Some new aspects of islands

Eszter K. Horváth, Szeged

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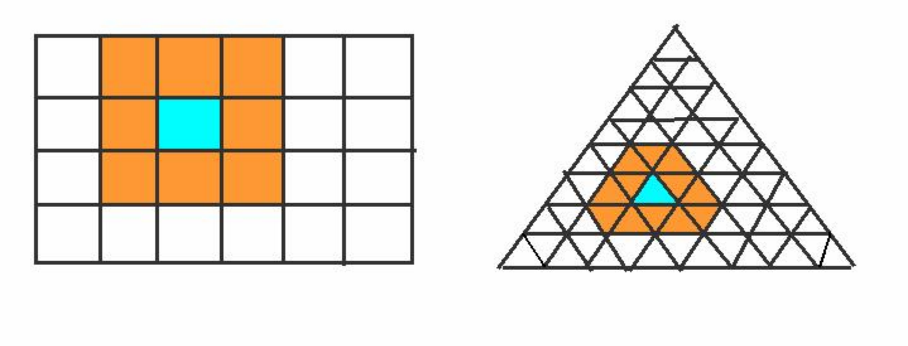
AAA 77, Potsdam

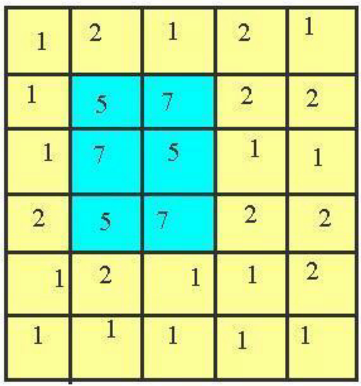
Definition/1

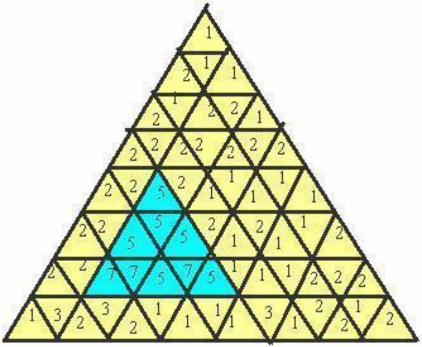
Grid, neighbourhood relation



Definition/2

We call a rectangle/triangle an *island*, if for the cell t, if we denote its height by a_t, then for each cell $\hat{t}$ neighbouring with a cell of the rectange/triangle T, the inequality $a_{\hat{t}} < min\{a_t : t \in T\}$ holds.





History/1

Coding theory

S. Földes and N. M. Singhi: On instantaneous codes, J. of Combinatorics, Information and System Sci., 31 (2006), 317-326.

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Rectangular islands

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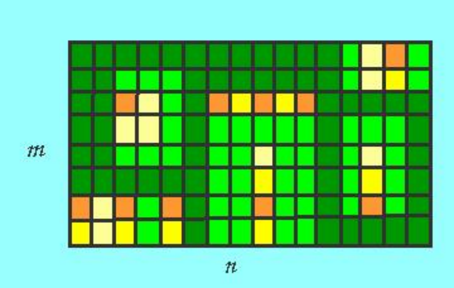
The maximum number of rectangular islands in a $m \times n$ rectangular board on square grid:

$$f(m,n) = [\frac{mn+m+n-1}{2}].$$



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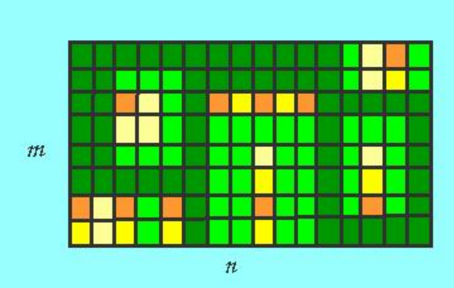

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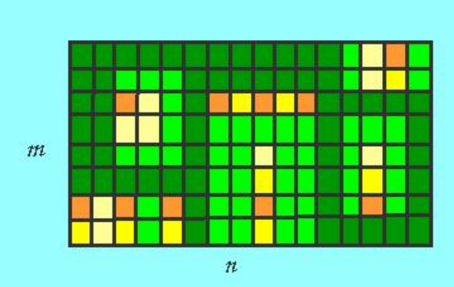

History/3

Rectangular islands in higher dimensions

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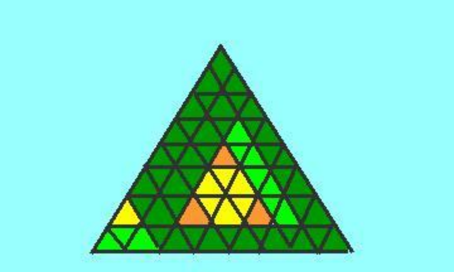
For the maximum number of triangular islands in an equilateral rectangle of side length n, $\frac{n^2+3n}{5} \leq f(n) \leq \frac{3n^2+9n+2}{14}$ holds.



History/4

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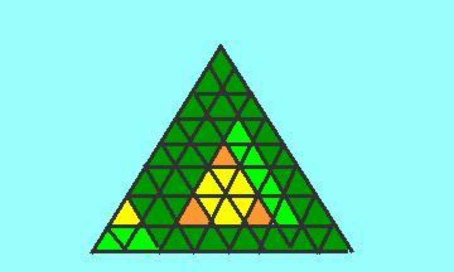

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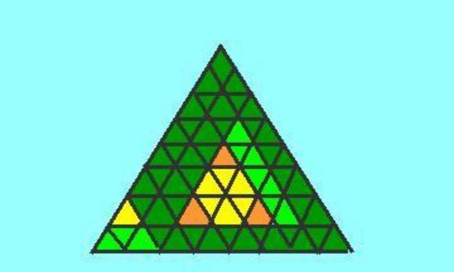

History/5

Square islands (also in higher dimensions)

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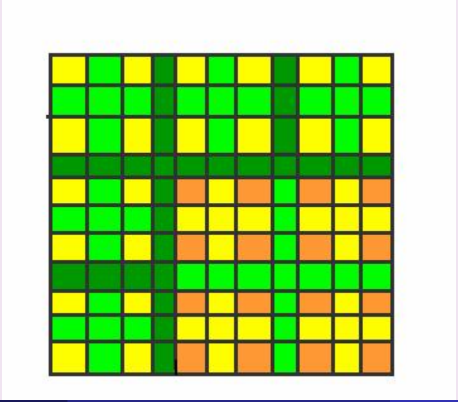

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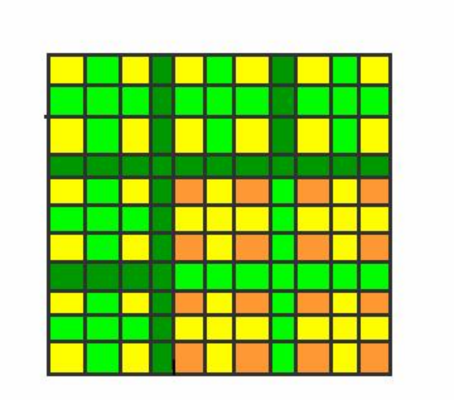

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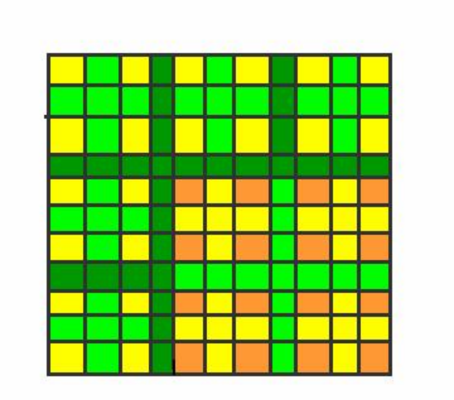

History/6

Some exact formulas

Peninsulas (semi islands) (J. Barát, P. Hajnal, E.K. Horváth):
$p(m, n) = f(m, n) = [(mn + m + n - 1)/2]$.

Cylindric board, rectangular islands (J. Barát, P. Hajnal, E.K. Horváth):
If $n \geq 2$, then $h_1(m, n) = [\frac{(m+1)n}{2}]$.

Cylindric board, cylindric and rectangular islands (J. Barát, P. Hajnal, E.K. Horváth):
If $n \geq 2$, then $h_2(m, n) = [\frac{(m+1)n}{2}] + [\frac{(m-1)}{2}]$.

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History/7

Further results on rectangular islands

Zs. Lengvárszky: The minimum cardinality of maximal systems of rectangular islands, European Journal of Combinatorics, available online 16 April 2008.

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Islands in Boolean algebras, i.e. in hypercubes /1

Joint work with Péter Hajnal

The board consists of all vertices of a hypercube, i.e. the elements of a Boolean algebra $BA = \{0, 1\}^n$.

We consider two cells neighbouring if their Hamming distance is 1.

We denote the maximum number of islands in $BA = \{0, 1\}^n$ by $b(n)$.

Theorem 1
$b(n) = 1 + 2^{n-1}$.

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Proof:

$b(n) \geq 1 + 2^{n-1}$ because we can put one-cell islands to all vertices with an odd number of 1-s.

We show $b(n) \leq 1 + 2^{n-1}$ by induction on n. For $n = 0, 1$ the statement is easy to check.
For $n \geq 2$, we cut the hypercube into two half-hypercubes, of size 2^{n-1}. If one of them is an island, then the other cannot contain island.
If neither of them is an island, then by the induction hypothesis, in both half-hypercubes, the maximum cardinality of a system of islands is at most 2^{n-2}.

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Rectangular fuzzy relations/1

Joint work with Branimir Šešelja and Andreja Tepavčević

Let A and B nonempty sets and L a lattice. Then a *fuzzy relation* ρ is a mapping from $A \times B$ to L.

For every $p \in L$, cut relation is an ordinary relation ρ_p on $A \times B$ defined by

$$(x, y) \in \rho_p \text{ if and only if } \rho(x, y) \geq p.$$

We consider special lattice valued fuzzy relations:
The set $\{1, 2, ..., m\} \times \{1, 2, ..., n\}$, $m, n \in \mathbb{N}$, is called *a* table of size $m \times n$. Such a table is the domain of a fuzzy relation Γ:

$$\Gamma : \{1, 2, ..., m\} \times \{1, 2, ..., n\} \to \mathbb{N}.$$

The co-domain is the lattice $(\mathbb{N}, \leq)$, where $\mathbb{N}$ is the set of natural numbers under the usual ordering $\leq$ and suprema and infima are max and min, respectively.

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Rectangular fuzzy relations/2

We say that two rectangles $\{\alpha, ..., \beta\} \times \{\gamma, ..., \delta\}$ and $\{\alpha_1, ..., \beta_1\} \times \{\gamma_1, ..., \delta_1\}$ are *distant* if they are disjoint and for every two cells, namely (a, b) from the first rectangle and (c, d) from the second, we have $(a - c)^2 + (b - d)^2 \geq 4$.

Fuzzy relation Γ is called *rectangular* if for every $p \in \mathbb{N}$, every nonempty p-cut of Γ is a union of distant rectangles.

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Rectangular fuzzy relations/3

5	5	3	5	5
4	4	2	4	4
2	2	1	2	2

$S_1 = \{1,2,3,4,5\} \times \{1,2,3\}$,
$S_2 = \{1,2,3,4,5\} \times \{1,2,3\} \setminus \{(3,1)\}$,
$S_3 = \{(1,2),(1,3),(2,2),(2,3),(3,3),(4,2),(4,3),(5,2),(5,3)\}$,
$S_4 = \{(1,2),(1,3),(2,2),(2,3),(4,2),(4,3),(5,2),(5,3)\}$ and
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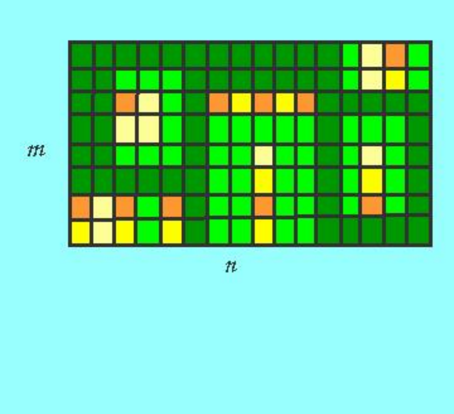
Rectangular fuzzy relations/4

Theorem 2

For every fuzzy relation $\Gamma : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \rightarrow \mathbb{N}$, there is a rectangular fuzzy relation $\Phi : \{1, 2, ..., n\} \times \{1, 2, ..., m\} \rightarrow \mathbb{N}$, having the same islands.



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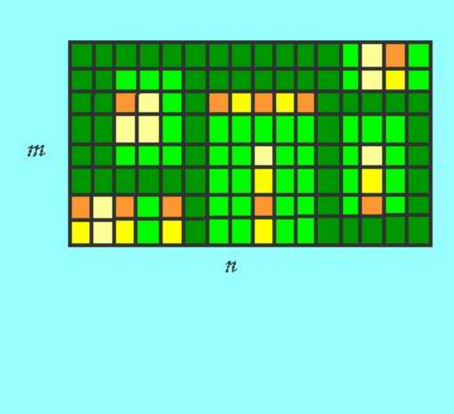

Rectangular fuzzy relations/5

Theorem 3

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If a fuzzy rectangular relation Ψ has the property that each island appears exactly in one cut, then we call it *standard fuzzy rectangular relation.* We denote by $\Lambda(m, n)$ the maximum number of different p-cuts of a standard fuzzy rectangular relation on the rectangular table of size $m \times n$.

Theorem 4

$\Lambda(m, n) = m + n - 1.$

Rectangular fuzzy relations/5

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We have further results, e.g.
Characterisation Theorem for rectangular fuzzy relations,
Constructing Algorithm which constructs rectangular fuzzy relation for a given arbitrary fuzzy relation with the same islands.

Probably we present more details on next conferences.

Thanks for the attention.

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