MBN172G: ELEMENTARY MATHEMATICS HOMEWORK

1. Basic algebraic laws

Problem 1.1. Factor the following formulas
  (1) $a^3 - 2a^2 - 4a + 8$;
  (2) $x^4 - 7x^2 + 1$;
  (3) $x^4 + x^3 + 2x^2 + x + 1$.

Problem 1.2. Simplify the following expressions:
  (1) $\left(\frac{1}{p^2-pq} - \frac{3q^2}{p^4-pq^3} - \frac{q}{p^3+p^2q+pq^2}\right) \cdot \left(q + \frac{p^2}{p+q}\right)$;
  (2) $\frac{1}{(x-y)(x-z)} + \frac{1}{(z-x)(z-y)} + \frac{1}{(y-x)(y-z)}$;
  (3) $\left(\frac{x-1}{3x+(x-1)^2} - \frac{1-3x+x^2}{x^3-1} - \frac{1}{1-x}\right) \div \frac{x^2+1}{1-x}$.

Problem 1.3. Prove, that if $abc = 1$ and $1 + a + ab \neq 0$, then

$$\frac{1}{1+a+ab} + \frac{1}{1+b+bc} + \frac{1}{1+c+ac} = 1.$$

Problem 1.4. Prove, that if $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$, $abc \neq 0$ and $xyz \neq 0$, then

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

2. Radical expressions

Problem 2.1. Calculate the value of the following expressions without using a calculator:
  (1) $\sqrt{7+2\sqrt{6}} - \sqrt{7-2\sqrt{6}}$,

  (2) $\sqrt[4]{28+16\sqrt{3}} - \sqrt[4]{28-16\sqrt{3}}$,

  (3) $\sqrt[2]{\frac{3^{\frac{2}{3}}}{\sqrt[3]{\frac{\sqrt{2}}{2^3 \sqrt[3]{3^2}}}}}$.

Problem 2.2. Simplify the following radical expressions:
  (1) $\frac{a-\sqrt{a}-2}{a-5\sqrt{a}+6}$,

  (2) $\frac{x\sqrt[3]{x}-1}{\sqrt[3]{x^2}-1} - \frac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}+1}$,

  (3) $\frac{a^{-\frac{2}{3}} \cdot a^{\frac{3}{4}} \cdot \sqrt[5]{a^2}}{\sqrt{a^{-3}} \cdot \sqrt[4]{a} \cdot a^{\frac{5}{6}}}$.

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3. Word problems

Problem 3.1. At 6 o'clock the hands of the clock are in line but point to opposite directions. When does this happen again after 6 o'clock (and before 6 o'clock)?

Problem 3.2. 37 people participated in a dance party. The first girl had danced with 8 boys, the second with 9 boys, and so on, while the last girl had danced with all boys. How many girls and boys were at the party?

Problem 3.3. A company wants to cut down pine trees in a forest. The local environmentalists started to protested, to which the director of the company said: "Currently 99% of the trees in the forest are pine trees. After the cutting 98% of the trees will be pine trees." What percentage of the forest does the company want to cut down?