

2. feladatsor – Rang, alterek MEGOLDÁSOK

2.1. Feladat. Megoldások:

- (a) $x_1 = 2, x_2 = -3, x_3 = -1,$
vektor alakban: $(2, -3, -1);$
- (b) nincs megoldás;
- (c) $x_4 = 1, x_1 = 2x_2 - x_3, x_2, x_3 \in \mathbb{R},$
vektor alakban: $(2x_2 - x_3, x_2, x_3, 1), x_2, x_3 \in \mathbb{R}$
- (d) $x_1 = 17 - 3x_2 + 3x_4, x_3 = 4 + x_4, x_2, x_4 \in \mathbb{R},$
vektor alakban: $(17 - 3x_2 + 3x_4, x_2, 4 + x_4, x_4), x_2, x_4 \in \mathbb{R}$
- (e) nincs megoldás;
- (f) $x_1 = 4 - 7x_4, x_3 = 2 - 4x_4, x_2, x_4 \in \mathbb{R},$
vektor alakban: $(4 - 7x_4, x_2, 2 - 4x_4, x_4), x_2, x_4 \in \mathbb{R};$
- (g) $x_1 = \frac{5}{17} + 3x_2, x_3 = \frac{7}{17}, x_4 = \frac{6}{17}, x_2 \in \mathbb{R},$
vektor alakban: $(\frac{5}{17} + 3x_2, x_2, \frac{7}{17}, \frac{6}{17}), x_2 \in \mathbb{R}.$

2.2. Feladat. Rang, lineáris függetlenség

- (a) $r = 3,$ lineárisan független;
(b) $r = 2,$ lineárisan függő;
(c) $r = 3,$ lineárisan független;
(d) $r = 2,$ lineárisan függő;
(e) $r = 2,$ lineárisan függő.

2.3. Feladat. Rang, maximális nemeltűnő aldetemináns:

- (a) $r = 2, \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix};$
- (b) $r = 3, \begin{vmatrix} 1 & 3 & 9 \\ 2 & 4 & 8 \\ 9 & 3 & 1 \end{vmatrix};$
- (c) $r = 1,$ tetszőleges nem nulla elemből álló 1×1 -es mátrix determinánása megfelelő.

2.4. Feladat. $r = 3.$

2.5. Feladat. Az U alterek elemei:

- (a) $U = \{(\bar{0}, \bar{0}, \bar{0}), (\bar{1}, \bar{0}, \bar{0}), (\bar{0}, \bar{1}, \bar{0}), (\bar{1}, \bar{1}, \bar{0})\};$
- (b) $U = \{(\bar{0}, \bar{0}, \bar{0}), (\bar{1}, \bar{2}, \bar{1}), (\bar{2}, \bar{1}, \bar{2}), (\bar{2}, \bar{0}, \bar{1}), (\bar{0}, \bar{2}, \bar{2}), (\bar{1}, \bar{1}, \bar{0}), (\bar{1}, \bar{0}, \bar{2}),$
 $(\bar{2}, \bar{2}, \bar{0}), (\bar{0}, \bar{1}, \bar{1})\};$
- (c) $U = \{(\bar{0}, \bar{0}, \bar{0}), (\bar{1}, \bar{1}, \bar{2}), (\bar{2}, \bar{2}, \bar{1})\};$
- (d) $U = \{(\bar{0}, \bar{0}, \bar{0}, \bar{0}), (\bar{0}, \bar{1}, \bar{0}, \bar{0}), (\bar{0}, \bar{2}, \bar{0}, \bar{0}), (\bar{1}, \bar{0}, \bar{0}, \bar{1}), (\bar{1}, \bar{1}, \bar{0}, \bar{1}), (\bar{1}, \bar{2}, \bar{0}, \bar{1}),$
 $(\bar{2}, \bar{0}, \bar{0}, \bar{2}), (\bar{2}, \bar{1}, \bar{0}, \bar{2}), (\bar{2}, \bar{2}, \bar{0}, \bar{2})\}.$

2.6. Feladat. Előállítások és koordinátasorok:

- (a) $(\bar{1}, \bar{0}, \bar{0}) = (\bar{0}, \bar{1}, \bar{0}) + (\bar{1}, \bar{1}, \bar{0}),$ koordinátasora: $(\bar{0}, \bar{1}, \bar{1});$

(b) $(\bar{1}, \bar{0}, \bar{1}) = (\bar{1}, \bar{1}, \bar{1}) + (\bar{0}, \bar{1}, \bar{0})$, koordinátasora: $(\bar{1}, \bar{1}, \bar{0})$.

2.7. Feladat. Dimenziók és bázisok:

(a) $\dim U = 3$, bázis: $(0, 1, 2, 4), (2, -1, 2, 2), (1, -1, 1, 2)$;

(b) $\dim U = 2$, bázis: $(1, 2, 4, 1), (0, 0, 3, -1)$;

(c) $\dim U = 2$, bázis: $(\bar{1}, \bar{4}, \bar{2}, \bar{3}), (\bar{2}, \bar{3}, \bar{4}, \bar{2})$.

2.8. Feladat. Az U alterek és dimenziójuk:

(a) $U = [(1, 0, 2), (0, 1, 0)]$, $\dim U = 2$;

(b) $U = [(\bar{4}, \bar{1}, \bar{3})]$, $\dim U = 1$;

(c) $U = [(\bar{1}, \bar{2}, \bar{1}, \bar{0}), (\bar{2}, \bar{2}, \bar{0}, \bar{1})]$, $\dim U = 2$;

(d) $U = [(1, -1, 0, 1)]$, $\dim U = 1$;

(e) $U = [(2, 1, 3, 0), (0, 0, 0, 1)]$, $\dim U = 2$;

(f) $U = [(3, 1, 0, 0), (1, 0, 1, 0), (0, 0, 0, 1)]$, $\dim U = 3$;

(g) $U = [(\bar{0}, \bar{1}, \bar{0}, \bar{0}), (\bar{1}, \bar{0}, \bar{0}, \bar{1})]$, $\dim U = 2$.

2.9. Feladat. Az alterek:

(a) $\{(x_1, x_2, x_3) : x_1 = x_3\}$;

(b) $\{(x_1, x_2, x_3) : x_1, x_2, x_3 \in \mathbb{R}\}$;

(c) $\{(x_1, x_2, x_3, x_4) : -x_1 + 2x_2 + x_3 = 0, -5x_1 + 3x_2 + x_4 = 0\}$;

(d) $\{(x_1, x_2, x_3, x_4) : x_4 = 2x_2\}$;

(e) $\{(x_1, x_2, x_3, x_4, x_5) : 2x_2 + 2x_3 + x_4 = \bar{0}, 2x_2 + x_3 + x_5 = \bar{0}\}$.

2.10. Feladat. Az $U_1 + U_2$ és $U_1 \cap U_2$ alterek dimenziói, bázisai:

(a) $\dim(U_1 + U_2) = 3$, bázis: $(1, 2, 1, 0), (0, 3, 2, 1), (0, 0, 1, 2)$

$\dim(U_1 \cap U_2) = 1$, bázis: $(2, 1, 0, -1)$;

(b) $\dim(U_1 + U_2) = 3$, bázis: $(1, 2, 1, 3), (0, -2, 3, -1), (0, 0, 1, -3)$

$\dim(U_1 \cap U_2) = 1$, bázis: $(0, 2, -3, 1)$;

(c) $\dim(U_1 + U_2) = 4$, bázis: $(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)$

$\dim(U_1 \cap U_2) = 1$, bázis: $(3, -3, 2, 1)$;

(d) $\dim(U_1 + U_2) = 3$, bázis: $(1, 0, 0, 0), (0, 0, 0, 1), (0, 2, 1, 0)$

$\dim(U_1 \cap U_2) = 1$, bázis: $(-3, 2, 1, -3)$;

(e) $\dim(U_1 + U_2) = 4$, bázis: $(\bar{1}, \bar{0}, \bar{0}, \bar{0}), (\bar{0}, \bar{1}, \bar{0}, \bar{0}), (\bar{0}, \bar{0}, \bar{1}, \bar{0}), (\bar{0}, \bar{0}, \bar{0}, \bar{1})$

$\dim(U_1 \cap U_2) = 1$, bázis: $(\bar{1}, \bar{2}, \bar{2}, \bar{1})$;

(f) $\dim(U_1 + U_2) = 5$, bázis: $(\bar{1}, \bar{0}, \bar{0}, \bar{0}, \bar{0}), (\bar{0}, \bar{1}, \bar{0}, \bar{0}, \bar{0}), (\bar{0}, \bar{0}, \bar{1}, \bar{0}, \bar{0}),$

$(\bar{0}, \bar{0}, \bar{0}, \bar{1}, \bar{0}), (\bar{0}, \bar{0}, \bar{0}, \bar{0}, \bar{1})$

$\dim(U_1 \cap U_2) = 0$;

(g) $\dim(U_1 + U_2) = 3$, bázis: $(\bar{1}, \bar{0}, \bar{1}, \bar{0}), (\bar{0}, \bar{1}, \bar{1}, \bar{0}), (\bar{0}, \bar{0}, \bar{0}, \bar{1})$

$\dim(U_1 \cap U_2) = 1$, bázis: $(\bar{1}, \bar{1}, \bar{2}, \bar{1})$.

2.11. Feladat. A V vektortér és U_1, U_2 altereinek megadott dimenziói esetén határozzuk meg az $U_1 + U_2$ és az $U_1 \cap U_2$ alterek dimenziójának összes lehetséges értékét.

(a) $\dim(U_1 + U_2) = 6, \dim(U_1 \cap U_2) = 2,$

$\dim(U_1 + U_2) = 5, \dim(U_1 \cap U_2) = 3;$

(b) $\dim(U_1 + U_2) = 5, \dim(U_1 \cap U_2) = 2,$

$\dim(U_1 + U_2) = 4, \dim(U_1 \cap U_2) = 3;$

(c) $\dim(U_1 + U_2) = 7, \dim(U_1 \cap U_2) = 0,$

$\dim(U_1 + U_2) = 6, \dim(U_1 \cap U_2) = 1,$

$\dim(U_1 + U_2) = 5, \dim(U_1 \cap U_2) = 2.$