Abstract

Béla Csaba

University of Szeged

Packing

Let G and H be two graphs on n vertices. Let $\Theta(G) = \max_{xy \in E(G)}\{d(x) + d(y)\}$, this is the Ore-degree of G. A theorem of Kostochka and Yu states that if $\Theta(G)\Delta(H) < n$ (here $\Delta(H)$ is the maximum degree of H), then there is an edge-disjoint placement of G and H into K_n. We show a strengthening of the above result when $\Theta(G) \leq 5$.

Michelle Delcourt

University of Illinois at Urbana-Champaign

A Discrete Approach to Bernoulli Convolutions

In this talk we consider a discrete version of the Bernoulli convolution problem traditionally studied via functional analysis. We develop an algorithm which bounds the Bernoulli sequences and gives a significant improvement on the best known bound.

Joint work with Neil Calkin, Jula Davis, Zebediah Engberg, Jobby Jacob, and Kevin James.

Péter Hajnal

University of Szeged

Combinatorics of poly-Bernoulli numbers

The $B_n^{(k)}$ poly-Bernoulli numbers — a natural generalization of classical Bernoulli numbers ($B_n = B_n^{(1)}$) — were introduced by Kaneko in 1997. When the parameter k is negative then $B_n^{(k)}$ is a natural number. Brewbaker was the first to give combinatorial interpretation of these numbers. He proved that $B_n^{(-k)}$ counts the, so called, lonesum 0-1 matrices of size $n \times k$. Several other interpretations were pointed out. We survey these and give new ones. Our new interpretation, for example, gives a transparent, combinatorial explanation of Kaneko's recursive formula for poly-Bernoulli numbers.

Joint work with Bea Bényi.

Ping Hu

University of Illinois at Urbana-Champaign

Mantel's Theorem for Random Hypergraphs

A cornerstone result in extremal graph theory is Mantel's Theorem, which states that every maximum triangle-free subgraph of K_n is bipartite. A sparse version of Mantel's Theorem is that, for sufficiently large p, every maximum triangle-free subgraph of $G(n,p)$ is w.h.p. bipartite. Recently, DeMarco and Kahn proved this for $p > K\sqrt{\log n/n}$ for some constant K, and apart from the value of the constant, this bound is best possible. We study an extremal problem of this type in random hypergraphs. Denote by F_5 the 3-uniform hypergraph with vertex set $\{a,b,c,d,e\}$ and edge set $\{abc, ade, bde\}$. Frankl and Furedi proved that the maximum 3-uniform hypergraph on n vertices containing no copy of F_5 is tripartite for $n > 3000$. A natural question is that for what p is every maximum F_5-free subhypergraph of $G^3(n,p)$ w.h.p. tripartite. We show this holds for $p > K\log n/n$ for some constant K and does not hold if $p = 0.1\sqrt{\log n}/n$.

Joint work with József Balogh, Jane Butterfield and John Lenz.

Mladen Kovačević

University of Novi Sad

Error-correcting codes in spaces of multisets

Abstract: We introduce the notion of multiset codes and argue that these constructions are appropriate for error-correction in channels that randomly permute the transmitted sequence of symbols. The motivation for studying such channels comes from the effect of out of order delivery of packets in some types of packet networks. Some of the basic properties of multiset codes are established, among which their equivalence to integer codes under the Manhattan metric. Perfect multiset codes are enumerated for all parameters for which they exist, and nonexistence is proven in the remaining cases.

Joint work with Dejan Vukobratović.

Gábor Kun

Alfréd Rényi Institute of Mathematics

Approximation of property (T) groups and expanders

We say that a sequence of d-regular graphs G_n converges to the Cayley graph of a d-generated group if for every $\epsilon > 0$ and r, if n is large enough, then at least $(1-\epsilon)$ portion of the r-balls in G_n is isomorphic to the r-ball in the Cayley graph. We solve Bowen's question that if G_n coverges to the Cayley graph of a Kazhdan property (T) group, then every G_n is a disjoint union of expanders $\pm o(n)$ edges. (Groups with property (T) and expanders are connected at many points starting with Margulis' expander constructions.) We give

combinatorial/spectral/ergodic characterisation of such graph sequences.

Hong Liu

University of Illinois at Urbana-Champaign

On the number of K_4-saturating edges

Let G be a K_4-free graph, an edge in its complement is a K_4-*saturating* edge if the addition of this edge to G creates a copy of K_4. Erdős and Tuza conjectured that for any n-vertex K_4-free graph G with $\lfloor n^2/4 \rfloor + 1$ edges, one can find at least $(1+o(1))\frac{n^2}{16}$ K_4-saturating edges. We construct a graph with only $\frac{2n^2}{33}$ K_4-saturating edges. Furthermore, we prove that it is best possible, i.e., one can always find at least $(1+o(1))\frac{2n^2}{33}$ K_4-saturating edges in an n-vertex K_4-free graph with $\lfloor n^2/4 \rfloor + 1$ edges.

Joint work with József Balogh.

András London

University of Szeged

Slowing down and accelerating games

The in the usual setup of positional Maker-Breaker games the players take the elements of a hypergraph and Maker wins upon getting a whole edge, otherwise Breaker is the winner. Slowing down means that the players may change the distribution of their move; instead of placing a stone, they may use two smaller ones, or even a unit of “sand”. We show some Erdős-Selfridge type theorems, and solve the recycled version of Box game with sand.

In graph games the players take the edges of some graph and want to get (or prevent) an object like long path, spanning tree, large clique etc. Following the ideas of Krivelevich and Frieze, we investigate a spanning tree game in which Maker’s subgraph should be connected in every steps. The Maker’s win graph can be described, and in fact their structure is quite restrictive. While the probabilistic intuition, or “Erdős paradigm” suggest that for $G(n, 1/2)$ Maker w.h.p. wins, it is a Breaker win for almost surely. We show that it changes with accelerating the game, that is if both player may take two edges in each steps.

Mirjana Mikalački

University of Novi Sad

Avoider-Enforcer star games

We study $(1 : b)$ Avoider-Enforcer games played on the edge set of the complete graph on n vertices. For every constant $k > 2$ we analyze the k-star game, where Avoider tries to avoid claiming k edges incident to the same vertex. We analyze both versions of Avoider-Enforcer games – the strict and the monotone – and for each provide explicit winning strategies for both players.

This is joint work with Andrzej Grzesik, Zoltán Lóránt Nagy, Alon Naor, Balázs Patkós and Fiona Skerman.

Rajko Nenadov

University of Novi Sad, ETH Zürich

Universality of random graphs

In this talk we present few simple partitioning lemmas for graphs. As applications, we show how to derive new bounds for the edge probability p for which a typical random graph $G = G(n,p)$ is universal with respect to some families of graphs. In particular, we improve the result of Johannsen, Krivelevich and Samotij for the universality of $G(n,p)$ with respect to the family of spanning bounded degree forests. In addition, we improve a result of Dellamonica, Kohayakawa, Rödl and Ruciński for the universality of $G(n,p)$ with respect to the family of bounded degree graphs with density d much smaller than the maximum degree, where the density of a graph H is defined as the maximum average degree over all subgraphs of H.

Joint work with Asaf Ferber and Ueli Peter.

László Ozsvárt

University of Szeged

It is a common work with Ivan Vrabski and András Pluhár.

Balázs Patkós

Alfréd Rényi Institute of Mathematics

Two Combinatorial Search Problems

The talk will consist of two independent parts. In the first part, we consider the following q-analog of the basic combinatorial search problem: let q be a prime power and GF(q) the finite field of q elements. Let V denote an n-dimensional vector space over GF(q) and let $\mathbf{v}$ be an unknown 1-dimensional subspace of V. We will be interested in determining the minimum number of queries that is needed to find $\mathbf{v}$ provided all queries are subspaces of V and the answer to a query U is YES if $\mathbf{v} \leqslant U$ and NO if $\mathbf{v} \not\leqslant U$. This number will be denoted by $A(n,q)$ in the adaptive case (when for each queries answers are obtained immediately and later queries might depend on previous answers) and $M(n,q)$ in the non-adaptive case (when all queries must be made in advance)

In the case $n = 3$ we prove $2q - 1 = A(3, q) < M(3, q)$ if q is large enough. While for general values of n and q we establish the bounds

$$n \log q \leq A(n, q) \leq (1 + o(1))nq$$

and

$$(1 - o(1))nq \leq M(n, q) \leq 2nq,$$

provided both n and q tend to infinity. The results of this part of the talk are joint with Tamás Héger, Marcella Takáts.

In the second part of the talk, we address the following problem: a ball $B_r(v)$ of radius r at vertex v in a graph G is the set of vertices in G that have distance at most r from v. For a subset X of vertices of G, we have $B_r(X) = \cup_{v \in X} B_r(v)$. An $(r, \leq l)$-*identifying code* of the graph G is a set C of vertices such that for every pair of distinct subsets $X, Y \subset V(G)$ with $|X|, |Y| \leq l$ the sets $B_r(X) \cap C$ and $B_r(Y) \cap C$ are different and non-empty. Determining the minimum size $i_r^{(l)}(G)$ of an identifying code in G is a much studied problem which also has an adaptive variant when we are allowed to ask balls $B_r(c)$ one by one. The minimum number of balls needed is denoted by $a_r^{(l)}(G)$. However, note that both in the original and in the adaptive case to be able to succed we need that $B_r(X) \neq B_r(Y)$ hold for all pairs of subsets of vertices with $|X|, |Y| \leq l$.

At the 5th Emléktábla Workshop, Gyula Katona suggested that balls of radius *at most* r should be allowed as queries. As balls of radius 0 consist only of the centers, this would make the definition valid for all graphs. We will be only interested in the $l = 1$ case and the minimum number of queries needed to determine the unknown vertex in this modified settings will be denoted by $M(G, r)$ and $A(G, r)$ in the non-adaptive and the adaptive case, respectively. Thus clearly we have $M(G, r) \leq i_r^{(1)}(G)$ and $A(G, r) \leq a_r^{(1)}(G)$. We consider the problem in this new setting for hypercubes, the Erdős-Rényi random graph model and graphs of bounded maximum degree. Results of the second part are joint with Younjin Kim, Mohit Kumbhat, Zoltán Lóránt Nagy, Alexey Pokrovskiy and Máté Vizer.

András Pluhár

University of Szeged

Slowing down and accelerating games

The in the usual setup of positional Maker-Breaker games the players take the elements of a hypergraph and Maker wins upon getting a whole edge, otherwise Breaker is the winner. Slowing down means that the players may change the distribution of their move; instead of placing a stone, they may use two smaller ones, or even a unit of "sand". We show some Erdős-Selfridge type theorems, and solve the recycled version of Box game with sand.

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It is a common work with Ivan Vrabski and András London.

Marko Savić

University of Novi Sad

Linear time algorithm for optimal feed-link placement

Given a polygon representing a transportation network together with a point p in its interior, we aim to extend the network by inserting a line segment, called a feed-link, which connects p to the boundary of the polygon. Once a feed link is fixed, the geometric dilation of some point q on the boundary is the ratio between the length of the shortest path from p to q through the extended network, and their Euclidean distance. The utility of a feed-link is inversely proportional to the maximal dilation over all boundary points. We give a linear time algorithm for computing the feed-link with the minimum overall dilation, thus improving upon the previously known algorithm of complexity that is roughly $O(n \log n)$.

Joint work with Miloš Stojaković.

Maryam Sharifzadeh

University of Illinois at Urbana-Champaign

Subdivisions of a large clique in C_6-free graphs

Mader conjectured that every C_4-free graph has a subdivision of a clique of order linear in its average degree. We show that every C_6-free graph has such a subdivision of a large clique.

We also prove the dense case of Mader's conjecture in a stronger sense, i.e. for every c, there is a c' such that every C_4-free graph with average degree $cn^{1/2}$ has a subdivision of a clique K_ℓ with $\ell = \lfloor c'n^{1/2} \rfloor$ where every edge is subdivided exactly 3 times.

Joint work with Jozsef Balogh and Hong Liu.

Miloš Stojaković

University of Novi Sad

Maker-Breaker H-game on random graphs

We will look at various properties of the Maker-Breaker H-game played on the edge set of the random graph $G(n, p)$. In this game two players, Maker and Breaker, alternately claim unclaimed edges of $G(n, p)$, until all the edges are claimed. Maker wins if he claims all the edges of a copy of a fixed graph H; Breaker wins otherwise.

Joint work with Tibor Szabó, Tobias Müller, Rajko Nenadov and Angelika Steger.

Géza Tóth

Alfréd Rényi Institute of Mathematics

Saturated topological graphs

A *simple topological graph* G is a graph drawn in the plane so that any pair of edges have at most one point in common, which is either an endpoint or a proper crossing. G is called *saturated* if no further edge can be added without violating this condition. We construct saturated simple topological graphs with n vertices and $O(n)$ edges. These constructions are nearly optimal: it is shown that every saturated simple topological graph with n vertices has at least cn edges for some constant $c \geq 1.5$.

For any fixed $k \geq 1$, we prove a similar statement for *k-simple topological graphs*, that is, for graphs drawn in the plane so that any two edges have at most k points in common. We consider many other generalizations and related problems.

Joint work with Jan Kynčl, Radoš Radoičić, and János Pach.

Balázs Udvari

University of Szeged

Abstract

Jan Volec

University of Warwick

Subcubic triangle-free graphs have fractional chromatic number at most 14/5

We prove that every subcubic triangle-free graph has fractional chromatic number at most 14/5, thus confirming a conjecture of Heckman and Thomas.

This is a joint work with Zdeněk Dvořák and Jean-Sébastien Sereni.