number of triples.

Proposition

Let $G = (V, E)$ be a simple graph, and T the set of those triples of the vertices, whose induced subgraph has an **odd number of edges.** Then (V, T) is a two-graph.

We call (V, T) the associated two-graph of G.

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Two graphs are switching equivalent if and only if they have the same associated two-graph.

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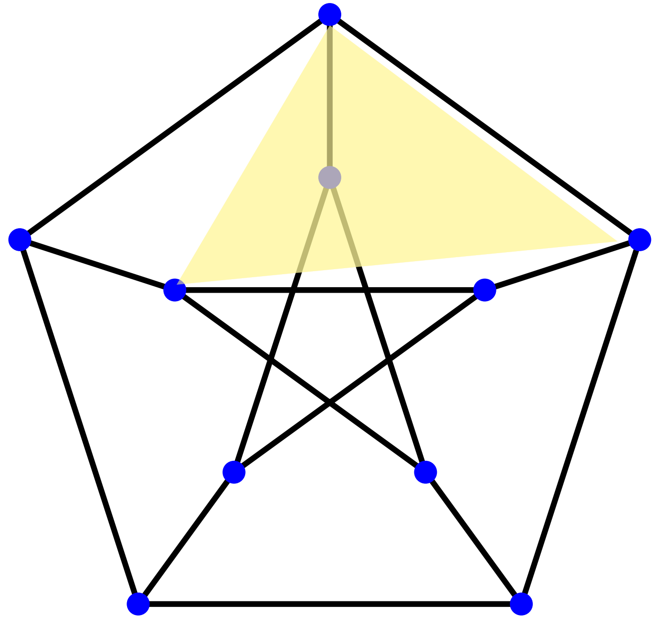
The two-graph of the Petersen graph

- "...triples with an **odd number of edges**..."
- no triangles
- $|T| = 15 \times 4 = 60$
- the graph automorphism group is $H = S_5$
- the automorphism group of the two-graph is $G = \mathrm{P\Sigma L}(2,9) \cong S_6$
- G is 2-transitive on V
- $H = S_5$ is a transitive maximal subgroup of G



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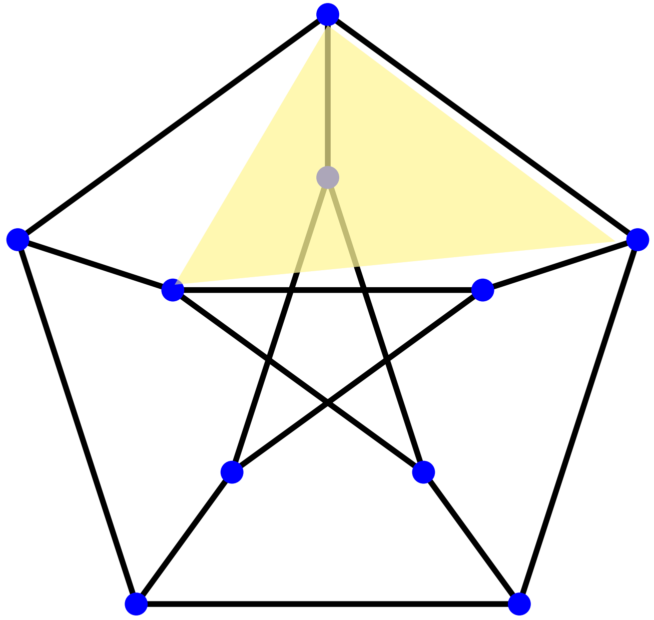

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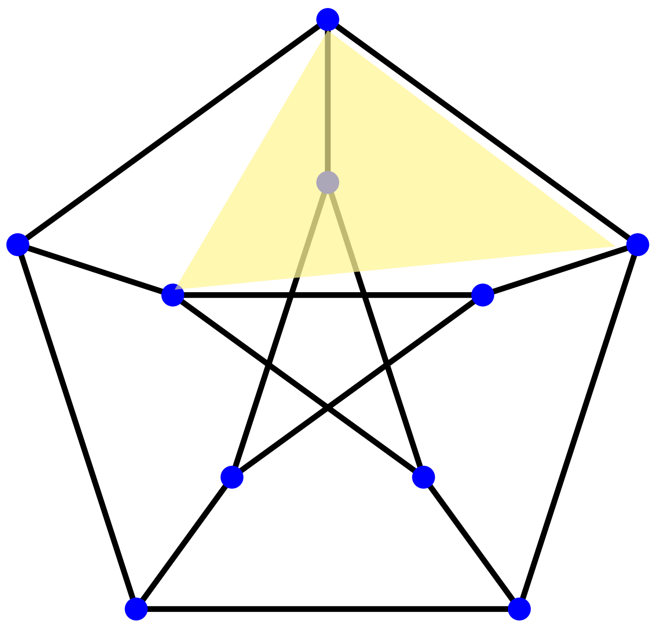

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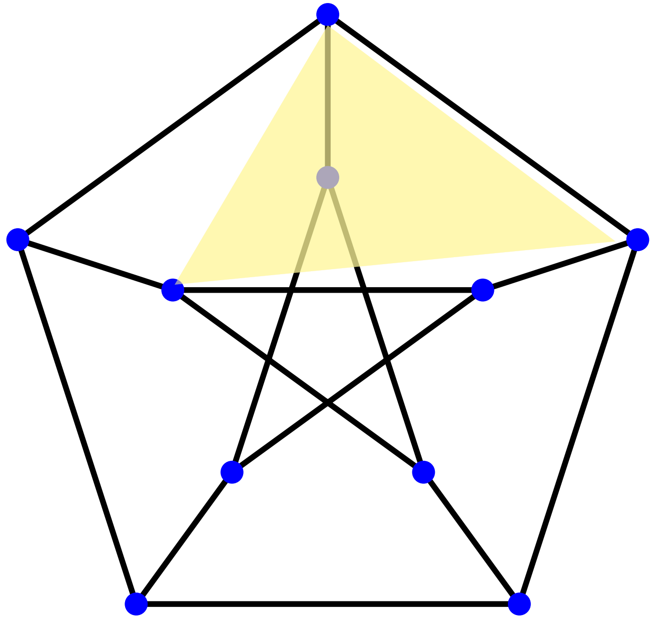

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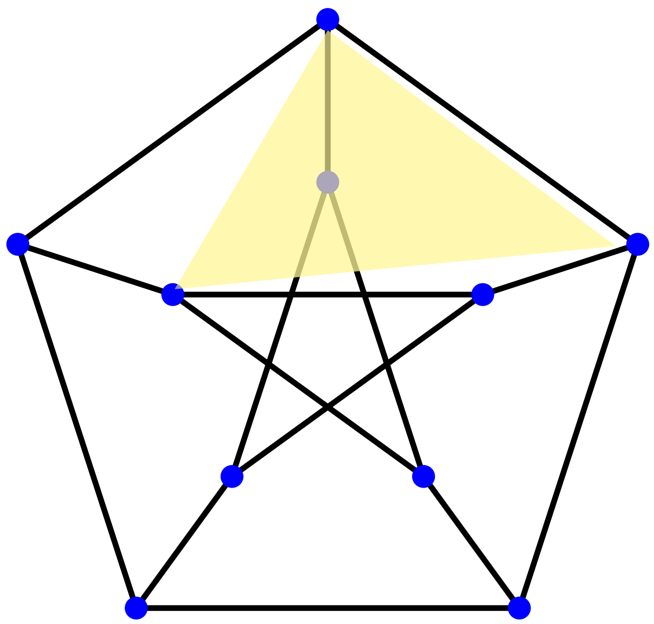

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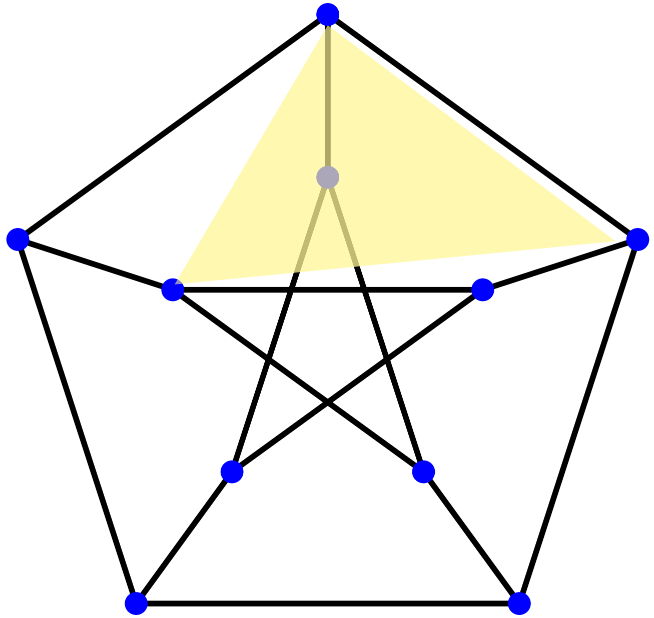

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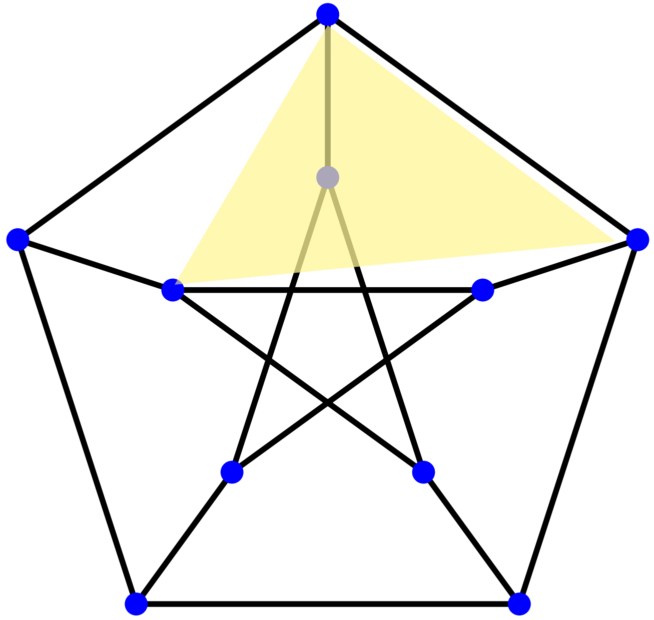

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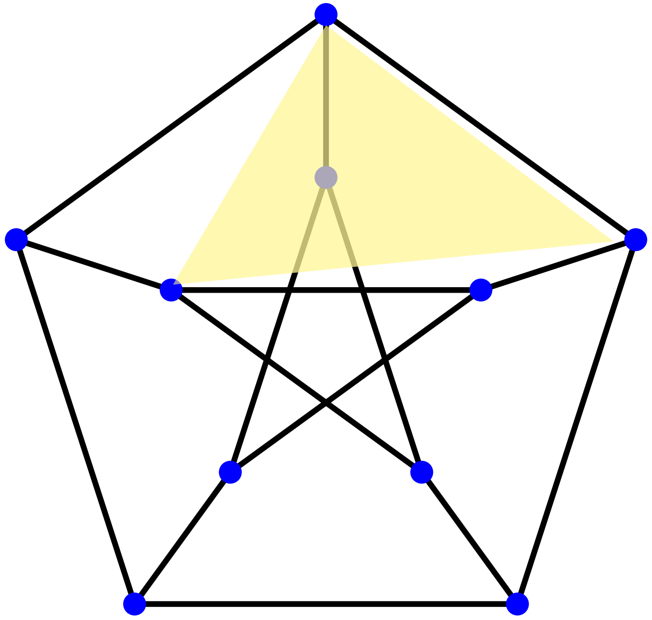

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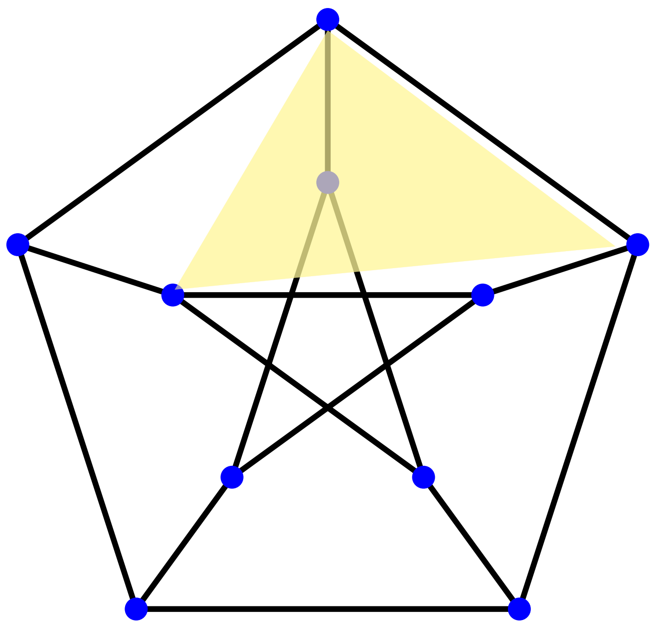

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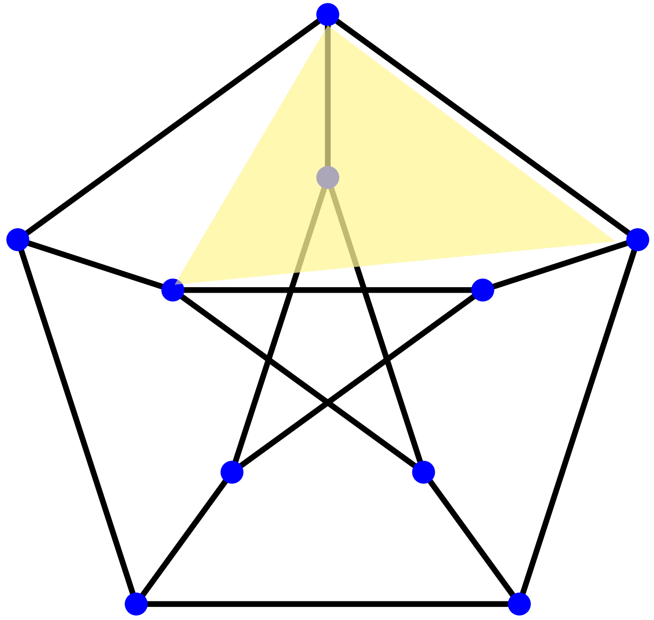

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2-transitive two-graphs

Finite two-graphs with 2-transitive automorphism groups G have been classified by Taylor (1991):

1. **Affine polar type:** $G \cong \mathbb{F}_2^{2m} \rtimes Sp(2m, 2)$, $m \geq 2$;
2. **Symplectic type:** $G \cong Sp(2m, 2)$, $m \geq 3$;
3. **Linear type:** $G \cong P\Sigma L(2, q)$, for $q \equiv 1 \pmod 4$;
4. **Unitary type:** $G \cong P\Gamma U(3, q)$, for $q \geq 5$ odd;
5. **Ree type:** $G \cong Ree(q) \rtimes \mathrm{Aut}(\mathbb{F}_q)$, for $q = 3^{2e+1}$, $e \geq 1$;
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Exceptional and sporadic graphs

G	H	Γ	Remark
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$Sp(6,2)$	$G_2(2)$	$U_3(3)$-graph	$\|V\| = 36$
$Sp(8,2)$	S_{10}	$J(10,3,2)$	$\|V\| = 120$
	$PSL(2,17)$	Bailey, Crnković-Švob	$\|V\| = 136$
$P\Gamma U(3,5)$	S_7	Goethals	$\|V\| = 126$; param. uniqueness; H is max. in G not containing $PSU(3,5)$
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G	H	Γ	Remark
$P\Sigma L(2,9)$	S_5	Petersen graph & its complement	$P\Sigma L(2,9)$ has two conjugacy classes of S_5
$Sp(6,2)$	$G_2(2)$	$U_3(3)$-graph	$\|V\| = 36$
$Sp(8,2)$	S_{10}	$J(10,3,2)$	$\|V\| = 120$
	$PSL(2,17)$	Bailey, Crnković-Švob	$\|V\| = 136$
$P\Gamma U(3,5)$	S_7	Goethals	$\|V\| = 126$; param. uniqueness; H is max. in G not containing $PSU(3,5)$
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Infinite classes of graphs

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$ASp(2m,2)$	$\mathbb{F}_2^{2m} \rtimes O^{\pm}(2m,2)$	$VO^{\pm}_{2m}(2)$	$m \geq 2$
$Sp(2m,2)$	$O^{\pm}(2m,2)$	$NO^{\pm}_{2m}(2)$	$m \geq 3$
	$Sp(m,4) \rtimes C_2$	$NO^{\pm}_{m+1}(4)$	$m \geq 4$ even
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Outline

1 SRGs, two-graphs, Seidel switchings

2 Two-graphs with doubly transitive automorphism groups

3 SRGs in the switching class of linear type two-graphs

Linear type two-graphs and their graphs

- Let q be a prime power with $q \equiv 1 \pmod 4$.
- There is a **unique** two-graph $\mathcal{TL}$ with

$$\mathrm{Aut}(\mathcal{TL}) = P\Sigma L(2, q).$$

- $\mathcal{TL}$ is isomorphic to its complement.
- Any SRG in the switching class of $\mathcal{TL}$ has parameters

$$\left(q+1, \frac{q \pm \sqrt{q}}{2}, \frac{(\sqrt{q} \pm 1)^2}{4} - 1, \frac{(\sqrt{q} \pm 1)^2}{4}\right).$$

- If q is a square, then such graphs are known to exist.

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Are there SRGs in the switching class of $\mathcal{TL}$ with **transitive** automorphism groups?

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Strongly regular graphs in two-graphs of linear type

$p^f = q \equiv 1 \pmod 4$, $f \geq 2$.

Lemma (Transitive maximal subgroups of $P\Sigma L(2,q)$)

Let $G = P\Sigma L(2,q)$. Let H be a **transitive maximal subgroup** not containing $PSL(2,q)$.

- (i) **NEGATIVE:** If $p \equiv 1 \pmod 4$, then there is not such H.
- (ii) **EXCEPTIONAL:** If $q = 9$, then there are **2 conjugacy classes** of $H \cong S_5$.
- (iii) **POSITIVE:** If $q > 9$, $p \equiv 3 \pmod 4$, then there is a **unique conj class** of $H = C_{\frac{q+1}{2}} \rtimes C_{2f}$.

Theorem

Let $p \equiv 3 \pmod 4$, $q = p^{2e}$.
Let $\mathcal{TL}$ be the **linear two-graph** with automorphism group $G = P\Sigma L(2,q)$, $q > 9$.
Then there is (up to isomorphism) a unique pair $\Gamma, \bar{\Gamma}$ of complementary strongly regular graphs in the switching class of $\mathcal{TL}$, admitting

$$H = C_{\frac{q+1}{2}} \rtimes C_{4e}$$

as **transitive automorphism group.**

THANK YOU FOR YOUR ATTENTION!

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- **Michel Lavrauw,** University of Primorska (Slovenia)