

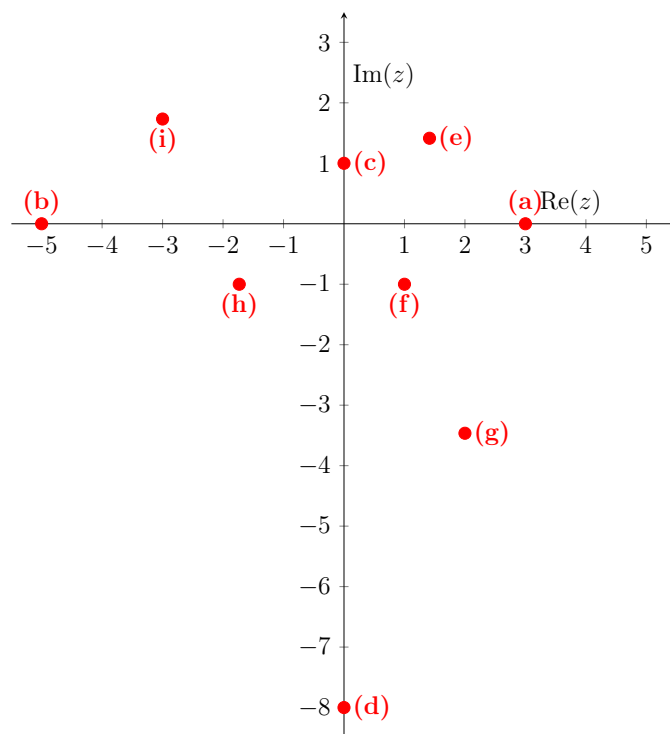
4. feladatsor – Komplex számok

4.1. Feladat megoldása.

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|------------------|---|
| (a) $-i$; | (f) $-\frac{11}{17} + \frac{27}{17}i$; |
| (b) -1 ; | (g) $-\frac{3}{13} - \frac{11}{13}i$; |
| (c) $41 - 11i$; | (h) $\frac{11}{10} - \frac{23}{10}i$; |
| (d) $17 - 2i$; | (i) $-\frac{2}{5} + \frac{3}{10}i$. |
| (e) $-15 - 5i$; | |

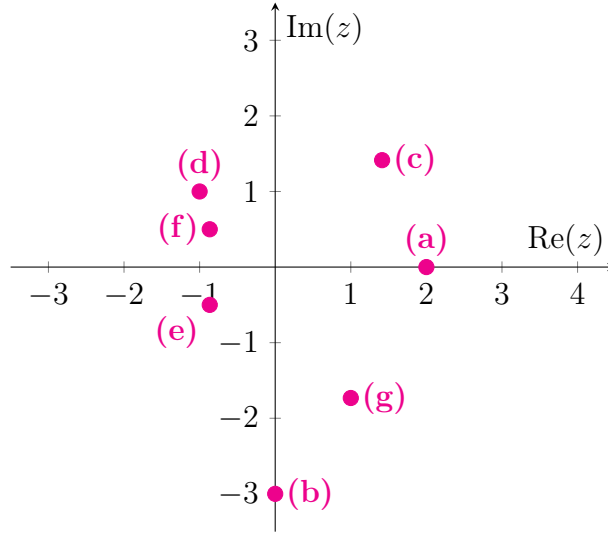
4.2. Feladat megoldása.

- | | |
|---|---|
| (a) $3 = 3 \cdot (\cos 0 + i \cdot \sin 0)$; | (f) $1 - i = \sqrt{2}(\cos \frac{7\pi}{4} + i \cdot \sin \frac{7\pi}{4})$; |
| (b) $-5 = 5 \cdot (\cos \pi + i \cdot \sin \pi)$; | (g) $2 - 2\sqrt{3}i = 4(\cos \frac{5\pi}{3} + i \cdot \sin \frac{5\pi}{3})$; |
| (c) $i = \cos \frac{\pi}{2} + i \cdot \sin \frac{\pi}{2}$; | (h) $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \cdot \sin \frac{7\pi}{6})$; |
| (d) $-8i = 8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2})$; | (i) $-3 + \sqrt{3}i = 2\sqrt{3}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6})$. |
| (e) $\sqrt{2} + \sqrt{2}i = 2(\cos \frac{\pi}{4} + i \cdot \sin \frac{\pi}{4})$; | |



4.3. Feladat megoldása.

- | | |
|---|--|
| (a) $2(\cos 0 + i \sin 0) = 2$; | (e) $\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$; |
| (b) $3(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}) = -3i$; | (f) $\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$; |
| (c) $2(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}) = \sqrt{2} + \sqrt{2}i$; | (g) $2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}) = 1 - \sqrt{3}i$. |
| (d) $\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = -1 + i$; | |



4.4. Feladat megoldása.

- (a) $8(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = -8i$;
- (b) $20(\cos 0 + i \cdot \sin 0) = 20$;
- (c) $\cos \pi + i \cdot \sin \pi = -1$;
- (d) $2^{67}(\cos \frac{5\pi}{6} + i \cdot \sin \frac{5\pi}{6}) = -2^{66}\sqrt{3} + 2^{66}i$;
- (e) $2^{611}(\cos \frac{3\pi}{2} + i \cdot \sin \frac{3\pi}{2}) = -2^{611}i$;
- (f) $6^{1526}(\cos \frac{2\pi}{3} + i \cdot \sin \frac{2\pi}{3}) = -3 \cdot 6^{1525} + 3\sqrt{3} \cdot 6^{1525}i$.

4.5. Feladat megoldása.

- (a) $3 = 2(\cos 0 + i \sin 0)$
 $-3 = 2(\cos \pi + i \sin \pi)$
- (b) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$
 $-2i = 2(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2})$
- (c) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i = \cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$
 $-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i = \cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}$
- (d) $-2\sqrt{2} + 2\sqrt{2}i = 4(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})$
 $2\sqrt{2} - 2\sqrt{2}i = 4(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$
- (e) $\sqrt{3} + i = 2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$
 $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$
- (f) $-2 = 2(\cos \pi + i \sin \pi)$
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$
 $1 - \sqrt{3}i = 2(\cos(-\frac{\pi}{3}) + i \sin(-\frac{\pi}{3}))$
- (g) $2i = 2(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$,
 $-\sqrt{3} - i = 2(\cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6})$,
 $\sqrt{3} - i = 2(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6})$.
- (h) $\sqrt{2}\sqrt[6]{2} + \sqrt{2}\sqrt[6]{2}i = 2\sqrt[6]{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$,

- $2\sqrt[6]{2}(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12})$,
- $2\sqrt[6]{2}(\cos \frac{11\pi}{12} + i \sin \frac{11\pi}{12})$,
- (i) $\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}$,
 $\cos \frac{5\pi}{8} + i \sin \frac{5\pi}{8}$,
 $\cos \frac{9\pi}{8} + i \sin \frac{9\pi}{8}$,
 $\cos \frac{13\pi}{8} + i \sin \frac{13\pi}{8}$.
- (j) $2 = 2(\cos 0 + i \sin 0)$,
 $1 + \sqrt{3}i = 2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$,
 $-1 + \sqrt{3}i = 2(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$,
 $-2 = 2(\cos \pi + i \sin \pi)$,
 $-1 - \sqrt{3}i = 2(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$,
 $1 - \sqrt{3}i = 2(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})$,
- (k) $\sqrt[4]{2}(\frac{1}{2} + \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$,
 $\sqrt[4]{2}(-\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$,
 $\sqrt[4]{2}(-\frac{1}{2} - \frac{\sqrt{3}}{2}i) = \sqrt[4]{2}(\cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3})$,
 $\sqrt[4]{2}(\frac{\sqrt{3}}{2} - \frac{1}{2}i) = \sqrt[4]{2}(\cos \frac{11\pi}{6} + i \sin \frac{11\pi}{6})$.

4.6. Feladat megoldása.

- (a) **Harmadik** egységgyökök: $1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
 Primitív harmadik egységgyökök: $-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$.
- (b) **Negyedik** egységgyökök: $1, i, -1, -i$.
 Primitív negyedik egységgyökök: $i, -i$.
- (c) **Hatodik** egységgyökök: $1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$.
 Primitív hatodik egységgyökök: $\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i$.
- (d) **Nyolcadik** egységgyökök: $1, \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -1, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, -i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.
 Primitív nyolcadik egységgyökök: $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i, \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$.

