

4. feladatsor – Relációk

4.1. Feladat megoldása.

$$\alpha \cap \beta = \{(1, 2), (3, 2)\}$$

$$\alpha \setminus \beta = \{(3, 4), (4, 4), (5, 5)\}$$

$$\alpha^{-1} = \{(2, 1), (2, 3), (4, 3), (4, 4), (5, 5)\}$$

$$\alpha\beta = \{(1, 2), (3, 2), (3, 3), (3, 5), (4, 3), (4, 5)\}$$

$$\beta\alpha = \{(4, 2), (4, 4), (4, 5)\}$$

$$\beta\alpha^{-1} = \{(1, 1), (1, 3), (2, 1), (2, 3), (3, 1), (3, 3), (4, 5)\}$$

$$\beta \cap \alpha^{-1} = \{(4, 3)\}$$

4.2. Feladat megoldása.

(a) $\alpha^{-1} = \{(y, x) \in \mathbb{E}^2 : x \text{ az } y \text{ gyermeke}\}$

$$\alpha\beta = \{(x, y) \in \mathbb{E}^2 : y \text{ az } x \text{ nagyapja}\}$$

$$\beta\alpha = \{(x, y) \in \mathbb{E}^2 : y \text{ az } x \text{ apai nagyszülője}\}$$

(b) $\alpha^{-1} = \{(y, x) \in \mathbb{R}^2 : x = 2y\}$

$$\alpha\beta = \{(x, y) \in \mathbb{R}^2 : x = 2^{y+1}\}$$

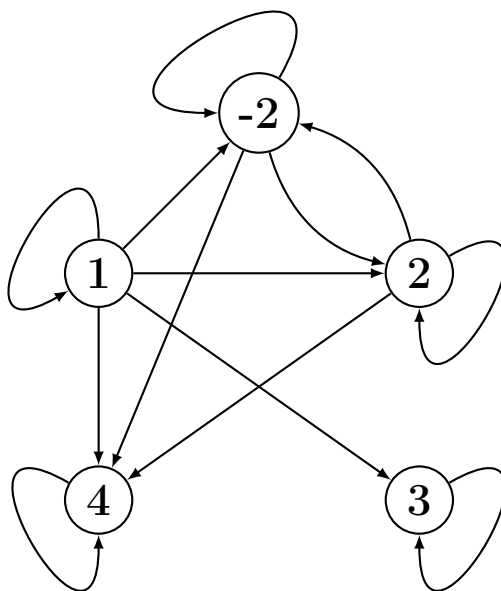
$$\beta\alpha = \{(x, y) \in \mathbb{R}^2 : x = 4^y\}$$

(c) $\alpha^{-1} = \{(y, x) \in \mathbb{R}^2 : y = x^2\}$

$$\alpha\beta = \{(x, y) \in \mathbb{R}^2 : x^2 = \frac{y-1}{3}\}$$

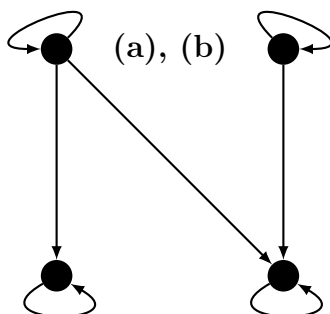
$$\beta\alpha = \{(x, y) \in \mathbb{R}^2 : (3x+1)^2 = y\}$$

4.3. Feladat megoldása.



Reflexív, nem szimmetrikus, nem antiszimmetrikus, tranzitív, nem dichotom.

4.4. Feladat megoldása.



(c): Nincs ilyen gráf, minden dichotom reláció reflexív.

4.5. Feladat megoldása.

	Refl.	Szim.	Antiszim.	Tranz.	Dich.	Ekv.	R.r.
(a)	✓	✓	✗	✗	✗	✗	✗
(b)	✓	✗	✓	✓	✓	✗	✓
(c)	✗	✗	✓	✓	✗	✗	✗
(d)	✓	✓	✗	✗	✗	✗	✗
(e)	✓	✗	✗	✓	✓	✗	✗
(f)	✓	✓	✗	✓	✗	✓	✗
(g)	✓	✓	✗	✓	✗	✓	✗
(h)	✓	✓	✗	✓	✗	✓	✗
(i)	✗	✗	✓	✓	✗	✗	✗
(j)	✓	✓	✗	✓	✗	✓	✗

4.6. Feladat megoldása.

$$\mathcal{C} = \{\{1, 2\}, \{3, 5\}, \{4\}\}$$

$$\varrho = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 5), (5, 3), (5, 5), (4, 4)\}$$

4.7. Feladat megoldása.

(a) $\{\{-3, -2, -1\}, \{1, 2, 3\}\}$

(b) $\{\{-3, 0, 3\}, \{-2, 1\}, \{-1, 2\}\}$

(c) $\{\{\emptyset\}, \{\{\emptyset\}, \{0\}\}, \{\{1, 2\}, \{a, b\}\}, \{\{1, 2, 3\}\}\}$

(d) $\{\{2, 8, 14, 26\}, \{3, 9, 15\}, \{19\}\}$

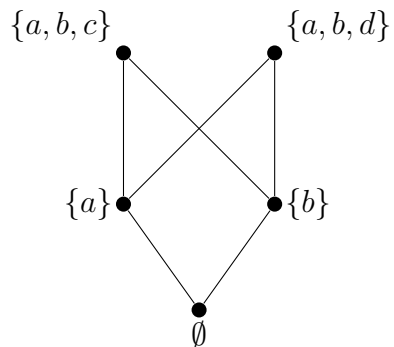
(e) $\{\{71, 602\}, \{301, 4, 121\}, \{216, 54, 315\}\}$

(f) $\{\{0\}, \{-1, 1\}, \{-2, 2\}, \{-3, 3\}, \dots\} = \{\{a, -a\}: a \in \mathbb{Z}\}$

(g) $\{\{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}, \{\dots, -3, -1, 1, 3, \dots\}\}$

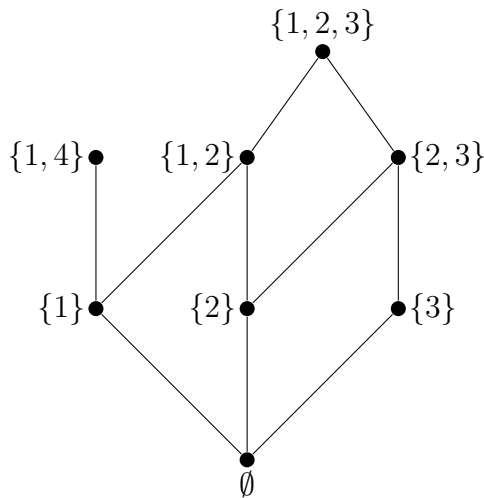
4.8. Feladat megoldása.

(a)



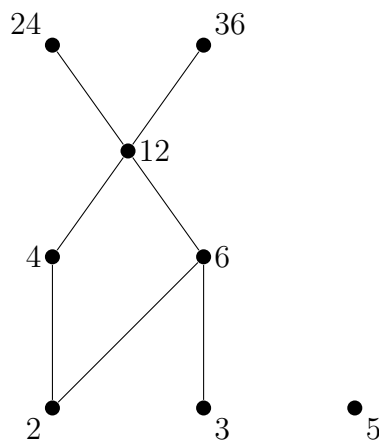
Legnagyobb elem: nincs.
 Legkisebb elem: \emptyset .
 Maximális elemek: $\{a, b, c\}$, $\{a, b, d\}$.
 Minimális elem: \emptyset .

(b)



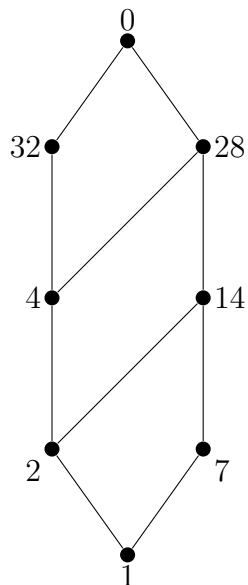
Legnagyobb elem: nincs.
 Legkisebb elem: \emptyset .
 Maximális elemek: $\{1, 2, 3\}$, $\{1, 4\}$.
 Minimális elem: \emptyset .

(c)



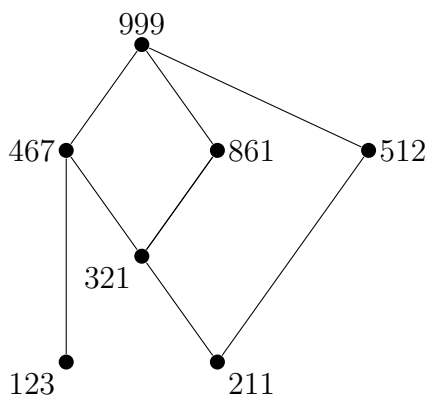
Legnagyobb elem: nincs.
 Legkisebb elem: nincs.
 Maximális elemek: 24, 36, 5.
 Minimális elemek: 2, 3, 5.

(d)



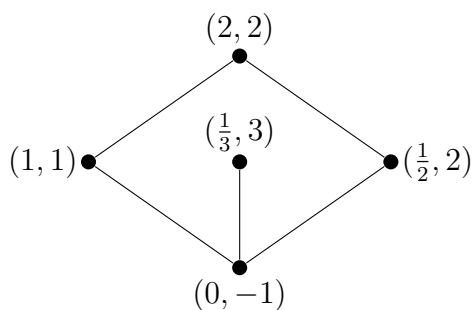
Legnagyobb elem: 0.
 Legkisebb elem: 1.
 Maximális elem: 0.
 Minimális elem: 1.

(e)



Legnagyobb elem: 999.
 Legkisebb elem: nincs.
 Maximális elem: 999.
 Minimális elemek: 123, 211.

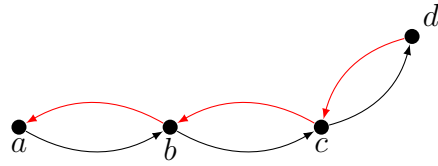
(f)



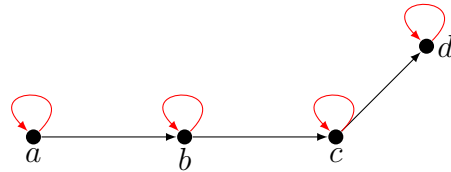
Legnagyobb elem: nincs.
 Legkisebb elem: $(0, -1)$.
 Maximális elemek: $(2, 2)$, $(\frac{1}{3}, 3)$.
 Minimális elem: $(0, -1)$.

4.9. Feladat megoldása.

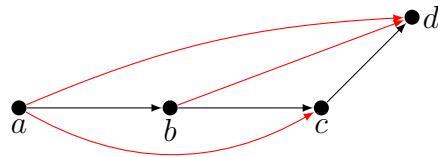
Szimmetrikus lezárt:



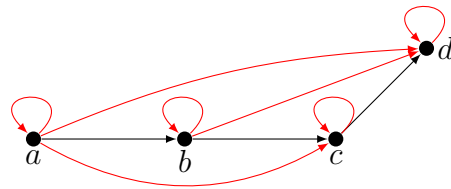
Reflexív lezárt:



Tranzitív lezárt:

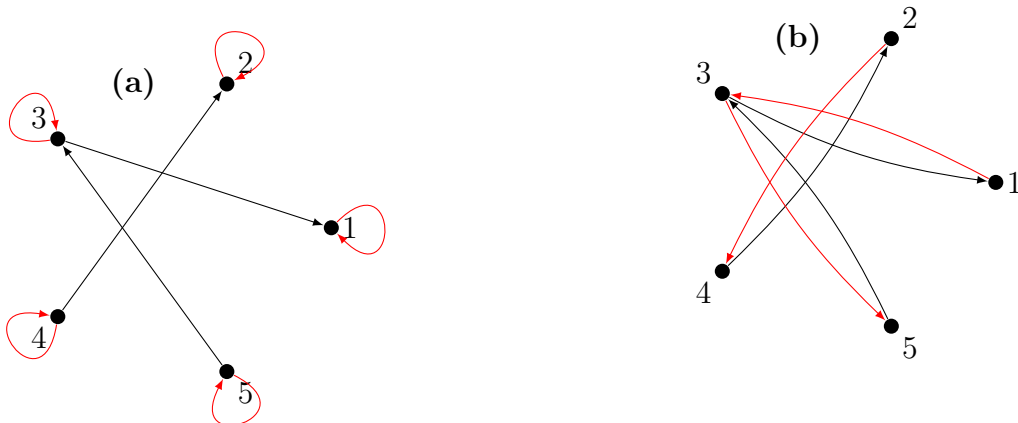


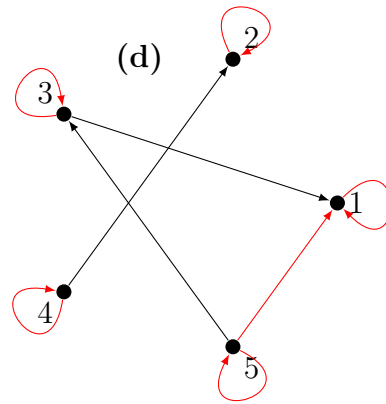
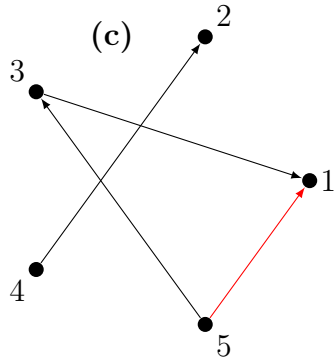
Reflexív és tranzitív lezárt:



4.10. Feladat megoldása.

- (a) ρ reflexív lezártja: $\{(4, 2), (3, 1), (5, 3), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$
- (b) ρ szimmetrikus lezártja: $\{(a, b) : |a - b| = 2\}$
- (c) ρ tranzitív lezártját: $\rho^+ = \rho \cup \{(5, 1)\}$
- (d) ρ reflexív és tranzitív lezártja: $\rho^* = \{(4, 2), (3, 1), (5, 3), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (5, 1)\}$





4.11. Feladat megoldása.

(a) $\rho^+ = \{(a, b) \in A^2 : |a - b| \text{ is even}\}$, ahol $A = \{1, 2, 3, 4, 5\}$

(b) $\tau^+ = \tau$

(c) $\alpha^+ = \{(a, b) \in \mathbb{R}^2 : \exists k \in \mathbb{Z}^+ : b = a^{2^k}\}$

(d) $\beta^+ = \{(x, y) \in \mathbb{R}^2 : \exists k \in \mathbb{Z}^+ : y - x = k\}$