Horn belief contraction: remainders, envelopes, complexity

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- belief contraction
- Horn formulas
- Horn belief contraction
- open problems

Belief change

- how to handle knowledge which is incomplete, imperfect and changing?
- revision: how to incorporate new information which contradicts previous knowledge?
  - scientific theory, knowledge base
- contraction: how to delete knowledge we do not believe anymore to be true?
- (Levi identity) revision with φ : contract $\neg\varphi$, add φ
- Alchourrón, Makinson, Gärdenfors (1985)
- Fermé, Hansson (2011): *AGM 25 years*

Belief contraction: an example

- knowledge base:
  - $\{a \to b,\ b \to c\}$
- consequence:
  - $a \to c$
- contract the consequence!
  - new knowledge base, version I:
    - $a \to b$
  - new knowledge base, version II:
    - $a, c \to b$
    - $b \to c$

Belief contraction: basic notions

- K: theory in full propositional logic (belief set), set of formulas closed under logical consequence
- φ: consequence of K to be contracted
- $\dot{-}$: contraction operator
- $K \dot{-} \varphi$: result of the contraction

Belief contraction: (basic) AGM postulates

- (closure) $K \dot{-} \varphi$ is a belief set
- (inclusion) $K \dot{-} \varphi \subseteq K$
- (vacuity) if $\varphi \notin K$ then $K \dot{-} \varphi = K$
- (success) if φ is not a tautology then $\varphi \notin K \dot{-} \varphi$
- (extensionality) if $\varphi \equiv \psi$ then $K \dot{-} \varphi = K \dot{-} \psi$
- (recovery) $K \subseteq Cn((K \dot{-} \varphi) \cup \{\varphi\})$

Partial meet contraction

- remainder of K with respect to φ: maximal subtheory of K not implying φ; add a single counter-model of φ to K
- $K \perp \varphi$: family of all remainders
- selection: $\gamma(\varphi) \subseteq K \perp \varphi$
- partial meet contraction:

$$K \dot{-} \varphi = \bigcap_{X \in \gamma(\varphi)} X$$

Theorem

(AGM) A contraction operator satisfies the AGM postulates iff it is a partial meet contraction.

Computational issues

- reasoning in propositional logic is computationally hard
- belief change is even harder: Eiter, Gottlob (1992), Nebel (1998), Liberatore (2000)
- consider belief change in computationally tractable fragments of propositional logic
- Flouris, Plexousakis, Antoniou (2004): belief contraction in arbitrary logics

Horn Formulas, entailment

- Horn clause: at most one unnegated variable, e.g. $C = \overline{a} \vee \overline{b} \vee c$, written as $a, b \rightarrow c$, $Body(C) = \{a, b\}$, $Head(C) = c$
- definite clause: exactly one unnegated variable
- (definite) formula: conjunction of (definite) Horn clauses
- Horn function: representable by a Horn formula
- entailment: $(a, b \rightarrow c) \wedge (c \rightarrow d) \models (a, b \rightarrow d)$
- implicate: $K \models C$
- prime implicate: no subclause is an implicate
- forward chaining - efficient

Horn logic in AI

- reasoning in Horn logic is computationally easy
- equivalent formalisms: closures, lattices, functional dependencies
- Horn logic is the framework for many applications, it is natural for human reasoning
- Poole - Mackworth: *Artificial Intelligence: Foundations of Computational Agents*, 2010:
  - ‘uses rational computational agents and Horn clause logic as unifying threads in this vast field’

Horn belief contraction

- Booth, Meyer, Varzinczak (2009)
- Booth, Meyer, Varzinczak, Wasserman (2010, 2011)
- Creignou, Papini, Pichler, Woltran (2012) + other fragments
- Delgrande (2008)
- Delgrande, Peppas (2011): revision
- Delgrande, Wassermann (2010, 2011)
- Fotinopoulos, Papadopoulos (2009)
- Langlois, Sloan, Szörényi, T. (2008)
- Ribeiro (2010)
- Wu, Zhang, Zhang (2011)
- Zhuang, Pagnucco (2010, 2010, 2011, 2012)

Our results

- positive computational result on finding remainders represented by counter-model
- negative computational result on the Horn formula size of the contraction

Horn formulas and intersection closure

- intersection of two vectors: $(1,0,1) \cap (0,1,1) = (0,0,1)$
- a Boolean function f is closed under intersection if $f(x) = f(y) = 1$ implies $f(x \cap y) = 1$ (or: $T(f)$ is closed under intersection)

Theorem

(McKinsey, 1943)
A Boolean function is Horn if and only if it is closed under intersection.

- $x \oplus y$ is not Horn: $1 \oplus 0 = 1,\ 0 \oplus 1 = 1,\ 0 \oplus 0 = 0.$

Horn envelope or Horn LUB

- $Env(\psi)$ is the conjunction of all Horn implicates of ψ
- $T(Env(\psi))$ is the closure of $T(\psi)$ under intersection
- $Env(x \oplus y) = \bar{x} \vee \bar{y}$

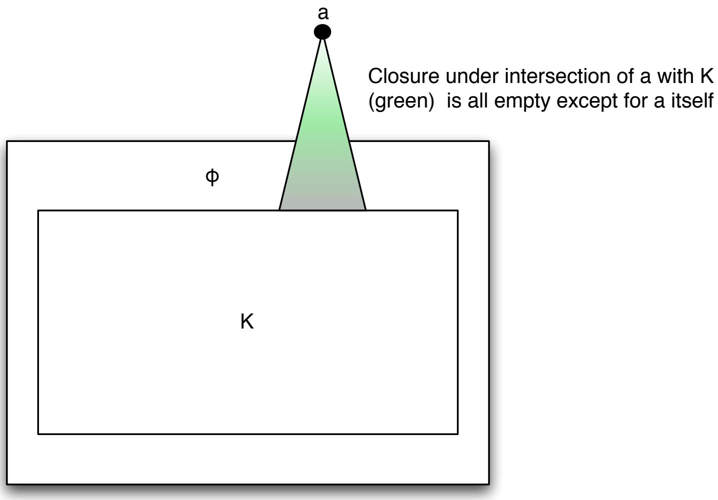
Remainders for Horn formulas

- Horn belief set K, consequence φ to be contracted
- remainder (reminder): a maximal subset of K which does not imply φ
- in terms of truth assignments: a minimal extension of $T(K)$ not contained in $T(\varphi)$; not all counter-models work!
- $Env(K \vee C_a) : T(Env(K \vee C_a)) \wedge F(\varphi) = \{a\}$
- $a = (1, 0, 1) : \; C_a = x \wedge \bar{y} \wedge z$

Question

- which counter-models of φ can be added as a single point extension?

Picture of what we need



Example

- variables a, b, c
- $K = a \wedge b, \ \varphi = a \wedge b$
- models: $110, 111$
- add counter-model 000: new formula is

$$(a \rightarrow b) \wedge (b \rightarrow a) \wedge (c \rightarrow a)$$

- add counter-model 001: envelope includes 000, new formula for envelope is

$$(a \rightarrow b) \wedge (b \rightarrow a)$$

  not maximal

Closure, body - building formula

Definition

(Closure) X: set of variables

$$Cl_K(X) = \{v : K \models (X \to v)\}$$

Definition

(Body-building formula)

$$K^\varphi = \bigwedge_{C \in \varphi} \bigwedge_{v \notin Cl_K(Body(C))} (Body(C), v \to Head(C))$$

Characterization of remainders

Theorem
$Env(K \vee C_a)$ is a remainder iff a satisfies K^{φ} and falsifies φ.

- new characterization of quasi-closed sets for closures

Corollary
Remainders represented by their 'generating' truth assignments can be listed with polynomial delay.

Example continued

- $K = a \wedge b, \ \varphi = a \wedge b$
- $Cl_K(\emptyset) = \{a, b\}$
- $K^\varphi = (c \to a) \wedge (c \to b)$
- remainders: $000, 010, 100,$ but not $001, 011, 101$

Partial meet contractions

- a partial meet contraction is an intersection of remainders
- $$Env\left(K \vee \bigvee_{a \in A} C_a\right)$$
  where $A \subseteq T(K^{\varphi}) \wedge F(\varphi)$
- maxichoice: singleton, full meet: equality

Question

- can the new belief sets can be computed efficiently?
- Eiter, Makino (2008): negative results for envelopes of Horn disjunctions

Formula that blows up after contraction

- belief set K_n

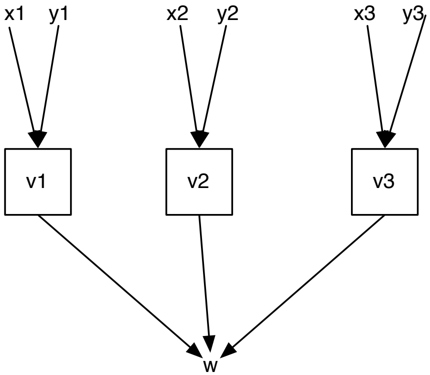
$$x_i \to v_i, \quad y_i \to v_i, \;\; 1 \leq i \leq n$$

$$v_1, \ldots, v_n \to w$$

- consequence φ_n to be contracted

$$x_1, y_1, \ldots, x_n, y_n, w \to v_1$$

Formula that blows up after contraction ($n = 3$)



A blow-up result

Theorem

- *Every Horn formula representation of the full meet contraction of φ_n from K_n has at least 2^n clauses.*
- *For every $\epsilon > 0$ and for almost all maxichoice contractions of φ_n from K_n, every Horn formula representation has at least $2^{((1/2)-\epsilon)n}$ clauses.*
- *For almost all partial meet contractions of φ_n from K_n, every Horn formula representation has at least 2^n clauses.*
- also applies to weak remainders

Lower bound lemma

- let A be a set of truth assignments such every u-variable and w is always set to 1, v_1 is always set to 0, and there are altogether k variables which are set to 0 in some $a \in A$

Theorem

Every Horn formula representing

$$Env\left(K \vee \bigvee_{a \in A} C_a\right)$$

contains at least 2^k clauses.

Open problems

- are there examples where every maxichoice/partial meet contraction is large?
- infra-remainders (Booth et al.), epistemic entrenchment (Zhuang, Pagnucco - Horn cores), semantic approaches
- Horn belief revision (Delgrande, Peppas)
- integration of reasoning, revision and learning into an efficient framework for developing knowledge bases (belief revision + theory revision?)