Optimization Methods practice session, for Master students

Exercises: KKT

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Use Karush—Kuhn–Tucker theorem to solve the following optimization problems:

1.
$$\begin{aligned} \min_{x_1,x_2,x_3} \quad & x_1^2 + x_2^2 + x_3^2 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 \leq -18. \end{aligned}$$

2.
$$\begin{aligned} \min_{x_1,x_2} \quad & x_1^2 + x_2^2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 5, \\ & x_1 + 2x_2 = 4, \\ & x_1, x_2 \geq 0. \end{aligned}$$

3. Using the KKT conditions find the closest point to $(0, 0)$ in the set defined by

$$M = \{x \in \mathbb{R}^2 : x_1 + x_2 \geq 4, 2x_1 + x_2 \geq 5\}.$$

   Can several points (solutions) exist?

4. Consider the problem

$$\begin{aligned} \min_{x_1,x_2,\ldots,x_n} \quad & \sum_{i=1}^{n} \log(\alpha_i + x_i) \\ \text{s.t.} \quad & \sum_{i=1}^{n} x_i, \\ & x_1, x_2, \ldots, x_n \geq 0, \end{aligned}$$

   where α_i's are given positive parameters. Using the KKT conditions find the solutions.