Lempel-Ziv-Welch algorithm

Peter Hajnal

Bolyai Institute, University of Szeged, Hungary

2023 fall

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H-tree and prefix-free encoding of characters

There is T rooted, binary, plane (0/1 labels at the two edges going to the two children) tree, and a bijective map between Σ and the leaves of T. The root-$\ell(b)$ path defines the code of the character "b" ($ell(b)$ is the leaf matched to the character b):

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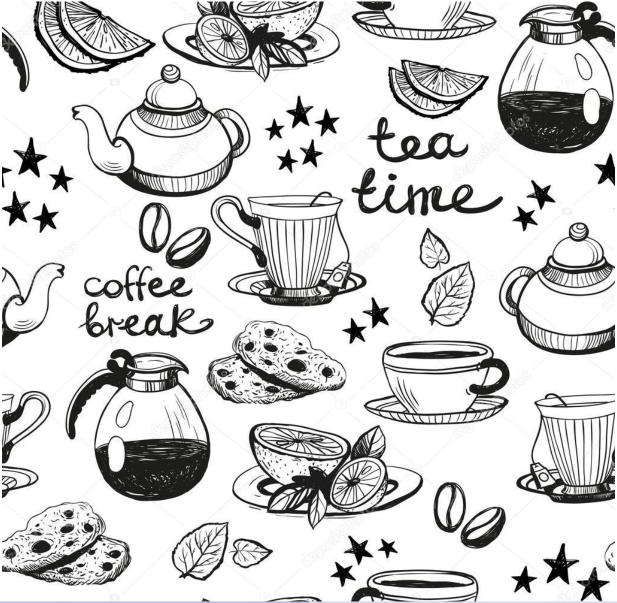
Break



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Example

We assume $\ell = 12$. The ASCII code of the letter 'a' is $97 \equiv 110\ 0001$. In the dictionary its code is $97 \equiv 0000\ 0110\ 0001$.

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`Stop:` If we processed the whole set we send "129".

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STOP	129	STOP	129
ma	130	ma	130
am	131	a?	131

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am	131	am	131
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sender —— 128 109 97 130 32 ——→ receiver

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am	131	am	131
ma␣	132	ma␣	132
␣m	133	␣?	133

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am	131	am	131
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␣m	133	␣m	133
ma␣m	134	ma␣?	134

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Before the whole text is encoded the receiver dictionary is the same as the sender dictionary except the last line, where the word''s last character is unknown.

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Corollary

After obtaining a new part of the code the receiver side can make up for the disadvantage in the previous dictionary.

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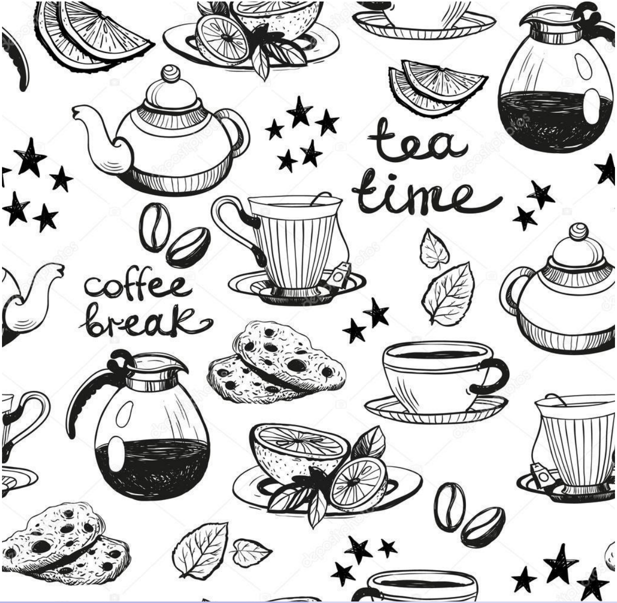

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The solution is a simple modification:

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This is a problem. If we set ℓ too large, then the final dictionary will be short compared to the possibility. If we set ℓ too small, then the dictionary might be full very soon.

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`The NEW extending the dictionary:` We add the word w^+ with the first available bit sequence in the dictionary. If the dictionary is full, then $\ell \leftarrow \ell + 1$. Available bit sequences will appear, the dictionary extension is possible.

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Proof: Easy. The dictionary on the receiver side has the same number of lines. The timing of the incrementation of the length depends on the number of lines.

This is the end!

Thank you for your attention!