```
   *----*    MuPAD 2.0.0 -- The Open Computer Algebra System
  /|   /|
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 |/   |/
 *----*                     UNREGISTERED VERSION
               Please contact info@sciface.com to register.

>> # Elemi függvények #
>> exp(ln(2));

                                     2
>> exp(2*ln(x));

                                     2
                                    x
>> sin(arcsin(2));

                                     2
>> sin(-x);

                                  -sin(x)
>> sin(PI/2);

                                     1
>> cos(x+PI);

                                  -cos(x)
>> cos(x+2*PI);

                                  cos(x)
>> sin(arctan(2));

                                     1/2
                                  2 5
                                  ------
                                    5
>> DIGITS:=100: sin(0.1);

0.09983341664682815230681419841062202698991538801798225999276686156165174\
4283292427609662443804063036727
>> DIGITS:=50: sin(1);

                                  sin(1)
>> DIGITS:=50: sin(1.0); DIGITS:=10:

           0.84147098480789650665250232163029899962256306079837
>> arccos(cos(3));

                                     3
>> log(10,100);

                                     2
>> sign(5);

                                     1
>> assume(x<0): sign(x*y)*sign(x);

                                  sign(y)
>> trunc(5.5);
```



```
                                     5
>> trunc(-5.5);

                                    -5
>> round(5.5);

                                     6
>> round(-5.5);

                                    -6
>> floor(5.5);

                                     5
>> floor(-5.5);

                                    -6
>> frac(1.234);

                                   0.234
>> frac(-1.234);

                                   0.766
>> sqrt(x);

                                    1/2
                                   x
>> binomial(20,10);

                                  184756
>> expand((a+b)^20);

 20    20         19        19           2  18          3  17          4  16
a   + b   + 20 a b   + 20 a   b + 190 a  b   + 1140 a  b   + 4845 a  b   +

          5  15           6  14           7  13            8  12
   15504 a  b   + 38760 a  b   + 77520 a  b   + 125970 a  b   +

           9  11            10  10            11  9             12  8
   167960 a  b   + 184756 a   b   + 167960 a   b  + 125970 a   b  +

          13  7           14  6           15  5          16  4
   77520 a   b  + 38760 a   b  + 15504 a   b  + 4845 a   b  +

         17  3         18  2
   1140 a   b  + 190 a   b
>> max(1,2,3);

                                     3
>> min(1,2,3);

                                     1
>>
>> # Határérték #
>> limit(sin(x)/x,x=0);

                                     1
>> limit((abs(x)-abs(0))/(x-0),x=0);

                                    -1
>> limit((abs(x)-abs(0))/(x-0),x=0,Left);

                                    -1
```

```
>>  ^[[4m^[[0m ^[[4m^[[0m ^[[4m^[[0m

^[[4m^[[1mlimit^[[0m -- compute a limit^[[0m

^[[4mIntroduction^[[0m

 ^[[1mlimit^[[0m^[[1m(f, x = x0)^[[0m computes the bidirectional limit lim(f(x),
 x = x0).

 ^[[1mlimit^[[0m^[[1m(f, x = x0, Left)^[[0m computes the one-sided limit lim(f(x
), x = x0-).

 ^[[1mlimit^[[0m^[[1m(f, x = x0, Right)^[[0m computes the one-sided limit lim(f(
x), x = x0+).

^[[4mCall(s)^[[0m

 ^[[1mlimit(f, x <= x0> <, dir>)^[[0m

^[[4mParameters^[[0m

^[[0m ^[[1mf^[[0m^[[0m   -^[[0m  an ^[[4marithmetical expression^[[0m representi
ng a function in ^[[1mx^[[0m
^[[0m ^[[1mx^[[0m^[[0m   -^[[0m  an ^[[4midentifier^[[0m

^[[0m ^[[1mx0^[[0m^[[0m  -^[[0m  the limit point: an ^[[4marithmetical expressio
n^[[0m, possibly ^[[1m^[[4minfinity^[[0m^[[0m
        or ^[[1m-infinity^[[0m


^[[4mOptions^[[0m

^[[0m ^[[1m^[[1mdir^[[0m^[[0m^[[0m  -^[[0m  either ^[[1mLeft^[[0m or ^[[1mRight
^[[0m. This controls the direction of the limit
        computation.

^[[4mReturns^[[0m

 an ^[[4marithmetical expression^[[0m, an interval of type ^[[1m^[[4mDom::Interv
al^[[0m^[[0m, an
 ^[[4mexpression^[[0m of type ^[[1m"limit"^[[0m, or ^[[1m^[[4mFAIL^[[0m^[[0m.

^[[4mSide Effects^[[0m

 The function is sensitive to the environment variable ^[[1m^[[4mORDER^[[0m^[[0m
, which
 determines the default number of terms in series computations (see ^[[1m^[[4mse
ries^[[0m^[[0m
 and example ^[[4m6^[[0m below).

 ^[[4mProperties^[[0m of identifiers set by ^[[1m^[[4massume^[[0m^[[0m are taken
 into account.

^[[4mOverloadable:^[[0m

 ^[[1mf^[[0m

^[[4mRelated Functions^[[0m

 ^[[1m^[[4masympt^[[0m^[[0m, ^[[1m^[[4mdiff^[[0m^[[0m, ^[[1m^[[4mdiscont^[[0m^[[
0m, ^[[1m^[[4mint^[[0m^[[0m, ^[[1m^[[4mO^[[0m^[[0m, ^[[1m^[[4mseries^[[0m^[[0m,
^[[1m^[[4mtaylor^[[0m^[[0m
```



```

^[[4mDetails^[[0m

 o ^[[1mlimit^[[0m^[[1m(f, x = x0)^[[0m computes the bidirectional limit of ^[[1
mf^[[0m when ^[[1mx^[[0m tends to
   ^[[1mx0^[[0m on the real axis. The limit point ^[[1mx0^[[0m may be omitted, i
n which case
   ^[[1mlimit^[[0m assumes ^[[1mx0 = 0^[[0m.

   If the limit point ^[[1mx0^[[0m is infinity or -infinity, then the limit is t
aken
   from the left to infinity or from the right to -infinity, respectively.

   If the left and right limits are different, then ^[[1m^[[4mundefined^[[0m^[[0
m is returned;
   see example ^[[4m2^[[0m.

 o ^[[1mlimit^[[0m^[[1m(f, x = x0, Left)^[[0m returns the limit when ^[[1mx^[[0m
 tends to ^[[1mx0^[[0m from the
   left. ^[[1mlimit^[[0m^[[1m(f, x = x0, Right)^[[0m returns the limit when ^[[1
mx^[[0m tends to ^[[1mx0^[[0m from
   the right. See example ^[[4m2^[[0m.

 o If the limit does not exist mathematically, but the system can assert
   that the function ^[[1mf^[[0m is bounded when ^[[1mx^[[0m approaches ^[[1mx0
^[[0m, then a bounding
   interval, of type ^[[1m^[[4mDom::Interval^[[0m^[[0m, for ^[[1mf(x)^[[0m in a
sufficiently small
   neighborhood of ^[[1mx0^[[0m is returned. This may happen, e.g., if ^[[1mf^[[
0m oscillates
   infinitesimally fast in the neighborhood of ^[[1mx0^[[0m; see example ^[[4m4
^[[0m. The
   boundaries are the limes inferior and the limes superior of ^[[1mf^[[0m for x
 ->
   x0.

 o If the limit cannot be computed, then the system returns a symbolic
   ^[[1mlimit^[[0m call (see example ^[[4m3^[[0m).

 o If ^[[1mf^[[0m contains parameters, then ^[[1mlimit^[[0m reacts to ^[[4mprope
rties^[[0m of those
   parameters set by ^[[1m^[[4massume^[[0m^[[0m; see example ^[[4m5^[[0m. If the
 limit cannot be
   computed without additional assumptions about the parameters, then
   ^[[1mlimit^[[0m indicates this by a warning.

 o Internally, ^[[1mlimit^[[0m tries to determine the limit from a series expans
ion
   of ^[[1mf^[[0m around ^[[1mx = x0^[[0m computed via ^[[1m^[[4mseries^[[0m^[[0
m. If the number of terms in the
   series expansion is too small to compute the limit, then ^[[1mlimit^[[0m retu
rns
   ^[[1m^[[4mFAIL^[[0m^[[0m. In such a case, it may be necessary to increase the
 value of the
   environment variable ^[[1m^[[4mORDER^[[0m^[[0m in order to find the limit (se
e example ^[[4m6^[[0m).

^[[4mExample 1^[[0m

 The following command computes lim( (1-cos(x)/x^2),x=0 ):

 >> limit((1 - cos(x))/x^2, x)
```

```
                                        1/2


 A possible definition of e is given by the limit of the sequence
 (1+1/n)^n for n->infinity:

 >> limit((1 + 1/n)^n, n = infinity)


                                      exp(1)


 Here is a more complex example:

 >> limit(
      (exp(x*exp(-x)/(exp(-x) + exp(-2*x^2/(x+1)))) - exp(x))/x,
      x = infinity
    )


                                      -exp(2)


^[[4mExample 2^[[0m

 The bidirectional limit of f(x)=1/x for x -> 0 does not exist:

 >> limit(1/x, x = 0)


                                     undefined


 You can compute the one-sided limits from the left and from the right by
 passing the options ^[[1mLeft^[[0m and ^[[1mRight^[[0m, respectively:

 >> limit(1/x, x = 0, Left),
    limit(1/x, x = 0, Right)


                                -infinity, infinity


^[[4mExample 3^[[0m

 If ^[[1mlimit^[[0m is not able to compute the limit, then a symbolic ^[[1mlimit
^[[0m call is
 returned:

 >> delete f: limit(f(x), x = infinity)


                             limit(f(x), x = infinity)


^[[4mExample 4^[[0m

 The function sin(x) oscillates for x -> infinity. The limes inferior and
 the limes superior are -1 and 1, respectively:
```



```
 >> limit(sin(x), x = infinity)


                                      [-1, 1]


 In fact, for x -> infinity the function ^[[1mf=sin(x)^[[0m assumes every value
in
 the returned interval infinitely often. This need not be the case in
 general.

^[[4mExample 5^[[0m

 ^[[1mlimit^[[0m is not able to compute the limit of x^n for x -> infinity witho
ut
 additional information about the parameter n:

 >> delete n: limit(x^n, x = infinity)


       Warning: cannot determine sign of n [stdlib::limit::limitMRV]

                                         n
                                  limit(x , x = infinity)


 However, for n > 0 the limit exists and equals infinity. We use ^[[1m^[[4massum
e^[[0m^[[0m to
 achieve this:

 >> assume(n > 0): limit(x^n, x = infinity)


                                      infinity


 Similarly, the limit is zero for n < 0:

 >> assume(n < 0): limit(x^n, x = infinity)


                                          0


^[[4mExample 6^[[0m

 It may be necessary to increase the value of the environment variable
 ^[[1m^[[4mORDER^[[0m^[[0m in order to find the limit, as in the following examp
le:

 >> limit((sin(tan(x)) - tan(sin(x)))/x^7, x = 0)


       Warning: ORDER seems to be not big enough for series \
       computation [stdlib::limit::lterm]

                                        FAIL


 >> ORDER := 8: limit((sin(tan(x)) - tan(sin(x)))/x^7, x)
```

```
                                          -1/30


^[[4mBackground^[[0m

 o If a limit cannot be computed, then ^[[1mlimit^[[0m issues a warning with a
   possible reason, as shown in examples ^[[4m5^[[0m and ^[[4m6^[[0m. You may wa
nt to suppress
   these warnings when you call ^[[1mlimit^[[0m from within your own procedures.
 You
   can control this by means of the procedure
   ^[[1mstdlib::limit::printWarnings^[[0m.

   The calls ^[[1mstdlib::limit::printWarnings(TRUE)^[[0m and
   ^[[1mstdlib::limit::printWarnings(FALSE)^[[0m switch the warnings that ^[[1ml
imit^[[0m
   issues on and off, respectively, and return the previous setting. The
   command ^[[1mstdlib::limit::printWarnings()^[[0m returns the current setting,

   which is ^[[1m^[[4mTRUE^[[0m^[[0m by default.

 o ^[[1mlimit^[[0m first tries a series computation to determine the limit. If t
his
   fails, then an algorithm based on the thesis of Dominik Gruntz: ``On
   Computing Limits in a Symbolic Manipulation System'', Swiss Federal
   Institute of Technology, Zurich, Switzerland, 1995, is used.

^[[4mChanges^[[0m

 o ^[[1mlimit^[[0m may return an interval of type ^[[1m^[[4mDom::Interval^[[0m^[
[0m.

--------------------------------------------------------------------------


?limit
>>
>> # Szummáció #
>> sum(1/n,n=1..infinity);

                                 infinity
>> sum(1/(n^2),n=1..infinity);

                                      2
                                    PI
                                    ---
                                     6
>> sum(1/(n^3),n=1..infinity);

                                  zeta(3)
>> sum(1/(n^4),n=1..infinity);

                                      4
                                    PI
                                    ---
                                    90
>> float(_plus(1/(n^3),n=1..1000));
Error:  [_plus]
>> DIGITS:=200: float(PI); DIGITS:=10:

3.141592653589793238462643383279502884197169399375105820974944592307816406\
28620899862803482534211706798214808651328230664709384460955058223172535940\
81284811174502841027019385211055596446229489549303819
```



```
>>
>>
>> # Differenciálás #
>> diff(E^x+c,x);

                                  exp(x)
>> diff(x^n,x);

                                    n - 1
                                 n x
>> diff(2^x,x);

                                    x
                                 2  ln(2)
>> diff(sin(2*x),x);

                                2 cos(2 x)
>> diff(%,x);

                                -4 sin(2 x)
>> diff(%,x);

                                -8 cos(2 x)
>> diff(%,x);

                                16 sin(2 x)
>> diff(%,x);

                                32 cos(2 x)
>> diff(x^100,x $ 30);

                                                                  70
      7791097137057804874587232499277321440358327700684800000000 x
>>
>> # Integrálás #
>> int(E^x,x);

                                  exp(x)
>> int(x^3-3*x^2+2,x);

                                          4
                                     3   x
                               2 x - x  + --
                                          4
>> int(1/(ln(x)*x),x);

                                 ln(ln(x))
>> int(sin(x)/x,x=1..2);
Warning: While integrating, we will assume x has property [1, 2] of Type::\
Real instead of given property < 0. [intlib::defInt]

                               Si(2) - Si(1)
>> float(%);

                               0.6593299064
>> T:=series(sin(x)/x,x=0,10);

                         2    4      6       8
                        x    x      x       x         9
                    1 - -- + --- - ---- + ------ + O(x )
                        6    120   5040   362880
>> int(T,x);
```

```
                      3    5      7        9
                     x    x      x        x          10
                 x - -- + --- - ----- + ------- + O(x  )
                     18   600   35280   3265920
>> int(T,x=1..2);
Warning: While integrating, we will assume x has property [1, 2] of Type::\
Real instead of given property < 0. [intlib::defInt]
Warning: While integrating, we will assume x has property [1, 2] of Type::\
Real instead of given property < 0. [intlib::defInt]

                        9
                int(O(x ), x = 1..2) + 75366677/114307200
>> float(%);

                           9
                   int(O(x ), x = 1..2) + 0.6593344689
>>
>> # Sorbafejtés #
>> taylor(cos(x)/x,x=0,10);
Error: does not have a Taylor series expansion, try 'series' [taylor]
>> taylor(exp(x),x=0,10);

              2    3    4    5     6      7      8        9
             x    x    x    x     x      x      x        x         10
     1 + x + -- + -- + -- + --- + --- + ---- + ----- + ------ + O(x  )
             2    6    24   120   720   5040   40320   362880
>>
>> # Gyökkeresés #
>> solve(x^4-x^2=1);

                    { --        1/2   1/2     1/2 -- }
                    { |        2    (5    + 1)     | }
                    { |  x = - ------------------  | }
                    { --               2          -- }
>> f:=x^5-x^4-3*x^3+2*x+5;

                                 3    4    5
                         2 x - 3 x  - x  + x  + 5
>> solve(f);

                                                       3      4      5
x in ]-infinity, 0[ intersect RootOf(2 X49 - 3 X49  - X49  + X49  + 5, X49)
>> numeric::solve(f);

{[x = -1.472991017], [x = 1.568770674], [x = 1.906476185],

   [x = - 0.5011279209 - 0.9401206818 I],

   [x = - 0.5011279209 + 0.9401206818 I]}
>> quit
```

```
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For details, type `warranty'.

Eltérés a(z) 1. lépésben: -0.858407346410206884
Eltérés a(z) 10. lépésben: 0.099753034660389872
Eltérés a(z) 100. lépésben: 0.009999750031239429
Eltérés a(z) 1000. lépésben: 0.000999999749998981
Eltérés a(z) 10000. lépésben: 0.000099999999758627
Eltérés a(z) 100000. lépésben: 0.000010000000073340
```

```
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i=1 n=6 m=0.866025403784438597
i=2 n=12 m=0.965925826289068312
i=3 n=24 m=0.991444861373810382
i=4 n=48 m=0.997858923238603479
i=5 n=96 m=0.999464587476365685
i=6 n=192 m=0.999866137909561803
i=7 n=384 m=0.999966533917401090
i=8 n=768 m=0.999991633444350603
i=9 n=1536 m=0.999997908358900123
i=10 n=3072 m=0.999999477089588362
i=11 n=6144 m=0.999999869272388597
i=12 n=12288 m=0.999999967318096594
i=13 n=24576 m=0.999999991829524149
i=14 n=49152 m=0.999999997957381037
i=15 n=98304 m=0.999999999489345259
i=16 n=196608 m=0.999999999872336343
i=17 n=393216 m=0.999999999968084086
i=18 n=786432 m=0.999999999992021049
i=19 n=1572864 m=0.999999999998005262
i=20 n=3145728 m=0.999999999999501288
i=21 n=6291456 m=0.999999999999875322
i=22 n=12582912 m=0.999999999999968914
i=23 n=25165824 m=0.999999999999992228
i=24 n=50331648 m=0.999999999999998002
i=25 n=100663296 m=0.999999999999999556
i=26 n=201326592 m=0.999999999999999889
i=27 n=402653184 m=1.000000000000000000
i=28 n=805306368 m=1.000000000000000000
i=29 n=1610612736 m=1.000000000000000000
```

```
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i=1 n=6 PI-T=0.543516442236477104
i=2 n=12 PI-T=0.141592653589793560
i=3 n=24 PI-T=0.035764112359544242
i=4 n=48 PI-T=0.008964040308555354
i=5 n=96 PI-T=0.002242450542925489
i=6 n=192 PI-T=0.000560702699283322
i=7 n=384 PI-T=0.000140181304330689
i=8 n=768 PI-T=0.000035045677935219
i=9 n=1536 PI-T=0.000008761441474547
i=10 n=3072 PI-T=0.000002190361742205
i=11 n=6144 PI-T=0.000000547590521371
i=12 n=12288 PI-T=0.000000136897635450
i=13 n=24576 PI-T=0.000000034224409085
i=14 n=49152 PI-T=0.000000008556102049
i=15 n=98304 PI-T=0.000000002139024957
i=16 n=196608 PI-T=0.000000000534755795
i=17 n=393216 PI-T=0.000000000133688616
i=18 n=786432 PI-T=0.000000000033421266
i=19 n=1572864 PI-T=0.000000000008355094
i=20 n=3145728 PI-T=0.000000000002088552
i=21 n=6291456 PI-T=0.000000000000521805
i=22 n=12582912 PI-T=0.000000000000130118
i=23 n=25165824 PI-T=0.000000000000031974
i=24 n=50331648 PI-T=0.000000000000007550
i=25 n=100663296 PI-T=0.000000000000001776
i=26 n=201326592 PI-T=0.000000000000000888
i=27 n=402653184 PI-T=0.000000000000000000
i=28 n=805306368 PI-T=0.000000000000000000
i=29 n=1610612736 PI-T=0.000000000000000000
```

```
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Eltérés a(z) 2. lépésben: 0.474925986923126153
Eltérés a(z) 3. lépésben: -0.325074013076873669
Eltérés a(z) 4. lépésben: 0.246354558351697506
Eltérés a(z) 5. lépésben: -0.198089886092747136
Eltérés a(z) 6. lépésben: 0.165546477543616621
Eltérés a(z) 7. lépésben: -0.142145830148691310
Eltérés a(z) 8. lépésben: 0.124520836517975297
Eltérés a(z) 9. lépésben: -0.110773281129083578
Eltérés a(z) 10. lépésben: 0.099753034660389872
Eltérés a(z) 11. lépésben: -0.090723155815800816
Eltérés a(z) 12. lépésben: 0.083189887662459938
Eltérés a(z) 13. lépésben: -0.076810112337540204
Eltérés a(z) 14. lépésben: 0.071338035810607714
Eltérés a(z) 15. lépésben: -0.066592998672150738
Eltérés a(z) 16. lépésben: 0.062439259392365276
Eltérés a(z) 17. lépésben: -0.058772861819755828
Eltérés a(z) 18. lépésben: 0.055512852465958495
Eltérés a(z) 19. lépésben: -0.052595255642149397
Eltérés a(z) 20. lépésben: 0.049968846921953247
Eltérés a(z) 21. lépésben: -0.047592128687802937
Eltérés a(z) 22. lépésben: 0.045431127126150717
Eltérés a(z) 23. lépésben: -0.043457761762738301
Eltérés a(z) 24. lépésben: 0.041648621215985226
Eltérés a(z) 25. lépésben: -0.039984031845239354
Eltérés a(z) 26. lépésben: 0.038447340703780419
Eltérés a(z) 27. lépésben: -0.037024357409427111
Eltérés a(z) 28. lépésben: 0.035702915317845640
Eltérés a(z) 29. lépésben: -0.034472523278645362
Eltérés a(z) 30. lépésben: 0.033324086890845983
Eltérés a(z) 31. lépésben: -0.032249683600957368
Eltérés a(z) 32. lépésben: 0.031242379891105898
Eltérés a(z) 33. lépésben: -0.030296081647355422
Eltérés a(z) 34. lépésben: 0.029405410889958183
Eltérés a(z) 35. lépésben: -0.028565603602795253
Eltérés a(z) 36. lépésben: 0.027772424566218756
Eltérés a(z) 37. lépésben: -0.027022095981726224
Eltérés a(z) 38. lépésben: 0.026311237351607009
Eltérés a(z) 39. lépésben: -0.025636814596444957
Eltérés a(z) 40. lépésben: 0.024996096795959932
Eltérés a(z) 41. lépésben: -0.024386619253422559
Eltérés a(z) 42. lépésben: 0.023806151830914946
Eltérés a(z) 43. lépésben: -0.023252671698496652
Eltérés a(z) 44. lépésben: 0.022724339795756165
Eltérés a(z) 45. lépésben: -0.022219480428963045
Eltérés a(z) 46. lépésben: 0.021736563527080754
Eltérés a(z) 47. lépésben: -0.021274189161091250
Eltérés a(z) 48. lépésben: 0.020831073996803617
Eltérés a(z) 49. lépésben: -0.020406039405258092
Eltérés a(z) 50. lépésben: 0.019998000998782128
```

```
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>> // Borwein kvadratikus algoritmusa
>>
>> N:=6:
>> DIGITS:=500:
>>
>> a:=sqrt(2): b:=0: p:=2+sqrt(2):
>> for i from 1 to N do
&>  uj_a:=(sqrt(a)+1/sqrt(a))/2:
&>  uj_b:=(sqrt(a)*(1+b))/(a+b):
&>  uj_p:=p*uj_b*(1+uj_a)/(1+uj_b):
&>  a:=uj_a:
&>  b:=uj_b:
&>  p:=uj_p:
&>
&>  elteres:=float(p-PI):
&>  tizedesjegy_pontossag:=trunc(abs(trunc(log(10,elteres)))):
&>  print(Unquoted, "Eltérés: ".
&>   expr2text(tizedesjegy_pontossag).
&>   " tizedesjegy."):
&>  end_for;

                          Eltérés: 2 tizedesjegy.

                          Eltérés: 8 tizedesjegy.

                         Eltérés: 18 tizedesjegy.

                         Eltérés: 40 tizedesjegy.

                         Eltérés: 83 tizedesjegy.

                         Eltérés: 170 tizedesjegy.

>> quit

real    0m22.197s
user    0m21.770s
sys     0m0.370s

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>> // Borwein kvartikus algoritmusa
>>
>> N:=5:
>> DIGITS:=1000:
>>
>> a:=6-4*sqrt(2): y:=sqrt(2)-1:
```



```
>> for i from 0 to N-1 do
&>  uj_y:=(1-(1-y^4)^(1/4))/(1+(1-y^4)^(1/4)):
&>  uj_a:=a*((1+uj_y)^4)-2^(2*i+3)*uj_y*(1+uj_y+uj_y^2):
&>  y:=uj_y:
&>  a:=uj_a:
&>  p:=1/a:
&>
&>  elteres:=float(p-PI):
&>  tizedesjegy_pontossag:=trunc(abs(trunc(log(10,elteres)))):
&>  print(Unquoted, "Eltérés: ".
&>   expr2text(tizedesjegy_pontossag).
&>   " tizedesjegy."):
&>  end_for;

                          Eltérés: 8 tizedesjegy.

                         Eltérés: 40 tizedesjegy.

                         Eltérés: 170 tizedesjegy.

                         Eltérés: 693 tizedesjegy.

                         Eltérés: 997 tizedesjegy.

>>
>> quit

real    0m34.248s
user    0m33.830s
sys     0m0.310s

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>> // Ooura kvadratikus algoritmusa a PI kiszámítására.
>> // A módszer a Gauss-Legendre-féle számtani-mértani közép (AGM) gyorsított
>> // változata; az eredeti programban a szorzásokat gyors
>> // Fourier-transzformációval (FFT) végzik (ami további gyorsítás).
>> // Copyright (C) 1999 Takuya Ooura <ooura@mmm.t.u-tokyo.ac.jp>
>> // http://momonga.t.u-tokyo.ac.jp/~ooura/fft.html
>>
>> DIGITS:=1000:                                // Itt kell megadni a pontosságot.
>> ELTERES:=10^(-DIGITS):                       //
>> SQRT_SQRT_ELTERES:=sqrt(sqrt(ELTERES)):      //
>> n:=1:                                        //
>>
>>     c := sqrt(0.125):
>>     a := 1 + 3 * c:
>>     b := sqrt(a):
>>     e := b - 0.625:
>>     b := 2 * b:
>>     c := e - c:
>>     a := a + e:
>>     npow := 4:
>>     while e > SQRT_SQRT_ELTERES do
&>         npow := 2 * npow:
&>         e := (a + b) / 2:
```

```
&>         b := sqrt(a * b):
&>         e := e - b:
&>         b := 2 * b:
&>         c := c - e:
&>         a := e + b:
&>      n := n + 1:                        //
&>      end_while:
>>     e := e * e / 4:
>>     a := a + b:
>>     p := (a * a - e - e / 2) / (a * c - e) / npow:
>>
>> float(PI);                                 //

3.141592653589793238462643383279502884197169399375105820974944592307816406\
2862089986280348253421170679821480865132823066470938446095505822317253594 0\
8128481117450284102701938521105559644622948954930381964428810975665933446 1\
2847564823378678316527120190914564856692346034861045432664821339360726024 9\
1412737245870066063155881748815209209628292540917153643678925903600113305 3\
0548820466521384146951941511609433057270365759591953092186117381932611793 1\
0511854807446237996274956735188575272489122793818301194912983367336244065 6\
6430860213949463952247371907021798609437027705392171762931767523846748184 6\
7669405132000568127145263560827785771342757789609173637178721468440901224 9\
5343014654958537105079227968925892354201995611212902196086403441815981362 9\
7747713099605187072113499999983729780499510597317328160963185950244594553 4\
6908302642522308253344685035261931188171010003137838752886587533208381420 6\
1717766914730359825349042875546873115956286388235378759375195778185778053 2\
171226806613001927876611195909216420199
>> print(Unquoted, "Eltérés: ".             //
&>  expr2text(float(p-PI)));                  //

Eltérés: 0.00000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000000000000000000000000000000\
0000000000000000000000000000000000000000000000000181356974046297225616400123\
6664560057794308624137627294438081615896203218000675796778246929718810423 7\
8613862530754105268853262416104155889038284135019746734242226210630025195 31\
3475192055521113546147471776075388877051514128005621203468813443376201414 3\
7120591940304021790643424126609834708802559122731591357256856051812589739 3\
1295858130142277505064933510485457499649635475678132465392307048950355632 0\
7957673142754973767394067650499191421865033896489755428820017339737273593 0\
1197049483232368073311609973222174970887631301867047291754941691385441739 3\
2695864237129424910593466272534894700870343813639188664373543187766617928 8\
0746190627825535829249555792530416524985786254831303114123009506918368875 1\
9511782216121274952391967691567859696438637283928345899365722274547015215 0\
5856881947179170443568971884691722053347739173122668170988847976868388587 2\
8702883456303576384720292397372074735826279058097852733360031884196716833 5\
3740270507230712272767577543195639203293904963924821782417743793433694830 6\
52634842994

>> print(Unquoted, "Lépésszám: ".           //
&>  expr2text(n));                            //
```



```
                              Lépésszám: 7

>> quit

real    0m1.227s
user    0m1.020s
sys     0m0.210s
```

```
i=1
n=6
m=.866025403784438646763723170752936183471402626905190314027903489 72\
596650845440001854057309337862428 7
x=1
PI-T=.54351644223647729817147387102069433378296151865953487889123412\
3129916880923008943006315545206244203

i=2
n=12
m=.96592582628906828674974319972889736763390483900840455040234307631\
0423213979855516347561741858070452
x=.51763809020504152469779767524809665669813780263986102762800641463\
0113949497603993844735949388499330
PI-T=.14159265358979323846264338327950288419716939937510582097494459\
2307816406286208998628034825342117070

i=3
n=24
m=.99144486137381041114455752692856287127773827444810227145877460352\
8922068405082531763265433453277397
x=.26105238444010318309681245579097802038748140962346450378123389667\
0265650459647259228480737984584243
PI-T=.03576411235954409027585733179092294400834258353593965520690610\
4527132709300585035559619129011121120

i=4
n=48
m=.99785892323860350673806979127277760453186639632596641807472562927\
4187850828698111037956547626966877
x=.13080625846028613363063111755035088288126460789010803566397530149\
2693814892777930572416034523039976
PI-T=.00896404030855504130089391378776663954739248389353177560013783\
2264628600770441887886265969527106256

i=5
n=96
m=.99946458747636564442983644624285994588364095315313089603368742859\
9674458291480489070820060615630369
x=.06543816564355228412731985263457625222992525450339239375619176751\
4637470426534785459604967089728326
PI-T=.00224245054292603132749656207108169504681881001251296503953735\
6483164848859538664890049996789158120

i=6
n=192
m=.99986613790956178286274714906010002195345847142775142566570436111\
7678718332025681806890704453565826
x=.03272346325297356328594384696834610047132981567239244974814148723\
7746659648045140570847433469849730
PI-T=.00056070269928360035129045681984277716075718321227092067773975\
1605217825812539296566996405035159048

i=7
n=384
m=.99996653391740110345760381057913911237003239152400201555378732091\
3251507865987093466996694261615110
x=.01636227920787425857039824658921801844533529416452341952057948786\
9395791650646528660662556483480802
PI-T=.00014018130433116301203407431827723894950709482543064515336181\
7484137080073875503826681212236549320
```



```
i=8
n=768
m=.99999163344435064914755862056883470820631843513005534979804061014\
6982405957798028383213196212535752
x=.00818120805246957918924210834302383685031261696817992854723965236\
1223834891446872068405136594911429
PI-T=.00003504567793559294618003814964334269279291978660927302368292\
1383824409362075495780823980513831880

i=9
n=1536
m=.99999790835890018104166372432685047011985442966963796400028652886\
5271534387576238320830495121637079
x=.00409061258232819022882611784626426005671130508942161074227241896\
9167503098218319552244744665660570
PI-T=.00000876144147482979367377955834953367712448359401325883491808\
5597863807970610124360462372896212424

i=10
n=3072
m=.99999947708958832761109823746401407779555602264324535182230850229\
5128091128280785921170509203640298
x=.00204530736067660908238592229206387848510904769203534158371388824\
5273331322007767142830480620072408
PI-T=.00000219036174314272418487734855116064288709069930877090972682\
3987174026854539582504070922115054280

i=11
n=6144
m=.99999986927238853704857515517161734697783685384035148725450231910\
2694748968570675021677998522479114
x=.00102265381402739500247163869946902080272386682694276940575245689\
8332487164632400376503588263738954
PI-T=.00000054759052168791786674266938553106967214440882114839041224\
7567979495682278667240416592913844424

i=12
n=12288
m=.99999996731809660020873887214910799212477104987677524490954984855\
3649378822556495841909467488782913
x=.00051132692372483462812329904167729802119501834726478547125933748\
2095516552932735964095631620218256
PI-T=.00000013689763579086976929851067097822945050700691820650339700\
0630415836535475042009011679142354120

i=13
n=24576
m=.99999999182952411667384663803013397534384502751287301637915404800\
1048780389112773181329855767815864
x=.00025566346395130948052344901602532864686870304453467621730677964\
0752647275554860534356069073112171
PI-T=.00000003422440928327309407121418384197497667378026388555757510\
2312962705067479235224474150763103432

i=14
n=49152
m=.99999999795738102708231552524588538297227930747251410367402745375\
3971026542949376732184460524640016
x=.00012783173223676626186947646649138260988152781056255781673371926\
3849165602625460130474715776617413
PI-T=.00000000855610234179050187436026447147454638813300446270923636\
6739286684268082752460658054996493512
```

```
i=15
n=98304
m=.99999999948934525664019474785353706261343768748187857046032191171\
0880362435970386388089786016708618
x=.00006391586615102207116070807251821846928420566418226833293283142\
8121132542711501922590836328451701
PI-T=.00000000213902558675838974278728386374874192698968491692705996\
3950722556162900832081419899607491784

i=16
n=196608
m=.99999999987233631415189967862122304051682339686835924045669061473\
4928158954721742046647376496906109
x=.00003195793307959090310938154255793333263503358446685194730153233\
1929528365772989529067714664526266
PI-T=.00000000053475639677152020286402868193989259348825272066041423\
7297909666930466499443247610061546696

i=17
n=393216
m=.99999999996808407853746560663390442836378158635102740491164743804\
9351972880568262872244496202009079
x=.00001597896654030543499584362445350196006406104280692607674654698\
0670958234914661662839321667722200
PI-T=.00000000013368909919800022366442455284282791194569199344511023\
4307459937338246333155412453657175240

i=18
n=786432
m=.99999999999202101963433456959463826986819401995350055251289058485\
2827106374053887731010152637630099
x=.00000798948327021646542806668183879235631019681169477246348191864\
5364721086824765902504069028704743
PI-T=.00000000003342227479982006672538951992225589519098372398983581\
6552059756184408791114680396210857160

i=19
n=1572864
m=.99999999999800525490858165289466969867207322580271743629306828098\
2591442829725410670919397800679915
x=.00000399474163511620120530147346356461485004455676682492653080263\
0979610315687865210185461784346668
PI-T=.00000000000835556869997501735692770532681989200345682046882253\
6573649529399059879588027721842648264

i=20
n=3145728
m=.99999999999950131372714528887966805780632370590547466559830610837\
5972574180527769713468547664336222
x=.00000199737081755909666405925404488658903363571572547377195911130\
7678422159435141726244357721880529
PI-T=.00000000000208889217499500438145569844692853212744519950276990\
5750916619246996022054953308273468616

i=21
n=6291456
m=.99999999999987532843178631444841705402175612885485402178834123760\
0675803340067844102572845624808516
x=.00000099868540877967283970569170986518056233278254800139575240199\
4414131573869502687735474268398585
PI-T=.00000000000052222304374882922300291039676302054352696227929675\
```



```
1994025028491452891022572848536315080

i=22
n=12582912
m=.99999999999996883210794657812638551597855984353287759923365203632\
7008895209545202976003140869391035
x=.00000049934270438985198331235394158794637574452005163238561622949\
1605364387787838330137800767018661
PI-T=.00000000000013055576093721218872816421119002021577116356292350\
7930528800923258113890041779079570632

i=23
n=25165824
m=.99999999999999220802698664450123895727420977504645635502646961534\
4503242757906208722145871905695479
x=.00000024967135219492793708861548164863083528480050429414620649761\
9640108500310109371363024819123382
PI-T=.00000000000003263894023430335236813084105425341222354143421225\
4788406672164019823180586453360927944

i=24
n=50331648
m=.99999999999999805200674666112341240046102555345081310469767343260\
4807475446605148537502236727643590
x=.00000012483567609746421172336255468185819030616389052416717182657\
6744164674337444248823774302175004
PI-T=.00000000000000815973505857585716616332202971628624198948156845\
1350886552207278762366786153711887560

i=25
n=100663296
m=.99999999999999951300168666528073451643666095766052816320573316371\
4801924619047356141900529540145714
x=.00000006241783804873213625906312907314414694658977754689172890296\
6849279417149026395476057937391809
PI-T=.00000000000000203993376464396548367399374281528864702820955572\
6166595012847633287317297547002733768

i=26
n=201326592
m=.99999999999999987825042166632017621762925302499534375168674982233\
8077164621358673883237636084383084
x=.00000003120891902436607192920429600309964886031266085579006387133\
9062187334749823040426615335770047
PI-T=.00000000000000050998344116099144542682113791548664782771939097\
8707138816326129188888616809553421512

i=27
n=402653184
m=.99999999999999996956260541658004359118981874284758507071836129123\
5103074359212235999796347979584938
x=.00000001560445951218303643956123943486579303830946734590076137071\
1513827585467290174658678939280714
PI-T=.00000000000000012749586029024786601347545336710109730763543465\
8981731033763542914286902889843483848

i=28
n=805306368
m=.99999999999999999239065135414501086884636127862431786818041902429\
3769385489015158272337012399131811
x=.00000000780222975609151827915050614659739327282312314419213435000\
5403842099828033627479586676853464
```

```
PI-T=.000000000000000031873965072561966794416998897289620364004346 89\
823915868802786988206032803808879533 6

i=29
n=1610612736
m=.99999999999999999809766283853625271540214698171310581363423524658\
43299347333363335015654560416855 23
x=.00000000390111487804575914699648887694425875179664923589163142535\
33276664365222764364037301885930 80
PI-T=.00000000000000000796849126814049171679475819654205181718949831\
8080487402693645273281446368993305800

i=30
n=3221225472
m=.99999999999999999952441570963406317873744653680684060001659554068\
8451637387488241517319412683695315
x=.00000000195055743902287957442589891392782464098322741904183452748\
5589166042258096929738223197487770
PI-T=.00000000000000001992122817035122930335596328649240876228993 17\
2535905149298116728405610012494226632

i=31
n=6442450944
m=.99999999999999999881103927408515794677293496162870409166310806 88\
3908460819275420723345521896385031
x=.00000000097527871951143978732890626639587422864790677350546878071\
1133423077439001986863590209121103
PI-T=.00000000000000000498030704258780732654955755881918214202112 62\
5016536801172050122171679180248278216

i=32
n=12884901888
m=.99999999999999999970275981852128948668881615413290118489199690 98\
6505704036935276484570048218240701
x=.00000000048763935975571989367894773437693235284413627643353924138\
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x=.00000000000000000000000000169183889035709430244326402539589738686\
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PI-T=.000000000000000000000000000000000000000000000370412477346007 20\
1642915992128322439242017934870013128

i=91
n=7427640235712281649394745344
m=.99999999999999999999999999999999999999999999999999999991055253653\
3602370104081608304909530913172711
x=.00000000000000000000000000084591944517854715122163201269794869343\
0417852278540104128717511673642661
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i=92
n=14855280471424563298789490688
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x=.00000000000000000000000000021147986129463678780540800317448717335\
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PI-T=.0000000000000000000000000000000000000000000409804685353226752 4\
0984450808308818992095493647544313032

i=94
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i=95
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m=.99999999999999999999999999999999999999999999999999999999965059584\
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PI-T=.00000000000000000000000000000000000000000267514626696231955193\
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i=97
n=475368975085586025561263702016
m=.99999999999999999999999999999999999999999999999999999999997816224\
```

```
03646490161386917989025658034455 98
x=.000000000000000000000000000001321749133091479923783800019840544833\
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PI-T=.000000000000000000000000000000000000000000492296774828670476711\
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i=98
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i=99
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m=.999999999999999999999999999999999999999999999999999999999999863514\
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x=.000000000000000000000000000000330437283272869980945950004960136208\
3712479444470583006622545513183263
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7333871606047415330708592907328120008

i=100
n=3802951800684688204490109616128
m=.999999999999999999999999999999999999999999999999999999999999965878\
5005697640877167059357852590678837
x=.000000000000000000000000000000165218641636434990472975002480068104\
1855937092946581907088875158449443
PI-T=.000000000000000000000000000000000000000066128684029500724763206\
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i=101
n=7605903601369376408980219232256
m=.999999999999999999999999999999999999999999999999999999999999991469\
6251424410219291764839463147669709
x=.000000000000000000000000000000082609320818217495236487501240034052\
0927514602540226559210841182011440
PI-T=.000000000000000000000000000000000000000238761373795213525096051\
4709639992616172412849126230299993288

i=102
n=15211807202738752817960438464512
m=.999999999999999999999999999999999999999999999999999999999999997867\
4062856102554822941209865786917427
x=.000000000000000000000000000000041304660409108747618243750620017026\
0462546784115274894715830198436969
PI-T=.000000000000000000000000000000000000001159469052545681793538302\
2598269343167420119000749951520040136

i=103
n=30423614405477505635920876929024
m=.999999999999999999999999999999999999999999999999999999999999999466\
8515714025638705735302466446729357
x=.000000000000000000000000000000020652330204554373809121875310008513\
0228852357747960677578734314080982
PI-T=.000000000000000000000000000000000000004842299767547554867307380\
7873639261721657087190004675592021192

i=104
n=60847228810955011271841753858048
```



```
m=.999999999999999999999999999999999999999999999999999999999999999866\
7128928506409676433825616611682339
x=.000000000000000000000000000000010326165102277186904560937655004256\
5109584110254626799231005586765487
PI-T=.000000000000000000000000000000000000019573622627555047162383713\
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i=105
n=121694457621910022543683507716096
m=.999999999999999999999999999999999999999999999999999999999999999966\
6782232126602419108456404152920584
x=.000000000000000000000000000000005163082551138593452280468827502128\
2552371020817636629836322008245241
PI-T=.000000000000000000000000000000000000034304945487562539457460047\
6358507097719831363245878322184283336

i=106
n=243388915243820045087367015432192
m=.999999999999999999999999999999999999999999999999999999999999999991\
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x=.000000000000000000000000000000002581541275569296726140234413751064\
1271343441789464775359799433847616
PI-T=.000000000000000000000000000000000000009323023692759250863776538 4\
3948089488109015462630192200585012424

i=107
n=486777830487640090174734030864384
m=.999999999999999999999999999999999999999999999999999999999999999997\
9173889507912651194278525259557536
x=.000000000000000000000000000000001290770637784648363070117206875532\
0616303446417318229446453435823792
PI-T=.000000000000000000000000000000000000564632568447832262080208079\
0748863020029879834776160196057064648

i=108
n=973555660975280180349468061728768
m=.999999999999999999999999999999999999999999999999999999999999999999\
4793472376978162798569631314889384
x=.000000000000000000000000000000000645385318892324181535058603437766\
0250046899776416640022887874611848
PI-T=.000000000000000000000000000000000003393046557569270782734864247\
2682342000455903436767176386463884488

i=109
n=1947111321950560360698936123457536
m=.999999999999999999999999999999999999999999999999999999999999999999\
8698368094244540699642407828722346
x=.000000000000000000000000000000000322692659446162090767529301718883\
0047550351978551687077658812905861
PI-T=.000000000000000000000000000000000010935483861893106837813947362\
4622752849903308610565505196351251656

i=110
n=3894222643901120721397872246915072
m=.999999999999999999999999999999999999999999999999999999999999999999\
9674592023561135174910601957180586
x=.000000000000000000000000000000000161346329723081045383764650859441\
4791355882260305944737474033252740
PI-T=.000000000000000000000000000000000005619010768783612316828844605 3\
6335313954177959826340639756119925960

i=111
```

```
n=778844528780224144279574449383O144
```

```
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Eltérés a(z) 1. lépésben: -0.306852819440054714
Eltérés a(z) 2. lépésben: 0.193147180559945286
Eltérés a(z) 3. lépésben: -0.140186152773387973
Eltérés a(z) 4. lépésben: 0.109813847226612027
Eltérés a(z) 5. lépésben: -0.090186152773387929
Eltérés a(z) 6. lépésben: 0.076480513893278701
Eltérés a(z) 7. lépésben: -0.066376628963864204
Eltérés a(z) 8. lépésben: 0.058623371036135796
Eltérés a(z) 9. lépésben: -0.052487740074975364
Eltérés a(z) 10. lépésben: 0.047512259925024614
Eltérés a(z) 11. lépésben: -0.043396830984066326
Eltérés a(z) 12. lépésben: 0.039936502349267045
Eltérés a(z) 13. lépésben: -0.036986574573809938
Eltérés a(z) 14. lépésben: 0.034441996854761459
Eltérés a(z) 15. lépésben: -0.032224669811905193
Eltérés a(z) 16. lépésben: 0.030275330188094807
Eltérés a(z) 17. lépésben: -0.028548199223669912
Eltérés a(z) 18. lépésben: 0.027007356331885668
Eltérés a(z) 19. lépésben: -0.025624222615482695
Eltérés a(z) 20. lépésben: 0.024375777384517350
Eltérés a(z) 21. lépésben: -0.023243270234530322
Eltérés a(z) 22. lépésben: 0.022211275220015092
Eltérés a(z) 23. lépésben: -0.021266985649550096
Eltérés a(z) 24. lépésben: 0.020399681017116533
Eltérés a(z) 25. lépésben: -0.019600318982883502
Eltérés a(z) 26. lépésben: 0.018861219478654934
Eltérés a(z) 27. lépésben: -0.018175817558382046
Eltérés a(z) 28. lépésben: 0.017538468155903653
Eltérés a(z) 29. lépésben: -0.016944290464785960
Eltérés a(z) 30. lépésben: 0.016389042868547365
Eltérés a(z) 31. lépésben: -0.015869021647581638
Eltérés a(z) 32. lépésben: 0.015380978352418362
Eltérés a(z) 33. lépésben: -0.014922051950611914
Eltérés a(z) 34. lépésben: 0.014489712755270445
Eltérés a(z) 35. lépésben: -0.014081715816158136
Eltérés a(z) 36. lépésben: 0.013696061961619654
Eltérés a(z) 37. lépésben: -0.013330965065407319
Eltérés a(z) 38. lépésben: 0.012984824408276863
Eltérés a(z) 39. lépésben: -0.012656201232748798
Eltérés a(z) 40. lépésben: 0.012343798767251224
Eltérés a(z) 41. lépésben: -0.012046445135187822
Eltérés a(z) 42. lépésben: 0.011763078674336014
Eltérés a(z) 43. lépésben: -0.011492735279152400
Eltérés a(z) 44. lépésben: 0.011234537448120308
Eltérés a(z) 45. lépésben: -0.010987684774101947
Eltérés a(z) 46. lépésben: 0.010751445660680647
Eltérés a(z) 47. lépésben: -0.010525150084000234
Eltérés a(z) 48. lépésben: 0.010308183249333136
Eltérés a(z) 49. lépésben: -0.010099980015973009
Eltérés a(z) 50. lépésben: 0.009900019984027009
```

```
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Eltérés a(z) 1. lépésben: -0.306852819440054714
Eltérés a(z) 10. lépésben: 0.047512259925024614
Eltérés a(z) 100. lépésben: 0.004975001249750255
Eltérés a(z) 1000. lépésben: 0.000499750000123034
Eltérés a(z) 10000. lépésben: 0.000049997499986798
Eltérés a(z) 100000. lépésben: 0.000004999974964170
```

```
j= 1 a=1.000000000000000 b=1.750000000000000 x=1.660219213410703
j= 2 a=1.000000000000000 b=1.660219213410703 x=1.601295972553822
j= 3 a=1.000000000000000 b=1.601295972553822 x=1.577911477805376
j= 4 a=1.000000000000000 b=1.577911477805376 x=1.571100496686157
j= 5 a=1.000000000000000 b=1.571100496686157 x=1.569347471831615
j= 6 a=1.000000000000000 b=1.569347471831615 x=1.568912399580561
j= 7 a=1.000000000000000 b=1.568912399580561 x=1.568805432179555
j= 8 a=1.000000000000000 b=1.568805432179555 x=1.568779194400333
j= 9 a=1.000000000000000 b=1.568779194400333 x=1.568772762293782
j=10 a=1.000000000000000 b=1.568772762293782 x=1.568771185705877
j=11 a=1.000000000000000 b=1.568771185705877 x=1.568770799278268
j=12 a=1.000000000000000 b=1.568770799278268 x=1.568770704564213
j=13 a=1.000000000000000 b=1.568770704564213 x=1.568770681349688
j=14 a=1.000000000000000 b=1.568770681349688 x=1.568770675659783
j=15 a=1.000000000000000 b=1.568770675659783 x=1.568770674265182
j=16 a=1.000000000000000 b=1.568770674265182 x=1.568770673923364
j=17 a=1.000000000000000 b=1.568770673923364 x=1.568770673839584
j=18 a=1.000000000000000 b=1.568770673839584 x=1.568770673819049
j=19 a=1.000000000000000 b=1.568770673819049 x=1.568770673814016
j=20 a=1.000000000000000 b=1.568770673814016 x=1.568770673812783
j=21 a=1.000000000000000 b=1.568770673812783 x=1.568770673812480
j=22 a=1.000000000000000 b=1.568770673812480 x=1.568770673812406
j=23 a=1.000000000000000 b=1.568770673812406 x=1.568770673812388
j=24 a=1.000000000000000 b=1.568770673812388 x=1.568770673812384
1.568770673812384
```

```
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>> /*++
&> A1_I_30_121 -- Az Analízis I. példatár I/30. feladatából a 121. megoldása
&>
&> A1_I_30_121 (x,n)
&>
&> x - bemenő x
&> n - bemenő n
&> ++*/
>>
>> A1_I_30_121 := proc (x,n)
&> begin
&>     float(cos(x/n)^n);
&>     end_proc:
>>
>> // Példa:
>> A1_I_30_121(0.2, 100);

                               0.9998000199
>>
>> ///////////////////////////////////////////////////////////////////////////////
/
>>
>> /*++
&> A1_I_30_121a -- Az Analízis I. példatár I/30./121. feladat közelítő megoldása
&>
&> A1_I_30_121a (x,h,m)
&>
&> x - bemenő x
&> h - Cauchy-féle hibaküszöb
&> m - maximális lépésszám
&> ++*/
>>
>> // Addig növeljük n-et, amíg a sorozat két szomszédos elemének különbsége
>> // nem csökken h alá. (Cauchy-féle konvergenciakritérium.)
>> A1_I_30_121a := proc (x,h,m)
&> local i,v,f,f_e,d;
&> begin
&>     if m<2 then
&>      return(FAIL);
&>      end_if;
&>     i:=1;
&>     v:=TRUE;
&>     while v and i<=m do
&>      f:=A1_I_30_121(x,i);
&>      if i>1 then
&>          d:=float(abs(f-f_e));
&>          if d < h then
&>              v:=FALSE;
&>              end_if; // float...
&>          end_if;  // i>1...
&>      i:=i+1;
&>      if v and i<=m then
&>          f_e:=f;
```



```
&>          end_if;
&>      end_while; // v and...
&>     i:=i-1;
&>     if not v then
&>      print(Unquoted,"Az n=".expr2text(i-1)." esetben |f(n+1)-f(n)|<h");
&>     else
&>      print(Unquoted,"Az n=".expr2text(i-1)." esetben |f(n+1)-f(n)|=".
&>          expr2text(d));
&>      end_if;
&>     print(Unquoted,"f(n)=".expr2text(f_e).", f(n+1)=".expr2text(f));
&>     if not v then
&>      return(f);
&>     else
&>      return(FAIL);
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_30_121a(0.5,0.0001,100);

                      Az n=35 esetben |f(n+1)-f(n)|<h

                  f(n)=0.9964348203, f(n+1)=0.9965336877

                               0.9965336877
>> A1_I_30_121a(0.5,0.00001,100);

              Az n=99 esetben |f(n+1)-f(n)|=0.00001261056921

                  f(n)=0.9987381652, f(n+1)=0.9987507757

                                   FAIL
>>
>> ///////////////////////////////////////////////////////////////////////////////
/
>>
>> /*++
&> A1_I_30_121b -- Az Analízis I. példatár I/30./121. feladat közelítő megoldása
&>
&> A1_I_30_121a (x,h,m)
&>
&> x - bemenő x
&> h - Cauchy-féle hibaküszöb
&> m - maximális lépésszám
&> ++*/
>>
>> // Addig növeljük n-et, amíg a sorozat szomszédos elemének különbsége
>> // nem csökken h alá. (Cauchy-féle konvergenciakritérium.)
>> // Az "a" verzióhoz képest annyit módosítunk, hogy n helyébe 2^n-et írunk.
>> A1_I_30_121b := proc (x,h,m)
&> local i,v,f,f_e,d;
&> begin
&>     if m<2 then
&>      return(FAIL);
&>      end_if;
&>     i:=1;
&>     v:=TRUE;
&>     while v and i<=m do
&>      f:=A1_I_30_121(x,2^i);
&>      if i>1 then
&>          d:=float(abs(f-f_e));
&>          if d < h then
```

```
&>              v:=FALSE;
&>              end_if; // float...
&>          end_if;  // i>1...
&>      i:=i+1;
&>      if v and i<=m then
&>          f_e:=f;
&>          end_if;
&>      end_while; // v and...
&>     i:=i-1;
&>     if not v then
&>      print(Unquoted,"Az n=2^".expr2text(i-1)." esetben |f(2n)-f(n)|<h");
&>     else
&>      print(Unquoted,"Az n=2^".expr2text(i-1)." esetben |f(2n)-f(n)|=".
&>          expr2text(d));
&>      end_if;
&>     print(Unquoted,"f(n)=".expr2text(f_e).", f(2n)=".expr2text(f));
&>     if not v then
&>      return(f);
&>     else
&>      return(FAIL);
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_30_121b(0.5,0.000000000001,100);

                     Az n=2^30 esetben |f(2n)-f(n)|<h

                            f(n)=1.0, f(2n)=1.0

                                    1.0
>>
>> ////////////////////////////////////////////////////////////////////////////
/
>>
>> // Ezután számos határérték-számítási feladatot gyorsan meg tudunk oldani:
>> A1_I_39_7 := proc (n)
&> local szorzat,i;
&> begin
&>     szorzat:=1;
&>     for i from 1 to 2*n-1 step 2 do
&>      szorzat:=szorzat*i;
&>      end_for;
&>     return(n/(szorzat^(1/n)));
&>     end_proc:
>>
>> // Lusták trükkje:
>> A1_I_30_121 := proc (x,n)
&> begin
&>     float(A1_I_39_7(n));
&>     end_proc:
>>
>> // Példa (sajnos, ennél a sorozatnál problémás...):
>> A1_I_30_121b(0.5,0.00001,10);

               Az n=2^9 esetben |f(2n)-f(n)|=0.0004596069263

                    f(n)=1.358221437, f(2n)=1.358681044

                                   FAIL
>> quit
```

```
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>> /*++
&> A1_I_40_1 -- Az Analízis I. példatár I/40. feladatából az 1. megoldása
&>
&> A1_I_40_1 (x,n)
&>
&> x - bemenő x (a feladatban 2)
&> n - bemenő n
&> ++*/
>>
>> A1_I_40_1 := proc (x,n)
&> begin
&>     if n=1 then sqrt(x)
&>      else sqrt(x*A1_I_40_1(x,n-1));
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_40_1(2, 10);

                                 1023/1024
                                2
>>
>> quit
```

```
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>> /*++
&> A1_I_40_3 -- Az Analízis I. példatár I/40. feladatából a 3. megoldása
&>
&> A1_I_40_3 (x,n)
&>
&> x - bemenő x (a feladatban c)
&> n - bemenő n
&> ++*/
>>
>> A1_I_40_3 := proc (x,n)
&> begin
&>     if n=1 then sqrt(x)
&>      else sqrt(x+A1_I_40_3(x,n-1));
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_40_3(2, 10);

          1/2     1/2     1/2     1/2     1/2     1/2     1/2     1/2
((((((((2    + 2)    + 2)    + 2)    + 2)    + 2)    + 2)    + 2)    + 2)^

   (1/2) + 2)^(1/2)
>> float(%);

                                1.999997647
>>
>> quit
```

```
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>> /*++
&> A1_I_41_1 -- Az Analízis I. példatár I/41. feladatából az 1. megoldása
&>
&> A1_I_41_3 (x,n)
&>
&> x - bemenő x
&> n - bemenő n
&> ++*/
>>
>> A1_I_41_1 := proc (x,n)
&> begin
&>     if n=1 then 0
&>      else x+A1_I_41_1(x,n-1)^2;
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_41_1(-2, 10);

                                     2
>> float(%);

                                    2.0
>>
>> quit
```

```
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>> /*++
&> A1_I_20 -- Az Analízis I. példatár I/20. feladatának megoldása
&>
&> A1_I_20 (a,m,n)
&>
&> a - bemenő a
&> m - monotonitási irány (1: növekvő, -1: csökkenő)
&> n - a vizsgált n-ek maximális száma
&> ++*/
>>
>> A1_I_20 := proc (a,m,n)
&> local v,i,f,f_e;
&> begin
&>     if abs(m) <> 1 or n=1 then
&>      return(FAIL);
&>      end_if;
&>
&>     v:=TRUE;
&>     i:=1;
&>     while v and (i<=n) do
&>      f:=i*(a^(1/i)-1);
&>      print(float(f));
&>      if i>1 then
&>          /*
&>           * Ha itt nem konvertálunk float-ba, akkor hibaüzenetet
&>           * kaphatunk, ugyanis csak valós számokat lehet összehasonlítani
&>           * egymással:
&>           */
&>          if float(f-f_e)*m<0 then v:=FALSE;
&>              end_if;
&>          end_if;
&>      f_e:=f;
&>      i:=i+1;
&>      end_while;
&>
&>     i:=i-1;
&>     if not v then
&>      i:=i-1;
&>      end_if;
&>     if m=-1 then
&>      print(Unquoted,"Monoton csökkenés n=".expr2text(i)."-ig");
&>      else
&>      print(Unquoted,"Monoton növekedés n=".expr2text(i)."-ig");
&>      end_if;
&>     end_proc:
>>
>> // Példa:
>> A1_I_20(3.5,-1,10);quit

                                       2.5

                                   1.741657387
```



```
                                   1.554883458

                                   1.471129599

                                   1.423675786

                                   1.393145575

                                   1.371861719

                                   1.356178365

                                   1.344142968

                                   1.334615817

                            Monoton csökkenés n=10-ig
```

```
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(PI/2)^1/1!=1.570796326794896558
(PI/2)^2/2!=1.233700550136169749
(PI/2)^3/3!=0.645964097506246171
(PI/2)^4/4!=0.253669507901047975
(PI/2)^5/5!=0.079692626246167034
(PI/2)^6/6!=0.020863480763352957
(PI/2)^7/7!=0.004681754135318687
(PI/2)^8/8!=0.000919260274839426
(PI/2)^9/9!=0.000160441184787360
(PI/2)^10/10!=0.000025202042373061
(PI/2)^11/11!=0.000003598843235212
(PI/2)^12/12!=0.000000471087477882
(PI/2)^13/13!=0.000000056921729220
(PI/2)^14/14!=0.000000006386603084
(PI/2)^15/15!=0.000000000668803511
(PI/2)^16/16!=0.000000000065659631
(PI/2)^17/17!=0.000000000006066936
(PI/2)^18/18!=0.000000000000529440
(PI/2)^19/19!=0.000000000000043771
(PI/2)^20/20!=0.000000000000003438
(PI/2)^21/21!=0.000000000000000257
(PI/2)^22/22!=0.000000000000000018
(PI/2)^23/23!=0.000000000000000001
(PI/2)^24/24!=0.000000000000000000
(PI/2)^25/25!=0.000000000000000000
```

```
125.00000
(PI/2)^1/1!=1.57079633
(PI/2)^2/2!=1.23370055
(PI/2)^3/3!=0.64596410
(PI/2)^4/4!=0.25366951
(PI/2)^5/5!=0.07969263
(PI/2)^6/6!=0.02086348
(PI/2)^7/7!=0.00468175
(PI/2)^8/8!=0.00091926
(PI/2)^9/9!=0.00016044
(PI/2)^10/10!=0.00002520
(PI/2)^11/11!=0.00000360
(PI/2)^12/12!=0.00000047
(PI/2)^13/13!=-0.00000017
(PI/2)^14/14!=-0.00000026
(PI/2)^15/15!=-0.00000041
(PI/2)^16/16!=-0.00000064
(PI/2)^17/17!=-0.00000100
(PI/2)^18/18!=-0.00000158
(PI/2)^19/19!=-0.00000248
(PI/2)^20/20!=-0.00000389
```

```
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n = 20
n=20 T(n)=1.050000000000000044
n=19 T(n)=1.055263157894736903
n=18 T(n)=1.058625730994152025
n=17 T(n)=1.062272101823185322
n=16 T(n)=1.066392006363949152
n=15 T(n)=1.071092800424263247
n=14 T(n)=1.076506628601733073
n=13 T(n)=1.082808202200133296
n=12 T(n)=1.090234016850011090
n=11 T(n)=1.099112183350001049
n=10 T(n)=1.109911218335000171
n=9 T(n)=1.123323468703888883
n=8 T(n)=1.140415433587986138
n=7 T(n)=1.162916490512569512
n=6 T(n)=1.193819415085428215
n=5 T(n)=1.238763883017085732
n=4 T(n)=1.309690970754271433
n=3 T(n)=1.436563656918090404
n=2 T(n)=1.718281828459045091
T(n)=2.718281828459045091
Eltérés: 0.000000000000000000
```

```
125.00000
1!=1, (PI/2)^1/1!=1.57079633
2!=2, (PI/2)^2/2!=1.23370055
3!=6, (PI/2)^3/3!=0.64596410
4!=24, (PI/2)^4/4!=0.25366951
5!=120, (PI/2)^5/5!=0.07969263
6!=720, (PI/2)^6/6!=0.02086348
7!=5040, (PI/2)^7/7!=0.00468175
8!=40320, (PI/2)^8/8!=0.00091926
9!=362880, (PI/2)^9/9!=0.00016044
10!=3628800, (PI/2)^10/10!=0.00002520
11!=39916800, (PI/2)^11/11!=0.00000360
12!=479001600, (PI/2)^12/12!=0.00000047
13!=-2147483648, (PI/2)^13/13!=-0.00000017
14!=-2147483648, (PI/2)^14/14!=-0.00000026
15!=-2147483648, (PI/2)^15/15!=-0.00000041
16!=-2147483648, (PI/2)^16/16!=-0.00000064
17!=-2147483648, (PI/2)^17/17!=-0.00000100
18!=-2147483648, (PI/2)^18/18!=-0.00000158
19!=-2147483648, (PI/2)^19/19!=-0.00000248
20!=-2147483648, (PI/2)^20/20!=-0.00000389
```

```
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2.72/1!=2.720000000000000195
2.72/2!=1.360000000000000098
2.72/3!=0.453333333333333366
2.72/4!=0.113333333333333341
2.72/5!=0.022666666666666668
2.72/6!=0.003777777777777778
2.72/7!=0.000539682539682540
2.72/8!=0.000067460317460317
2.72/9!=0.000007495590828924
2.72/10!=0.000000749559082892
2.72/11!=0.000000068141734808
2.72/12!=0.000000005678477901
2.72/13!=0.000000000436805992
2.72/14!=0.000000000031200428
2.72/15!=0.000000000002080029
2.72/16!=0.000000000000130002
2.72/17!=0.000000000000007647
2.72/18!=0.000000000000000425
2.72/19!=0.000000000000000022
2.72/20!=0.000000000000000001
2.72/21!=0.000000000000000000
2.72/22!=0.000000000000000000
2.72/23!=0.000000000000000000
2.72/24!=0.000000000000000000
2.72/25!=0.000000000000000000
```

```
(PI/2)^1/1!=1.570796327000000048
(PI/2)^2/2!=1.233700550458345591
(PI/2)^3/3!=0.645964097759282541
(PI/2)^4/4!=0.253669508033537494
(PI/2)^5/5!=0.079692626298195540
(PI/2)^6/6!=0.020863480779698192
(PI/2)^7/7!=0.004681754139597860
(PI/2)^8/8!=0.000919260275799670
(PI/2)^9/9!=0.000160441184975903
(PI/2)^10/10!=0.000025202042405968
(PI/2)^11/11!=0.000003598843240381
(PI/2)^12/12!=0.000000471087478620
(PI/2)^13/13!=0.000000056921729316
(PI/2)^14/14!=0.000000006386603095
(PI/2)^15/15!=0.000000000668803512
(PI/2)^16/16!=0.000000000065659631
(PI/2)^17/17!=0.000000000006066936
(PI/2)^18/18!=0.000000000000529440
(PI/2)^19/19!=0.000000000000043771
(PI/2)^20/20!=0.000000000000003438
(PI/2)^21/21!=0.000000000000000257
(PI/2)^22/22!=0.000000000000000018
(PI/2)^23/23!=0.000000000000000001
(PI/2)^24/24!=0.000000000000000000
(PI/2)^25/25!=0.000000000000000000
```

```
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>> /*
&>  * Trapézformula, ld. Móricz Ferenc: Numerikus módszerek az algebrában
&>  * és az analízben, 130. o. illetve Leindler László: Analízis, Nemzeti
&>  * könyvkiadó, 1993, 195. o.
&>  * A sum eljárás beépített függvény.
&>  */
>>
>> Trapez := proc (N)
&> begin
&>     return((b-a)/N*(f(a)/2+f(b)/2+sum(f(a+k*(b-a)/N),k=1..N-1)));
&> end_proc:
>>
>> factor(Trapez(6));

     /                  / a   b \       / a   2 b \       / 2 a   b \
1/12 | f(a) + f(b) + 2 f| - + - | + 2 f| - + --- | + 2 f| --- + - | +
     \                  \ 2   2 /       \ 3    3  /       \  3    3 /

      / a   5 b \       / 5 a   b \ \
   2 f| - + --- | + 2 f| --- + - | | (- a + b)
      \ 6    6  /       \  6    6 / /
>>
>> Simpson := proc (N)
&> begin
&>     return((b-a)/(6*N)*(f(a)+f(b)+
&>      4*sum(f(a+(2*k+1)*(b-a)/(2*N)),k=0..N-1)+
&>      2*sum(f(a+(2*k)*(b-a)/(2*N)),k=1..N-1)));
&> end_proc:
>>
>> factor(Simpson(3));

     /                  / a   b \       / a   2 b \       / 2 a   b \
1/18 | f(a) + f(b) + 4 f| - + - | + 2 f| - + --- | + 2 f| --- + - | +
     \                  \ 2   2 /       \ 3    3  /       \  3    3 /

      / a   5 b \       / 5 a   b \ \
   4 f| - + --- | + 4 f| --- + - | | (- a + b)
      \ 6    6  /       \  6    6 / /
>>
>> /*
&>  * Romberg-rekurzió
&>  */
>>
>> T := proc (k,m)
&> begin
&>     if m=Nil or m=0 then
&>      return(Trapez(k));
&>      end_if;
&>     return((4^m*T(2*k,m-1)-T(k,m-1))/(4^m-1));
&> end_proc:
>>
>> ////////////////////////////////////////////////////////////////////////////////
//
```



```
>>
>> f:=sin:
>> a:=0:
>> b:=PI:
>>
>> DIGITS:=20:
>>
>>
>> /*
&>  * Határozott integrál számítása (beépített függvény)
&>  */
>>
>> int(f(x),x=a..b);

                                    2
>>
>> /*
&>  * Illusztráljuk, hogy a trapézformula mennyire lassú konvergenciát ad
&>  * (a számolásigény is nagy): 1000-es egyenlő beosztásnál csak 5 tizedesjegy
&>  * pontosságot kapunk.
&>  */
>>
>> for i from 1 to 3 do
&>     print(float(Trapez(10^i)));
&>     end_for;

                          1.9835235375094545035

                          1.9998355038874435076

                          1.9999983550656625709

>>
>> /*
&>  * A Simpson-formula lényegesen gyorsabban közelít.
&>  */
>>
>> for i from 1 to 3 do
&>     print(float(Simpson(10^i)));
&>     end_for;

                          2.0000067844418011042

                          2.0000000006764718916

                          2.0000000000000676452

>>
>> /*
&>  * A Romberg-iteráció k=1 esetén már az m=4 esetben 8 jegy pontosságot ad.
&>  */
>>
>> for i from 1 to 4 do
&>     print(T(1,i));
&>     print(float(T(1,i)));
&>     end_for;

                                  2 PI
                                  ----
                                   3

                          2.0943951023931954923
```

```
                                  1/2
                          16 PI (2    + 1)   2 PI
                          ---------------- - ----
                                 45            9

                           1.9985707318238359863

               1/2                 1/2     1/2     1/2         1/2 1/2
2 PI   16 PI (2    + 1)   512 PI (2    + (2    + 2)    + (2 - 2   )    + 1)
---- - ---------------- + -------------------------------------------------
135          135                                 2835

                           2.0000055499796705137

        1/2                        1/2     1/2     1/2         1/2 1/2
16 PI (2    + 1)   2 PI   512 PI (2    + (2    + 2)    + (2 - 2   )    + 1)
---------------- - ---- - -------------------------------------------------
      2025         8505                          8505

      /          /      / PI \        / 3 PI \        / 5 PI \
    + | 65536 PI | 2 sin| -- | + 2 sin| ---- | + 2 sin| ---- | +
      \          \      \ 16 /        \  16  /        \  16  /

        / 7 PI \    1/2     1/2     1/2         1/2 1/2     \ \
   2 sin| ---- | + 2    + (2    + 2)    + (2 - 2   )    + 1 | | / 722925
        \  16  /                                            / /

                           1.9999999945872901717

>>
>> /*
&>  * k növelése tovább javítja a konvergencia sebességét.
&>  */
>>
>> for i from 1 to 4 do
&>     print(T(2,i));
&>     print(float(T(2,i)));
&>     end_for;

                                 1/2
                            PI (2    + 1)   PI
                            ------------- - --
                                  3         6

                           2.0045597549844209554

             1/2               1/2     1/2     1/2         1/2 1/2
   PI   PI (2    + 1)   8 PI (2    + (2    + 2)    + (2 - 2   )    + 1)
   -- - ------------- + -----------------------------------------------
   90        9                                 45

                           1.9999831309459855992

     1/2                      1/2     1/2     1/2         1/2 1/2
PI (2    + 1)    PI    8 PI (2    + (2    + 2)    + (2 - 2   )    + 1)
------------- - ---- - ----------------------------------------------- +
     135        5670                           135

   /        /      / PI \        / 3 PI \        / 5 PI \
   | 256 PI | 2 sin| -- | + 2 sin| ---- | + 2 sin| ---- | +
   \        \      \ 16 /        \  16  /        \  16  /
```



```
        / 7 PI \    1/2     1/2     1/2         1/2 1/2     \ \
   2 sin| ---- | + 2    + (2    + 2)    + (2 - 2   )    + 1 | | / 2835
        \  16  /                                            / /

                           2.0000000162880416574

               1/2               1/2     1/2     1/2         1/2 1/2
  PI      PI (2    + 1)   8 PI (2    + (2    + 2)    + (2 - 2   )    + 1)
------- - ------------- + -------------------------------------------------
1445850       8505                                  2025

      /        /      / PI \        / 3 PI \        / 5 PI \
    - | 256 PI | 2 sin| -- | + 2 sin| ---- | + 2 sin| ---- | +
      \        \      \ 16 /        \  16  /        \  16  /

        / 7 PI \    1/2     1/2     1/2         1/2 1/2     \ \
   2 sin| ---- | + 2    + (2    + 2)    + (2 - 2   )    + 1 | | / 8505 +
        \  16  /                                            / /

   /          /      / PI \        / 3 PI \        / 5 PI \
   | 32768 PI | 2 sin| -- | + 2 sin| ---- | + 2 sin| ---- | +
   \          \      \ 16 /        \  16  /        \  16  /

        / 7 PI \        / PI \        / 3 PI \        / 5 PI \
   2 sin| ---- | + 2 sin| -- | + 2 sin| ---- | + 2 sin| ---- | +
        \  16  /        \ 32 /        \  32  /        \  32  /

        / 7 PI \        / 9 PI \        / 11 PI \        / 13 PI \
   2 sin| ---- | + 2 sin| ---- | + 2 sin| ----- | + 2 sin| ----- | +
        \  32  /        \  32  /        \  32   /        \  32   /

        / 15 PI \    1/2     1/2     1/2         1/2 1/2     \ \
   2 sin| ----- | + 2    + (2    + 2)    + (2 - 2   )    + 1 | | / 722925
        \  32   /                                            / /

                           1.9999999999960339046

>>
>> /*
&>  * A Romberg-iteráció "elég gyors".
&>  */
>>
>> DIGITS:=50:
>> for i from 1 to 10 do
&>     elteres:=float(2-T(1,i));
&>     tizedesjegy_pontossag:=trunc(abs(trunc(log(10,elteres))));
&>     print(Unquoted, "Pontosság: ".
&>      expr2text(tizedesjegy_pontossag).
&>      " tizedesjegy.");
&>     end_for;

                         Pontosság: 1 tizedesjegy.

                         Pontosság: 2 tizedesjegy.

                         Pontosság: 5 tizedesjegy.

                         Pontosság: 8 tizedesjegy.

                        Pontosság: 11 tizedesjegy.
```

```
                          Pontosság: 16 tizedesjegy.

                          Pontosság: 20 tizedesjegy.

                          Pontosság: 26 tizedesjegy.

                          Pontosság: 32 tizedesjegy.

                          Pontosság: 38 tizedesjegy.

>>
>> quit
```

```
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>> /*
&>  * Ez a program szimbolikusan kiszámolja a Romberg-féle integrálásnál
&>  * fellépő T(k,m) kifejezéseket. Az eljárást részletesen ld.
&>  * Móricz Ferenc: Numerikus módszerek az algebrában és az analízisben c.
&>  * könyvében, 136. o.; az ott leírt eljárásnál olvasható jelölés helyett
&>  * itt T(k,m) az ottani T(2^m*k,m)-et jelöli.
&>  * Copyright (C) Kovács Zoltán, 2002/03/16
&>  */
>>
>> T := proc (k,m)
&> begin
&>     if m=Nil or m=0 then
&>      return(procname(k));
&>      end_if;
&>     return((4^m*T(2*k,m-1)-T(k,m-1))/(4^m-1));
&> end_proc:
>>
>> T(k,0);

                                   T(k)
>> factor(T(k,1));

                          1/3 (- T(k) + 4 T(2 k))
>> factor(T(k,2));

                    1/45 (T(k) - 20 T(2 k) + 64 T(4 k))
>> factor(T(k,3));

          1/2835 (- T(k) + 84 T(2 k) - 1344 T(4 k) + 4096 T(8 k))
>> factor(T(k,7));

1/49615367752825875 (- T(k) + 21844 T(2 k) - 95414592 T(4 k) +

   99158478848 T(8 k) - 25384570585088 T(16 k) +

   1600791219535872 T(32 k) - 24017731997138944 T(64 k) +

   72057594037927936 T(128 k))
>> quit
```