

**Optimization Methods practice session, for Master students**

**Exercises: Elementary optimization problems**

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Week 1.

1. **Exercise.** *Given a triangle, inscribe a parallelogram of maximum area in it such that two given sides of the triangle form two adjacent sides of the parallelogram.*
2. **Exercise.** *Given a circle. Inscribe a rectangle of maximum area in it.*
3. **Exercise.** *Given a fence of length  $L$ . Enclose a rectangle of maximum area with it.  
What is the answer if the enclosure must be designed next to a long wall, so that the wall can also form part of the boundary of our area (for which we do not need to use the fence)?*
4. **Exercise.** *Given a triangle. Choose one point on each of its sides such that the perimeter of the triangle determined by the three chosen points is minimal.*
5. **Exercise.** *A cowboy on the prairie, before heading home, wants to water his horse at a straight river. The rider starts from the same side of the straight river as his house is on. The cowboy rides at a constant speed  $v_0$ , and the time taken for watering is also known. Which path should he take to get home as soon as possible?*
6. **Exercise.** *(continued) Solve the previous problem if the river describes a large circular arc.*
7. **Exercise.** *(continued) Solve the original problem assuming that the cowboy skips the watering, BUT on the way home, the prairie consists of areas with tall grass and short grass. In the unmaintained area he rides at speed  $v_0$ , while in the maintained area he rides at speed  $v_1 (> v_0)$ . His house is in the maintained area, which is separated from the wild area by a straight line. He starts from the tall grass area. Which path should he take to get home as soon as possible?*
8. **Exercise.** *The two banks of a river form two parallel lines. On each side there is a town which we want to connect with a road and a bridge perpendicular to the banks at the river. How should we plan the road and the bridge to make the connecting route as short as possible?*
9. **Exercise.** *([Kepler]) Inscribe a cylinder of maximal volume in a given sphere.*
10. **Exercise.** *([Tartaglia]) Partition 8 into the sum of two non-negative numbers so that the product of these two numbers and their difference (a three-factor product) is as large as possible.*