A 8. feladatsor feladatainak megoldása

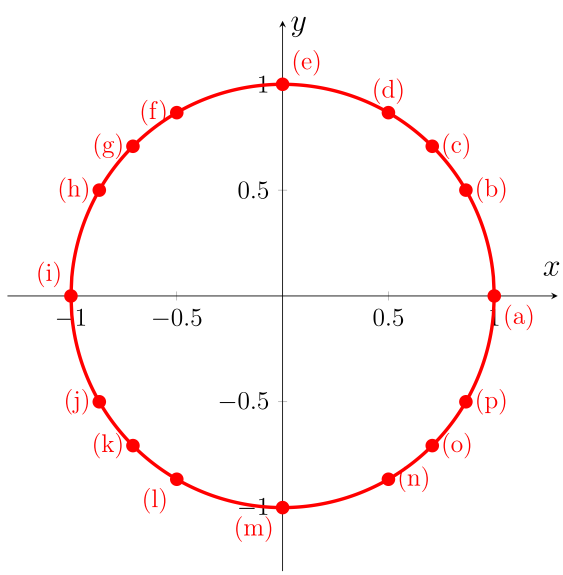
8.1. Feladat.

(a) $-i$, (b) -1, (c) $41-11i$, (d) $17-2i$,
(e) $-15-5i$, (f) $-\frac{11}{17}+\frac{27}{17}i$, (g) $-\frac{3}{13}-\frac{11}{13}i$, (h) $\frac{11}{10}-\frac{23}{10}i$,
(i) $-\frac{2}{5}+\frac{3}{10}i$.

$\rightsquigarrow$ **videók:** **8.1. Feladat: (a)–(e)**, **8.1. Feladat: (f)–(i)**

8.2. Feladat. Vegyük észre, hogy ha a φ szöget az egységkörön ábrázoljuk, akkor koordinátái: $(\cos\varphi, \sin\varphi)$. Ez indokolja, hogy a táblázatban először szerepel $\cos\varphi$ és utána $\sin\varphi$.

	fok	rad.	$\cos\varphi$	$\sin\varphi$		fok	rad.	$\cos\varphi$	$\sin\varphi$
(a)	0	0	1	0	(b)	30	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
(c)	45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	(d)	60	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
(e)	90	$\frac{\pi}{2}$	0	1	(f)	120	$\frac{2\pi}{3}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
(g)	135	$\frac{3\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	(h)	150	$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
(i)	180	π	-1	0	(j)	210	$\frac{7\pi}{6}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$
(k)	225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	(l)	240	$\frac{4\pi}{3}$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
(m)	270	$\frac{3\pi}{2}$	0	-1	(n)	300	$\frac{5\pi}{3}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
(o)	315	$\frac{7\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	(p)	330	$\frac{11\pi}{6}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$



$\rightsquigarrow$ **videó:** **8.2. Feladat: (b),(c),(d),(h),(k) és (n)**

8.3. Feladat.

(a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0) = 3e^0$,

(b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi) = 5e^{\pi i}$,

(c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2} = e^{\frac{\pi}{2} i}$,

(d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = 8e^{\frac{3\pi}{2} i}$,

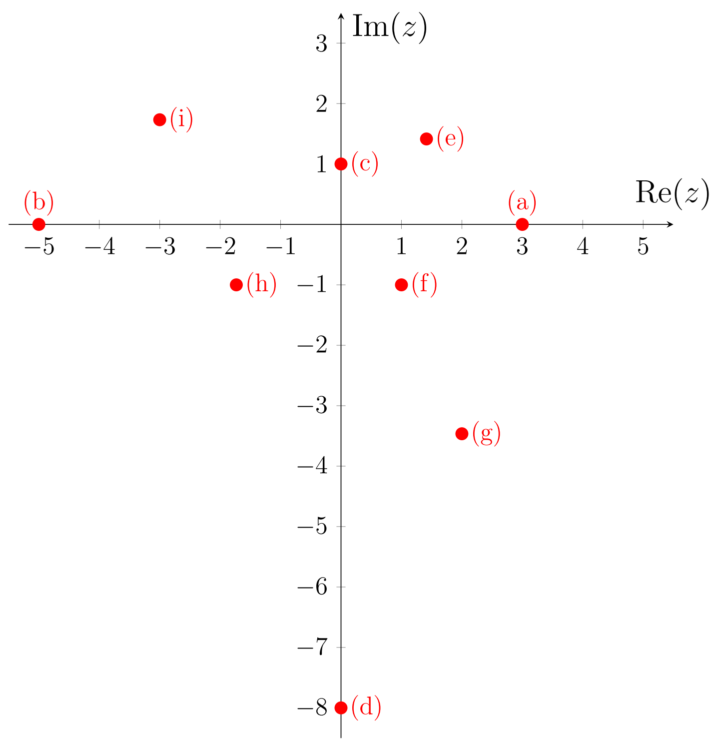
(e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4}) = 2e^{\frac{\pi}{4} i}$,

(f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4}) = \sqrt{2}e^{\frac{7\pi}{4} i}$,

(g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} + i \cdot \sin \frac{5\pi}{3}) = 4e^{\frac{5\pi}{3} i}$,

(h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6}) = 2e^{\frac{7\pi}{6} i}$,

(i) $-3 + \sqrt{3}i = 2\sqrt{3}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6}) = 2\sqrt{3}e^{\frac{5\pi}{6} i}$.



$\rightsquigarrow$ **videók:** **8.3. Feladat:(a), (c) és (f)**, **8.3. Feladat: (h)**

8.4. Feladat.

(a) $2(\cos 0 + i \sin 0) = 2$,

(b) $3e^{\frac{3\pi}{2} i} = -3i$,

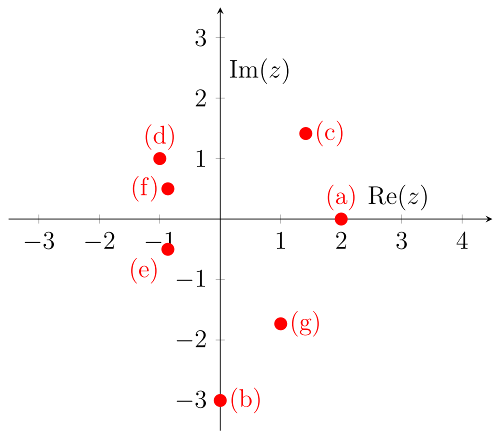
(c) $2e^{\frac{\pi}{4} i} = \sqrt{2} + \sqrt{2}i$,

(d) $\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -1 + i$,

(e) $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$,

(f) $e^{\frac{5\pi}{6} i} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$,

(g) $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i$.



$\rightsquigarrow$ **videók:** **8.4. Feladat: (a)–(b)**, **8.4. Feladat: (d) és (f)**

8.5. Feladat.

(a) $8(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2}) = -8i$,
(b) $20(\cos 0 + i \cdot \sin 0) = 20$,
(c) $\cos\pi + i \cdot \sin\pi = -1$,
(d) $2^{67}(\cos\frac{5\pi}{6} + i \cdot \sin\frac{5\pi}{6}) = -2^{66}\sqrt{3} + 2^{66}i$,
(e) $2^{611}(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2}) = -2^{611}i$,
(f) $6^{1526}(\cos\frac{2\pi}{3} + i \cdot \sin\frac{2\pi}{3}) = -3 \cdot 6^{1525} + 3\sqrt{3} \cdot 6^{1525}i$.

$\rightsquigarrow$ **videók:** **8.5. Feladat: (a)**, **8.5. Feladat: (d)**

8.6. Feladat.

(a) $3 = 3(\cos 0 + i\sin 0)$, $-3 = 3(\cos\pi + i\sin\pi)$,
(b) $2i = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$, $-2i = 2(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$,
(c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$, $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$,
(d) $-2\sqrt{2} + 2\sqrt{2}i = 4(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$, $2\sqrt{2} - 2\sqrt{2}i = 4(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$,
(e) $-1 - 4i$, $1 + 4i$,
(f) $\sqrt{3} + i = 2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$, $-\sqrt{3} - i = 2(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6})$
(g) $-2 = 2(\cos\pi + i\sin\pi)$, $1 + \sqrt{3}i = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$, $1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$,
(h) $2i = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$, $-\sqrt{3} - i = 2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$, $\sqrt{3} - i = 2\left(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6}\right)$,
(i) trigonometrikus alakban: $2\sqrt[6]{2}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3} + i\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right)$ $(k = 0, 1, 2)$, kanonikus alakban rendre: $\sqrt{2}\sqrt[6]{2} + \sqrt{2}\sqrt[6]{2}i$, $-\frac{\sqrt[6]{2}(1+\sqrt{3})}{\sqrt{2}} + \frac{\sqrt[6]{2}(-1+\sqrt{3})}{\sqrt{2}}i$, $\frac{\sqrt[6]{2}(-1+\sqrt{3})}{\sqrt{2}} - \frac{\sqrt[6]{2}(1+\sqrt{3})}{\sqrt{2}}i$.

$\rightsquigarrow$ **videók:** **8.6. Feladat: (b)**, **8.6. Feladat: (h)**

8.7. Feladat.

(a) $x_1 = -i$, $x_2 = i$
$x^2 + 1 = (x + i)(x - i)$

(b) $x_1 = -3 - i$, $x_2 = -3 + i$
$x^2 + 6x + 10 = (x + 3 + i)(x + 3 - i)$

(c) $x_{1,2} = -i$
$x^2 + 2xi - 1 = (x + i)^2$

(d) $x_1 = -2$, $x_2 = 1 + \sqrt{3}i$, $x_3 = 1 - \sqrt{3}i$
$x^3 + 8 = (x + 2)(x - 1 - \sqrt{3}i)(x - 1 + \sqrt{3}i)$

(e) $x_{1,2} = 0$, $x_3 = -3i$, $x_4 = 3i$
$x^4 + 9x^2 = x^2(x + 3i)(x - 3i)$

(f) $x_1 = -2$, $x_2 = 2$, $x_2 = -2i$, $x_3 = 2i$
$x^4 - 16 = (x + 2)(x - 2)(x + 2i)(x - 2i)$

(g) $x_{1,2} = -3i$, $x_{3,4} = 3i$
$x^4 + 18x^2 + 81 = (x + 3i)^2(x - 3i)^2$

$\rightsquigarrow$ **videó:** **8.7. Feladat: (b) és (g)**

Szabaduló szoba