BSc Mathematics for Computer Scientists 2: I. Foundations

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Goal of the first lecture

- You come from different parts of the world.

- You studied according to different curricula.

- Your secondary school leaving exams may have been completely different.

- The goal of the first lecture: to summarize what I assume to be known from the Hungarian secondary school mathematics curriculum.

IMPORTANT

If something is not clear, please indicate it: request a consultation, ask questions during the practical sessions.

Numbers: Natural numbers

- Natural numbers: the result of counting.

$$\mathbb{N} = \{0, 1, 2, \ldots, 2026, \ldots, \text{googol}, \ldots\}$$

- Different encodings: $2026 = 11111101001_2 = \text{MMXXVI} = \ldots$
- Basic operations: $+$, $\cdot$.
- $x + 5 = 3$ cannot be solved in $\mathbb{N}$.

Numbers: Integers

- Integers / signed numbers.

$$\mathbb{Z} = \{\dots, -2026, \dots, 0, 1, 2, \dots, 2026, \dots\}$$

- Basic operations: $+$, $\cdot$, $-$.

- $x + a = b$ is always solvable in $\mathbb{Z}$ and the solution is unique $\rightarrow b - a$.

- $5 \cdot x = 3$ cannot be solved in $\mathbb{Z}$. $\rightarrow$ number theory

Numbers: Rational numbers

- Rational numbers:

$$\mathbb{Q} = \{0, 1, -1, \frac{1}{2}, -\frac{1}{2}, 2, -2, \frac{1}{3}, -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \ldots\}$$

- They can be arranged into an infinite sequence.

- The same number, infinitely many different "names":

$$\text{half}, \frac{1}{2}, \frac{5}{10}, 0.5, 50\%, \frac{-1}{-2}, \frac{1013}{2026}, \ldots$$

- $ax = b$ is always solvable in $\mathbb{Q}$ IF $a \neq 0$ and the solution is unique $\rightarrow \frac{b}{a}$.

- $x^2 = 2$ cannot be solved in $\mathbb{Q}$.

Numbers: Real numbers

- Real numbers:

$$\mathbb{R} \ni -1, 0, 1, 2026, \frac{1}{2026}, -\frac{12}{34}, \sqrt{2}, \pi, e, c \text{ (Euler constant)}$$

- Real numbers concretely: infinite decimal expansions.

- They cannot be arranged into a single sequence ("there are too many").

- $x^2 = -1$ cannot be solved in $\mathbb{R}$.

- THE MOST IMPORTANT example. If during the course the word "number" is used, it should be understood as "real number".

Numbers: Complex numbers

- Complex numbers:

$$\mathbb{C} \ni \pi+2\sqrt{-1}, i=0+1\sqrt{-1}, -i=0+(-1)\sqrt{-1}, a+bi (a,b\in\mathbb{R}).$$

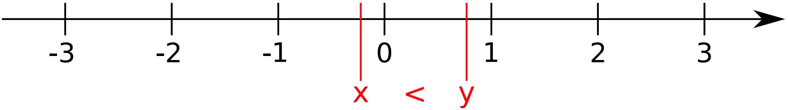
- Basic operations can be defined.
- Every polynomial has a root in $\mathbb{C}$.
- If you know them and can calculate with them, that is very good. In this course, we do not use them.

Numbers: Real numbers: Basic operations

• Addition, subtraction, multiplication, division (by a NON-0 number).

• The precise description is technical. We accept that they can be carried out.

• They cannot be arranged into an infinite sequence, but they can be associated with the points of a line:



(Source: Wikipedia)

- The real numbers are ordered.

Definition

$\alpha \in \mathbb{R}$ is positive $\Leftrightarrow$ $\alpha > 0$.

$\beta \in \mathbb{R}$ is negative $\Leftrightarrow$ $\beta < 0$.

- 0 is neither positive nor negative.

Notation

$\mathbb{R}_+$: set of non-negative numbers, R_{++}: set of positive numbers,
$\mathbb{R}_-$: set of non-positive numbers, R_{--}: set of negative numbers.

add	$-$	0	$+$
$-$	$-$	$-$	??
0	$-$	0	$+$
$+$	??	$+$	$+$

mult	$-$	0	$+$
$-$	$+$	0	$-$
0	0	0	0
$+$	$-$	0	$+$

div	$-$	0	$+$
$-$	$+$	nd	$-$
0	0	nd	0
$+$	$-$	nd	$+$

Numbers: Real numbers: Intervals

Notation ($a, b \in \mathbb{R}$)

$$[a, b] = \{x \in \mathbb{R} : a \le x \le b\},$$

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$$]a, b] = \{x \in \mathbb{R} : a \le x \le b\},$$

$$] - \infty, b] = \{x \in \mathbb{R} : a \le x \le b\},$$

$$] - \infty, b[= \{x \in \mathbb{R} : a \le x \le b\},$$

$$[a, +\infty[= \{x \in \mathbb{R} : a \le x \le b\},$$

$$]a, +\infty[= \{x \in \mathbb{R} : a \le x \le b\},$$

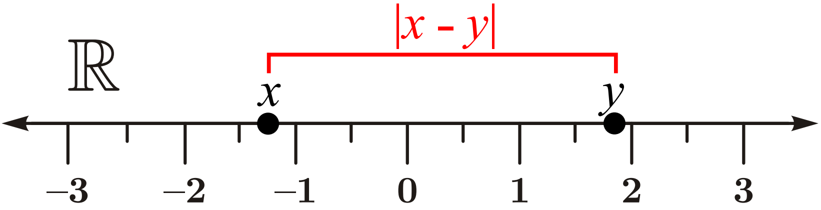
$$] - \infty, \infty[= \{x \in \mathbb{R} : a \le x \le b\}.$$

Definition

Let $x \in \mathbb{R}$. Then

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x = 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

- That is, $|x|$ is the distance of x from 0 on the number line.



(Source: Wikipedia)

Break

Real numbers: additional operations

Reminder: Basic operations with real numbers

Addition, subtraction, multiplication, division BY A NON-ZERO NUMBER.

Our goal is to introduce further important operations.

Real numbers: exponentiation

- Let $\alpha \in \mathbb{R}$.

Definition: Exponentiation, positive integer exponent k

$\alpha^1 = \alpha$, $\alpha^2 = \alpha \cdot \alpha$,

$$\alpha^k = \overbrace{\alpha \cdot \alpha \cdot \ldots \cdot \alpha}^{k}.$$

- $\alpha^{k+1} = \alpha \cdot \alpha^k$. (Recursive definition.)
- Let k, ℓ be two arbitrary positive integers. Then

$$\alpha^{k+\ell} = \alpha^k \cdot \alpha^\ell.$$

Theorem

Let $k > \ell$ be two arbitrary positive integers. Then

$$\frac{\alpha^k}{\alpha^\ell} = \alpha^{k-\ell}.$$

- $\alpha^{-3}? =? \frac{\alpha^2}{\alpha^5}$.

Real numbers: exponentiation II

Let α be a NON-0 real number, i.e. $\alpha \in \mathbb{R} - \{0\}$.

Definition: Exponentiation, integer exponent

$\alpha^0 = 1$, $\alpha^{-k} = \frac{1}{\alpha^k}$.

Theorem

Let k, ℓ be integers. Then

$$\alpha^k \cdot \alpha^\ell = \alpha^{k+\ell}.$$

$$\alpha^{\frac{1}{3}} \cdot \alpha^{\frac{1}{3}} \cdot \alpha^{\frac{1}{3}} \quad ?=? \quad \alpha^1.$$

Real numbers: exponentiation III

Let α be a POSITIVE number, i.e. $\alpha \in \mathbb{R}_{++}$. Furthermore let $k \in \mathbb{N}_+$.

Definition: Exponentiation, reciprocal exponent

Let $k \in \mathbb{N}_+$, then

$$\alpha^{\frac{1}{k}} = \sqrt[k]{\alpha}.$$

Definition: Exponentiation, exponent $\frac{\ell}{k}$, where $k \in \mathbb{N}_+, \ell \in \mathbb{Z}$

$$\alpha^{\frac{\ell}{k}} = \sqrt[k]{\alpha^\ell}.$$

Definition: Exponentiation, rational exponent

Let $r \in \mathbb{Q}$. Write r as $r = \frac{\ell}{k}$ $(k \in \mathbb{N}_+, \ell \in \mathbb{Z})$

$$\alpha^r = \sqrt[k]{\alpha^\ell}.$$

Real numbers: exponentiation IV

Theorem

The above definition is well-defined.
That is, if α is a POSITIVE number ($\alpha \in \mathbb{R}_{++}$), $r = \frac{\ell}{k} = \frac{\ell'}{k'}$ ($k, k' \in \mathbb{N}_+$, $\ell, \ell' \in \mathbb{Z}$), then

$$\sqrt[k]{\alpha^\ell} = \sqrt[k']{\alpha^{\ell'}}.$$

A technical remark:

Statement

Let $k \in \mathbb{N}_+$. Consider the following equation:

$$x^k = \alpha \qquad (\mathcal{E})$$

(1) IF k is odd, then $(\mathcal{E})$ has exactly one root; (2) IF k is even, then $(\mathcal{E})$ has exactly one non-negative root in the case $\alpha \in \mathbb{R}_+$, while it has no root if $\alpha < 0$. Moreover, the set of roots is closed under sign change.

Theorem

Let α be a positive real number, i.e. $\alpha \in \mathbb{R}_{++}$. ASSUME $\alpha > 1$. Let $\beta \in \mathbb{R}$ be an arbitrary exponent.

(a) If
$$\ell_1 < \ell_2 < \ell_3 < \ldots < \beta < \ldots < u_3 < u_2 < u_1,$$
then
$$\alpha^{\ell_1} < \alpha^{\ell_2} < \alpha^{\ell_3} < \ldots < \alpha^{u_3} < \alpha^{u_2} < \alpha^{u_1}.$$

(b) If
$$\ell_1 < \ell_2 < \ell_3 < \ldots < \beta < \ldots < u_3 < u_2 < u_1,$$
$0 < u_i - \ell_i$ can be arbitrarily small, then
$$\alpha^{\ell_1} < \alpha^{\ell_2} < \alpha^{\ell_3} < \ldots < \alpha^{u_3} < \alpha^{u_2} < \alpha^{u_1},$$
and moreover $\alpha^{u_i} - \alpha^{\ell_i}$ can also be arbitrarily small.

Real numbers: exponentiation VI

Consequence (we ASSUME $\alpha > 1$)

If

$$\ell_1 < \ell_2 < \ell_3 < \ldots < \beta,$$

$0 < \beta - \ell_i$ can be arbitrarily small, then the numbers α^{ℓ_i} have infinite decimal expansions that approach an infinite decimal expansion.

Real numbers: exponentiation: the definition (we ASSUME $\alpha > 1$)

(a) If

$$\ell_1 < \ell_2 < \ell_3 < \ldots < \beta < \ldots < u_3 < u_2 < u_1,$$

$0 < u_i - \ell_i$ can be arbitrarily small, then α^β is the unique real number such that

$$\alpha^{\ell_1} < \alpha^{\ell_2} < \alpha^{\ell_3} < \ldots < \alpha^\beta < \ldots < \alpha^{u_3} < \alpha^{u_2} < \alpha^{u_1}.$$

(b) The value defined above does not depend on the choice of

$$\ell_1 < \ell_2 < \ell_3 < \ldots < \beta < \ldots < u_3 < u_2 < u_1.$$

Theorem: Identities of exponentiation

Let $\alpha, \beta \in \mathbb{R}_{++}$, $x, y \in \mathbb{R}$.

(a)

$$a^x \cdot a^y = a^{x+y},$$

(b)

$$\frac{a^x}{a^y} = a^{x-y},$$

(c)

$$(a^x)^y = a^{x \cdot y},$$

(d)

$$a^x \cdot b^x = (ab)^x,$$

(e)

$$\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x,$$

Real numbers: the logarithm

Let $\alpha \in \mathbb{R}_{++}$, $1 \neq a \in \mathbb{R}_{++}$.

Theorem

Every positive number α can be written in the form a^x, where $0 < a \neq 1$ is a real number.

Definition: concept of the logarithm

$\log_a \alpha$ is the uniquely defined exponent x such that $\alpha = a^x$.

That is, $\log_a \alpha$ "thinks of" the number α as a power of a and "encodes" the number by the exponent.

Real numbers: logarithm: identities

Theorem: Identities of the logarithm function

Let $\alpha, \beta \in \mathbb{R}_{++}$, $1 \neq a \in \mathbb{R}$.

(a)

$$\log_a(X \cdot Y) = \log_a X + \log_a Y \qquad // \quad a^x \cdot a^y = a^{x+y},$$

(b)

$$\log_a\left(\frac{X}{Y}\right) = \log_a X - \log_a Y \qquad // \quad a^x \cdot a^y = a^{x+y},$$

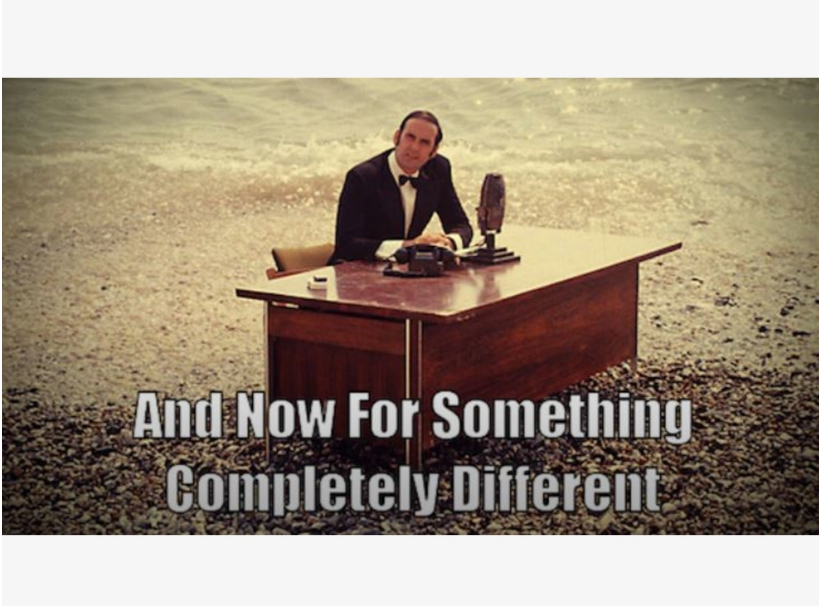
(c)

$$\log_a(X^y) = y \log_a X \qquad // \quad (a^x)^y = a^{x \cdot y},$$

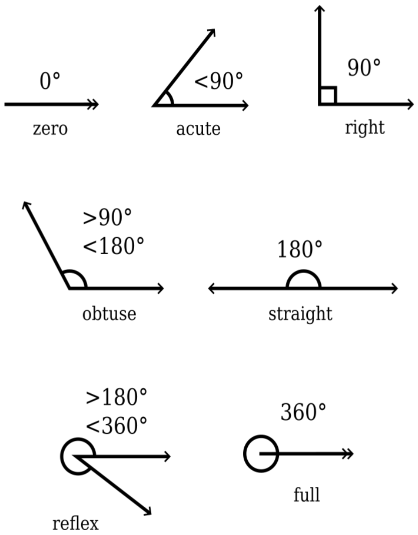
(d)

$$\log_B X = \frac{\log_a X}{\log_a B}, \quad \log_a B \log_B x = \log_a X \quad // \quad (a^b)^x = a^{bx}.$$

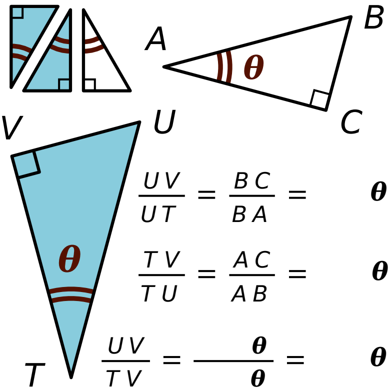
... and now for something completely different ...



Angles: Geometric angles and their types

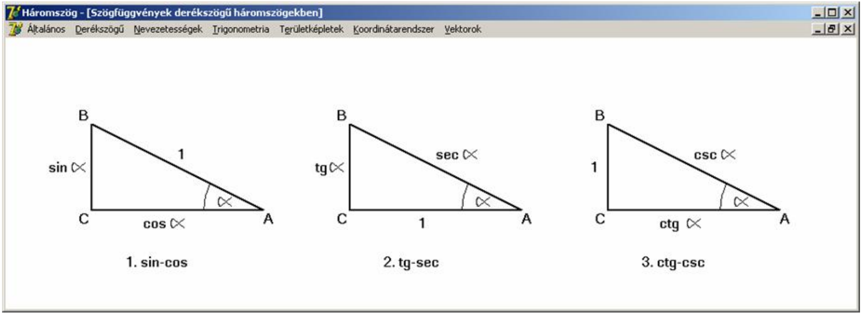


Angles: Trigonometric functions of acute angles



Source:
https://en.wikipedia.org/wiki/Trigonometricfunctions♯/media/File:Acade

Angles: Trigonometric functions of acute angles (continued)



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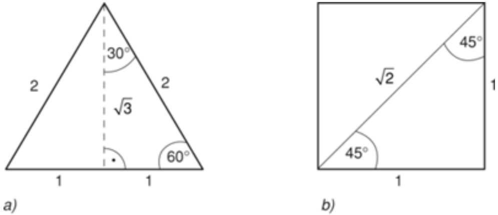
Angles: Special triangles, special trigonometric functions

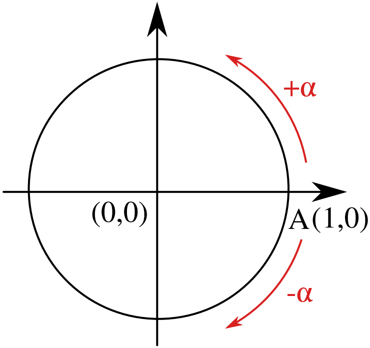


	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
tg	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
ctg	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$

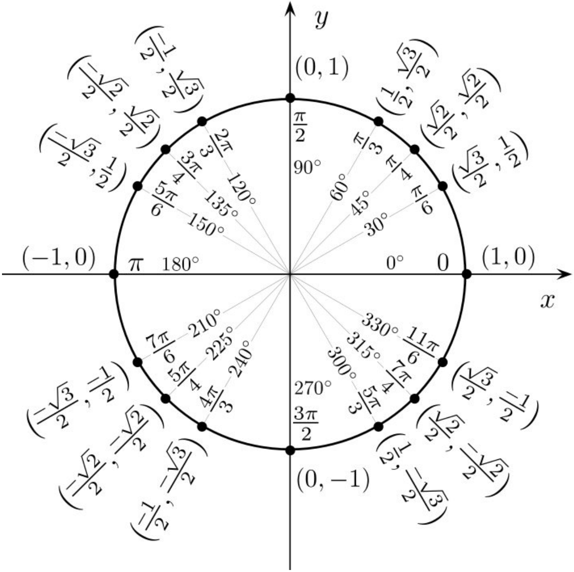
Source:
https://mersz.hu/object/trigonometriaiharomszogekestablazat.png/17025

Angles: Real numbers

Rotational angles and their measurement in radians:



Angles: Special angles



Source: https://okosodjal.webnode.hu/news/nevezetes-szogek/

Angles: Special trigonometric values

	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tg	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
ctg	$-$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$-$

Source: https://gorbem.hu/MT/Haromszog6elemei/image026.gif

Trigonometric functions

(a)

$$\sin : \mathbb{R} \to \mathbb{R}, x \mapsto \sin x,$$

(b)

$$\cos : \mathbb{R} \to \mathbb{R}, x \mapsto \cos x,$$

(c)

$$\tan : \mathbb{R} \to \mathbb{R}, x \mapsto \tan x = \frac{\sin x}{\cos x},$$

(d)

$$\cot : \mathbb{R} \to \mathbb{R}, x \mapsto \cot x = \frac{\cos x}{\sin x},$$

(e)

$$\sec : \mathbb{R} \to \mathbb{R}, x \mapsto \sec x = \frac{1}{\cos x},$$

(f)

$$\csc : \mathbb{R} \to \mathbb{R}, x \mapsto \csc x = \frac{1}{\sin x}.$$

Properties of trigonometric functions

(a)
$$\sin^2 x + \cos^2 x = 1,$$

(b)
$$\sin 2x = 2 \sin x \cos x,$$

(c)
$$\cos 2x = 1 - 2\cos^2 x,$$

(d)
$$\sin(x + k \cdot 2\pi) = \sin x, \quad \cos(x + k \cdot 2\pi) = \cos x \quad (k \in \mathbb{Z}),$$

(e)
$$\tan(x + k \cdot \pi) = \tan x, \quad \cot(x + k \cdot 2\pi) = \cot x \quad (k \in \mathbb{Z}).$$

(f)
$$\sin(-x) = \sin x, \quad \sin(\frac{\pi}{2} - x) = \cos x, \quad \sin(\pi - x) = \sin x,$$
$$\cos(-x) = \cos x, \quad \cos(\frac{\pi}{2} - x) = \sin x, \quad \cos(\pi - x) = -\cos x.$$

Break

Functions

Definition

By a function $f : D \to C$ we mean an assignment/rule/formula that assigns to EVERY element $x \in D$ a UNIQUELY DETERMINED element of C, which is denoted by $f(x)$. The value $f(x)$ is called the image of x. Here $x \in D$ is an input, while $f(x) \in C$ is a value or image.

The set D is called the domain, or the set of departure. Its notation is $dom(f)$. The set C is called the set of attainable values, or the codomain. Its notation is co-dom(f).

Definition

The set $\{f(x) : x \in D\}$ is called the set of attained values, the set of images, or the range. Its notation is im(f).

Definition

A function $f : \text{dom}(f) \to \text{co-dom}(f)$ is called a real function if $\text{dom}(f), \text{co-dom}(f) \subset \mathbb{R}$.

Properties of functions

Definition: surjective function

A function $f : \mathrm{dom}(f) \to \text{co-dom}(f)$ is called onto, or surjective, if $\text{co-dom}(f) = \mathrm{im}(f)$, that is, if for any possible value ($y \in \text{co-dom}(f)$) there exists an input ($x \in \mathrm{dom}(f)$) where the function takes the chosen value ($y = f(x)$).

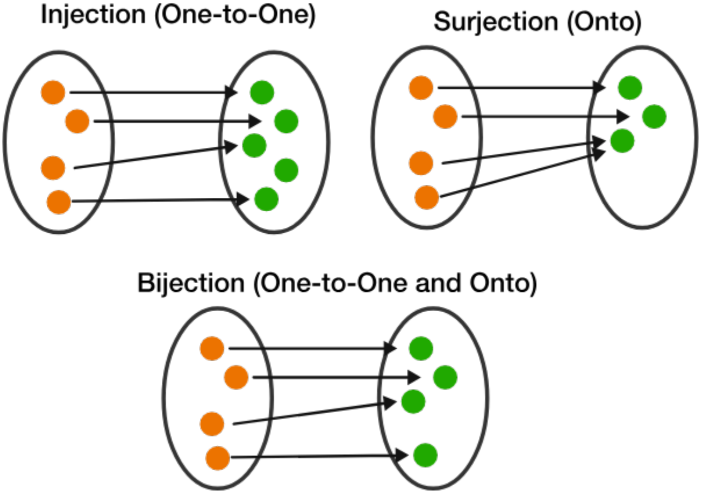
Definition: injective function

A function $f : \mathrm{dom}(f) \to \text{co-dom}(f)$ is called one-to-one, or injective, if for any attained value ($y \in \mathrm{im}(f)$) there exists exactly one input ($x \in \mathrm{dom}(f)$) where the function takes the chosen value ($y = f(x)$). Alternatively: f is injective if for any possible value ($y \in \text{co-dom}(f)$) there exists at most one input ($x \in \mathrm{dom}(f)$) where the function takes the chosen value ($y = f(x)$).

Definition: bijection function

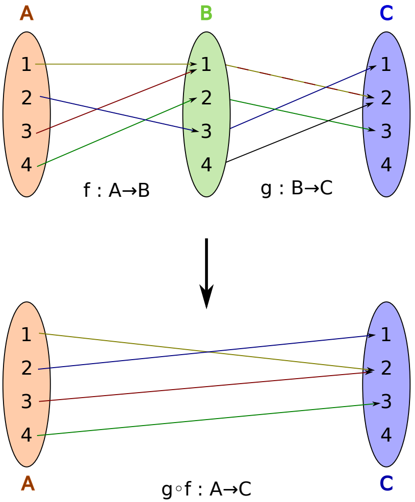
A function is called a bijection (or a pairing mapping), if it is injective and surjective. That is, for any possible value ($y \in \text{co-dom}(f)$) there exists exactly one place/input ($x \in \mathrm{dom}(f)$) where the function takes the chosen value ($y = f(x)$).

Representation of functions: Domain defined on a finite set



Source: https://brilliant.org/wiki/bijection-injection-and-surjection/

Composite functions, function composition



Source: https://en.wikipedia.org/wiki/Function_composition

Diagram of transformations defined on a finite set

If $\mathrm{dom}(f) = \text{co-dom}(f)$, then we (often) say that the function is also a transformation.

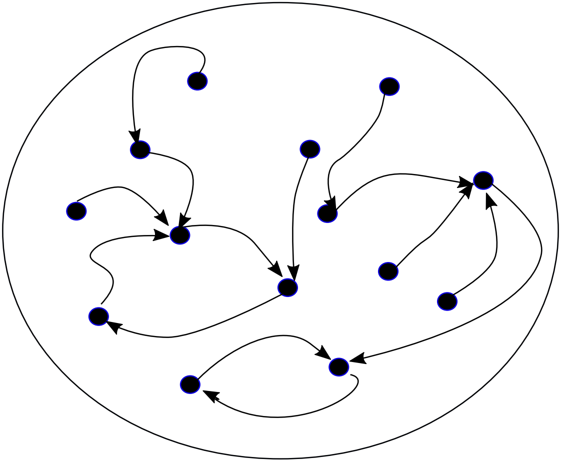
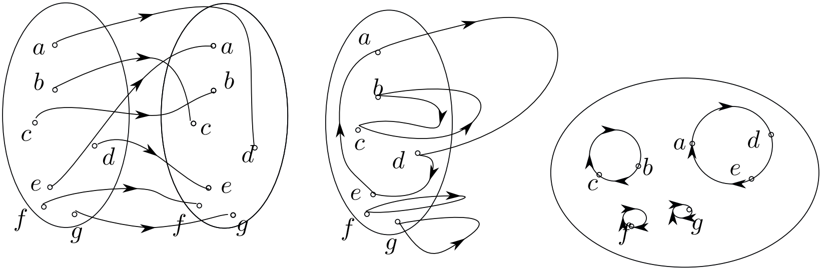


Diagram of bijective transformations defined on a finite set: Permutations



Inverse of bijective transformations

Definiton

Let f : dom$(f) \to$ co-com(f) be a bijective function. Then $f^{(-1)}$: co-dom$(f) \to$ dom(f) is the inverse function of f, that is defined as $f^{(-1)}(y) = x$ iff $f(x) = y$.

Examples of inverse functions

Observation

$\sin x : \mathbb{R} \to \mathbb{R}$, $\cos x : \mathbb{R} \to \mathbb{R}$, $\tan x : \mathbb{R} - \{\frac{2k+1}{2} \cdot \pi : k \in \mathbb{Z}\} \to \mathbb{R}$ are not bijective.

Claim

(1) $\sin_{\text{bij}} x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$ is bijective.

(2) $\cos_{\text{bij}} x : [0, \pi] \to [-1, 1]$ is bijective.

(3) $\tan_{\text{bij}} x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to \mathbb{R}$ is bijective.

Definition

$\arcsin x$, $\arccos x$ and $\arctan x$ are the inverse function of $\sin_{\text{bij}}$, $\cos_{\text{bij}}$ and $\tan_{\text{bij}}$, resp.

Examples of inverse functions II.

Observation

$x^2 : \mathbb{R} \to \mathbb{R}$, $2^x : \mathbb{R} \to \mathbb{R}$ are not bijective.

Claim

(1) $x^2_{\text{bij}} : \mathbb{R}_+ \to \mathbb{R}_+$ is bijective.

(2) $2^x_{\text{bij}} : \mathbb{R} \to \mathbb{R}_{++}$ is bijective.

Definition

(1) $\sqrt{x}$ is the inverse of x^2_{bij},

(2) $\log_2 x$ is the inverse of 2^x_{bij}.

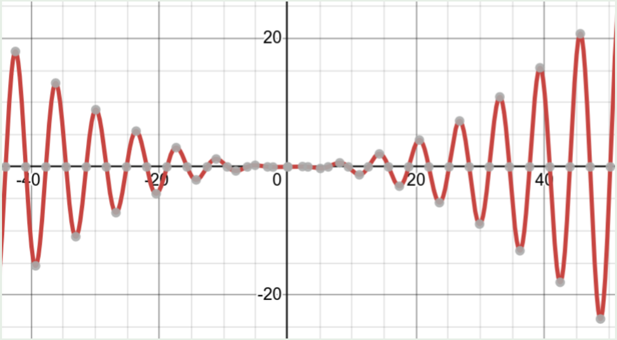
Graph of real functions

Definition

The graph of a function $f : \text{dom}(f) \to \text{co-dom}(f)$ is

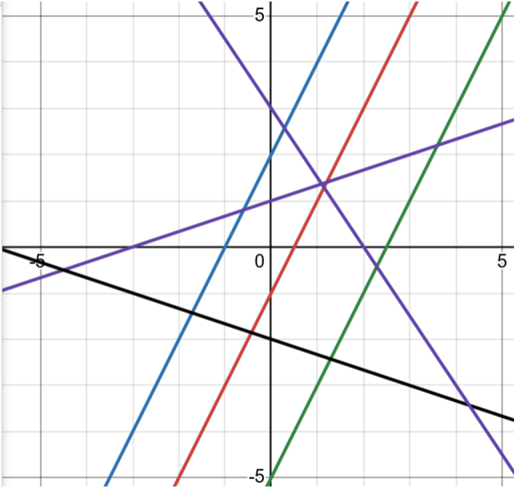
$$graph(f) = \{(x, f(x)) : x \in \text{dom}(f) \subset \mathbb{R}\} \subset \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$$

Example: $\frac{1}{100}x^2 \sin x$ (Source: https://www.desmos.com/calculator)

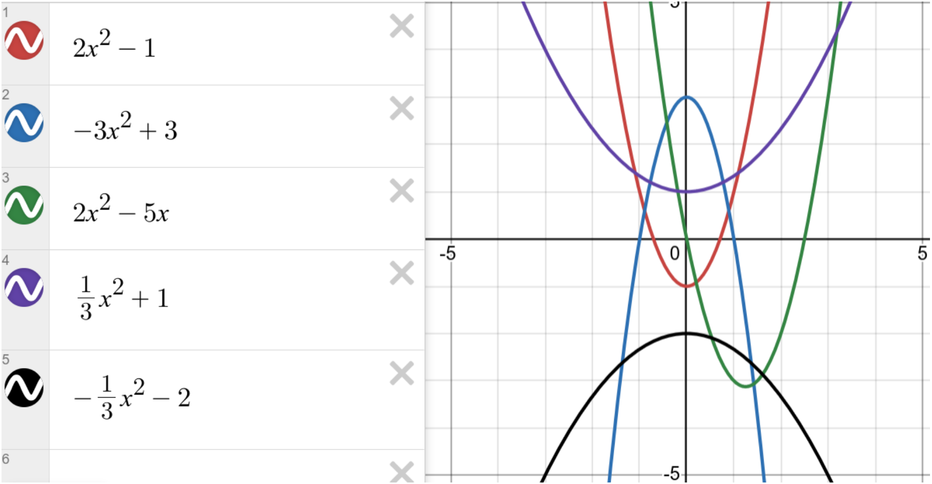


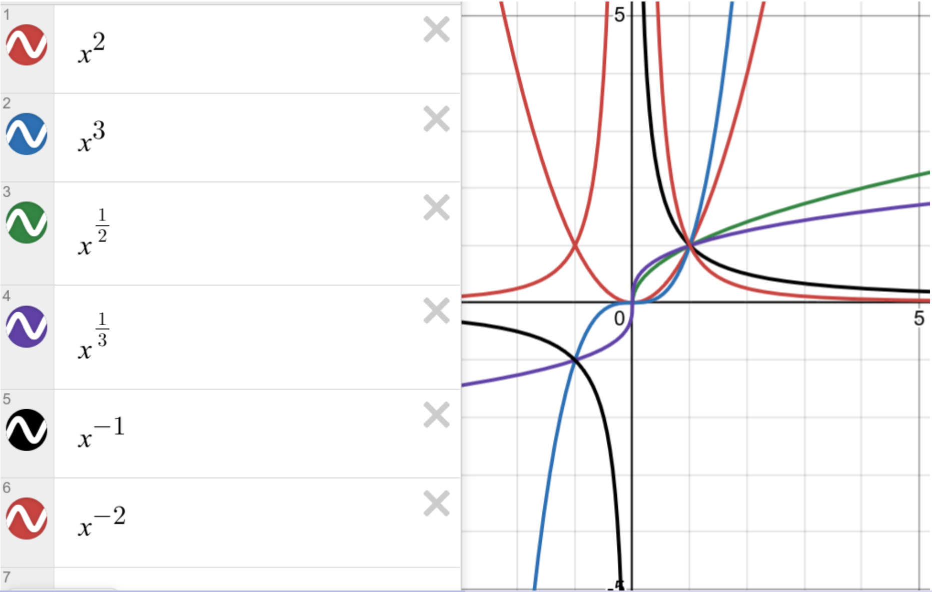
Linear functions

1. $2x-1$
2. $2x+2$
3. $2x-5$
4. $\frac{1}{3}x+1$
5. $-\frac{1}{3}x-2$
6. $\frac{x}{2}+\frac{y}{3}=1$



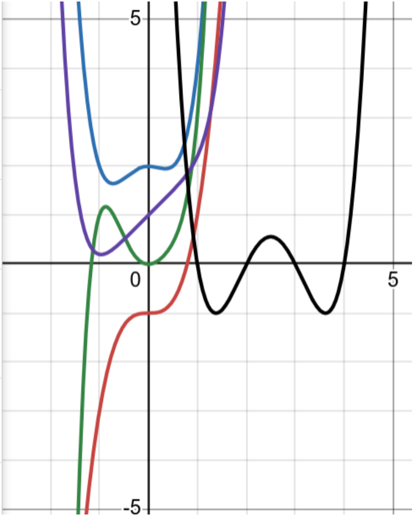
Quadratic functions




1
x^2
2
x^3
3
$x^{\frac{1}{2}}$
4
$x^{\frac{1}{3}}$
5
x^{-1}
6
x^{-2}
7
5
0
5
-5

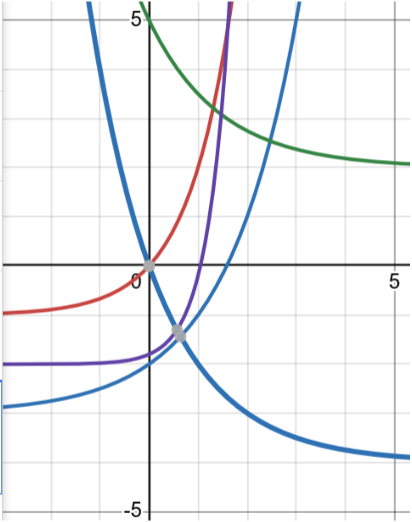
Polynomial functions

1. $2x^3 - 1$
2. $2x^4 + x^3 - x^2 + 2$
3. $2x^5 - x^3 + 2x^2$
4. $\frac{1}{5}x^6 + x + 1$
5. $(x-1)(x-2)(x-3)(x-4)$
6.

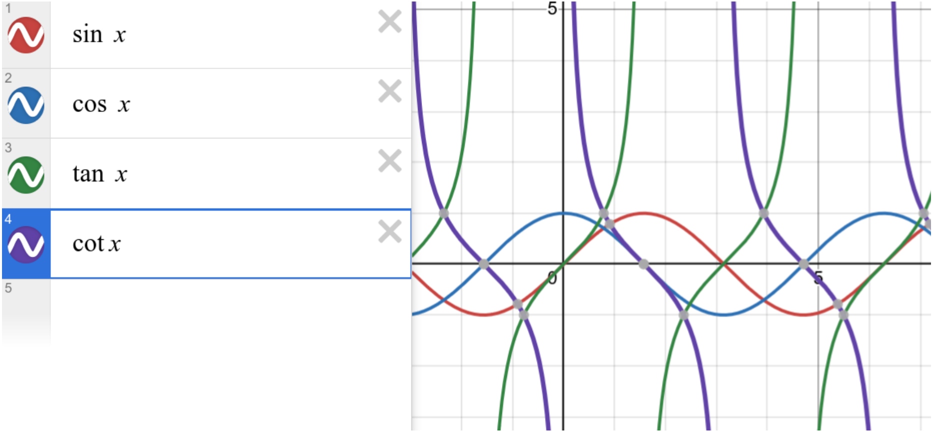


Exponential functions

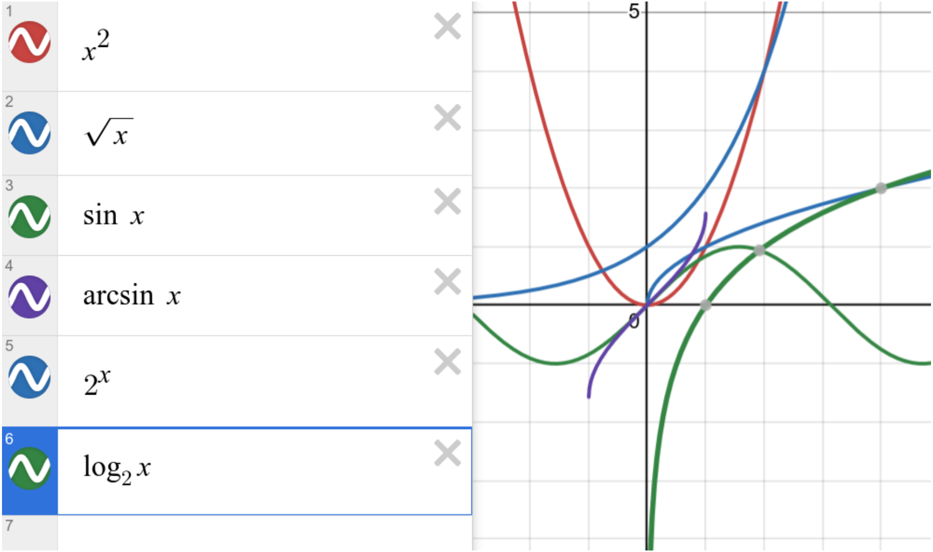
1. $3^x - 1$
2. $2^x - 3$
3. $3 \cdot 2^{-x} + 2$
4. $\frac{1}{5} 3^{2x} - 1 - 1$
5. $\left(\frac{1}{2}\right)^{(x-2)} - 4$
6.



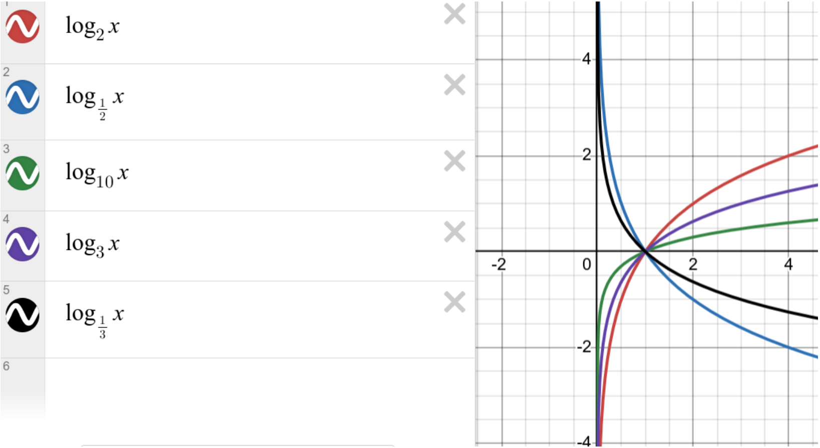
Trigonometric functions



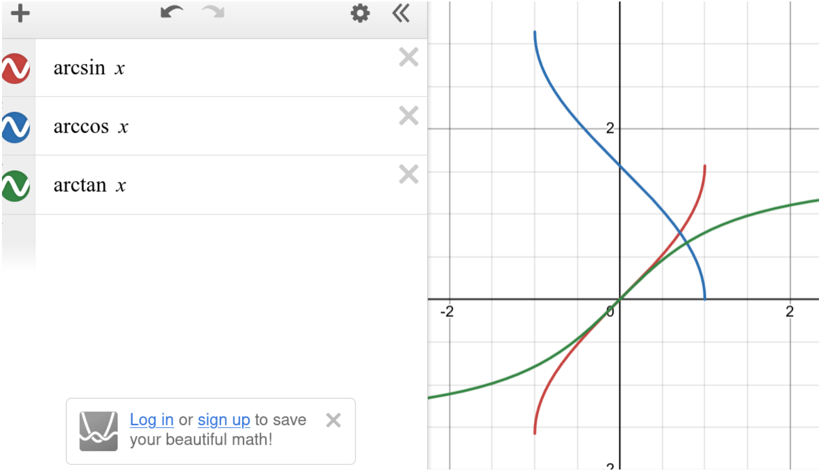
Inverse functions



Logarithmic functions

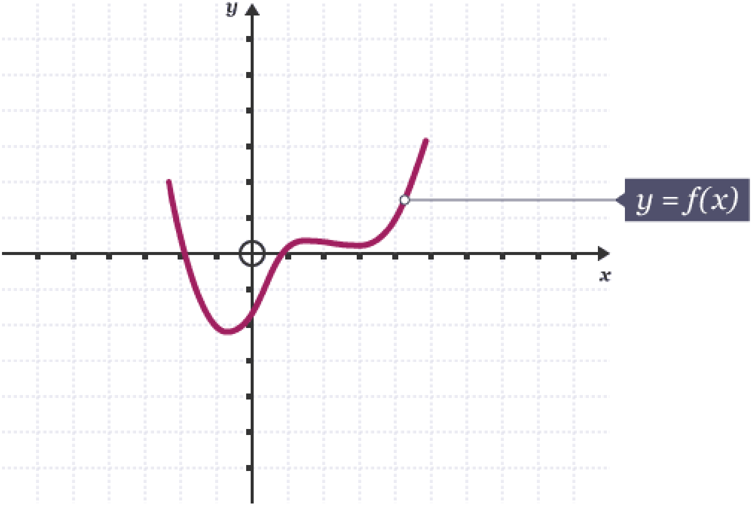


Inverse trigonometric functions



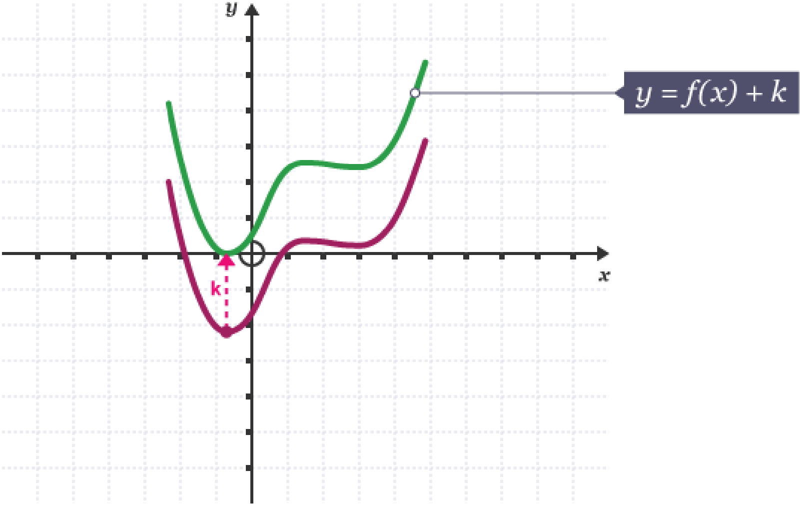
Break

Graph of a given function



Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

Drawing the graph of a given function I



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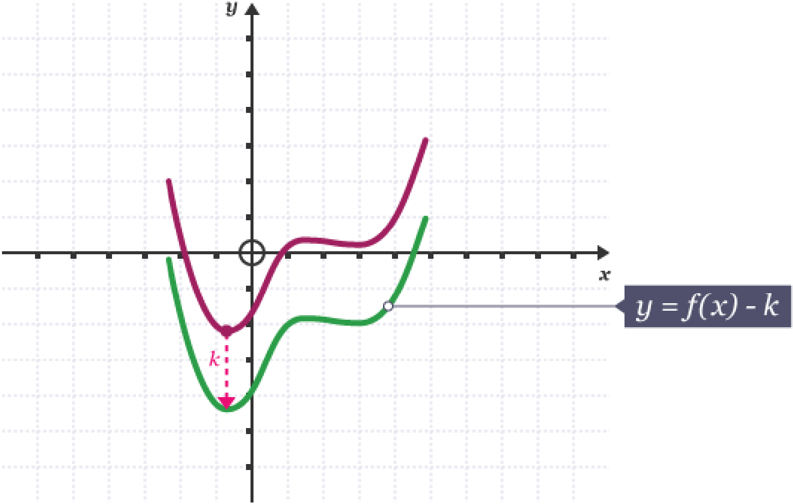
Drawing the graph of a given function II



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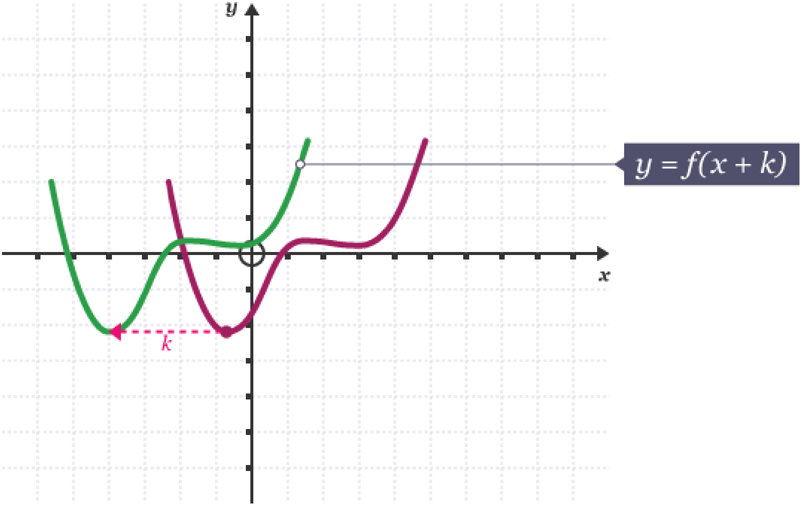
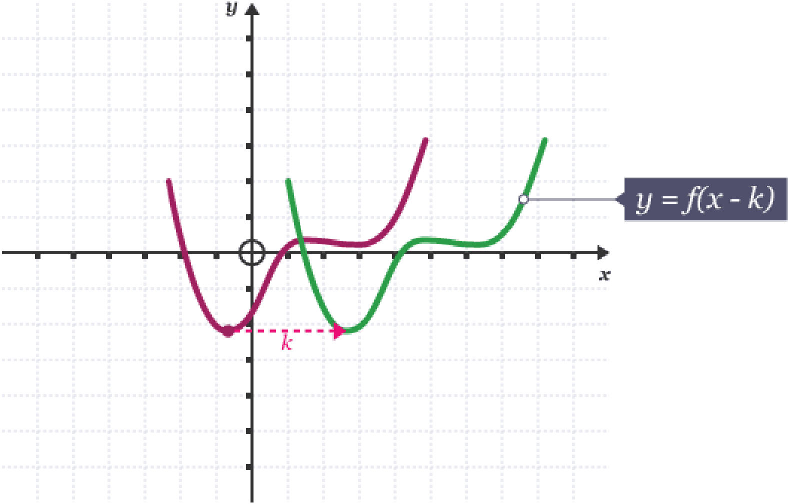
Drawing the graph of a given function III



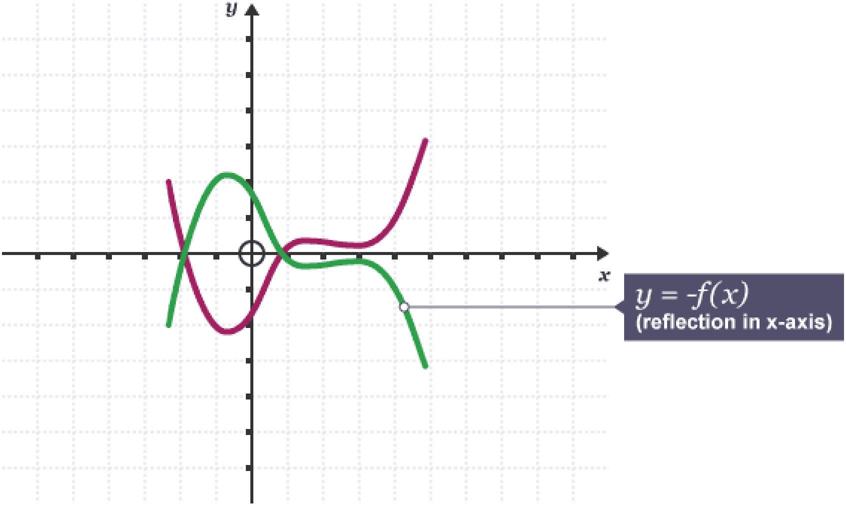
Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

Drawing the graph of a given function IV



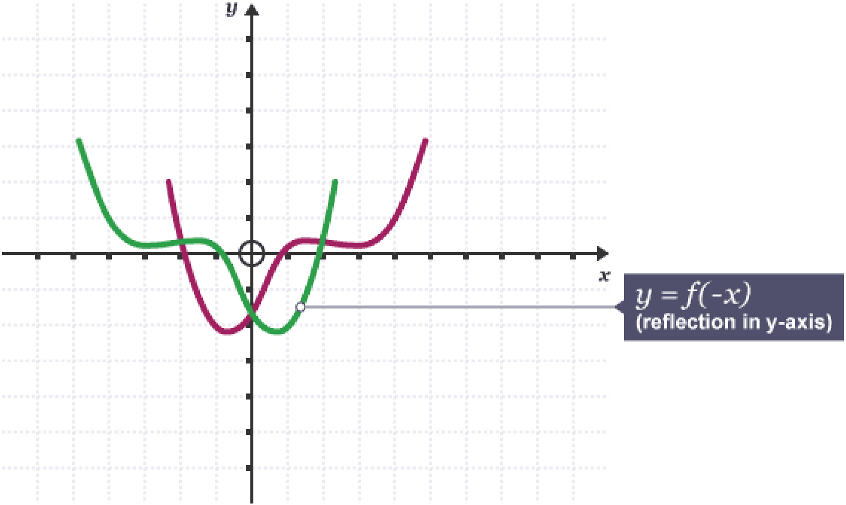
Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

Drawing the graph of a given function V



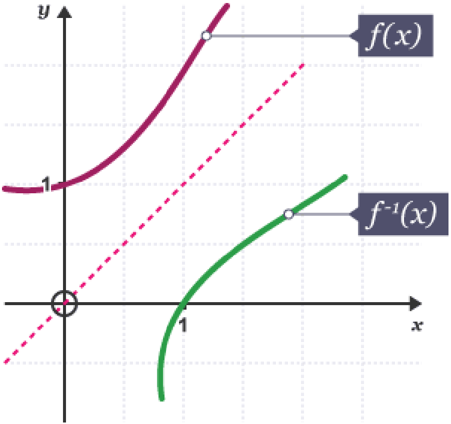
Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

Drawing the graph of a given function VI



Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

Drawing the graph of a given function VII



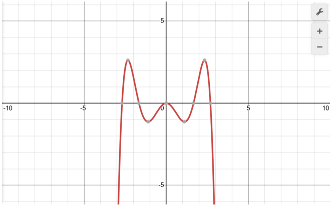
Source: https://www.bbc.co.uk/bitesize/guides/zc6hhyc/revision/1

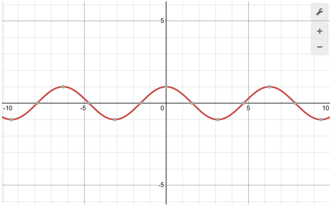
Properties of functions

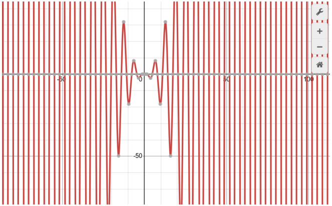
A function f is called

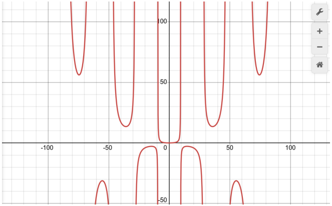
- **even** iff for all $x \in \mathbb{R}$ we have $f(x) = f(-x)$,
- **odd** iff for all $x \in \mathbb{R}$ we have $f(x) = -f(-x)$,
- **monotonically increasing** iff for all $x, y \in \mathbb{R}$ with $x < y$ we have $f(x) \leq f(y)$,
- **strictly monotonically increasing** iff for all $x, y \in \mathbb{R}$ with $x < y$ we have $f(x) < f(y)$,
- **monotonically decreasing** iff for all $x, y \in \mathbb{R}$ with $x < y$ we have $f(x) \geq f(y)$,
- **strictly monotonically decreasing** iff for all $x, y \in \mathbb{R}$ with $x < y$ we have $f(x) > f(y)$,
- **periodic with period** P iff for all $x \in \mathbb{R}$ we have $f(x + P) = f(x)$.

Reading the graph of a real function: Even functions

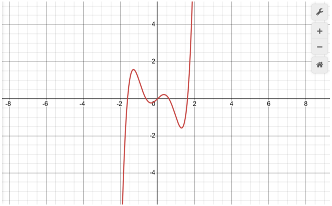


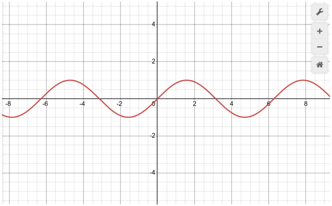


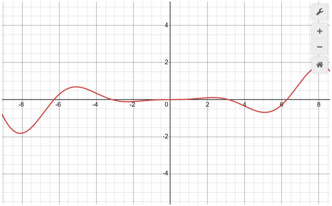


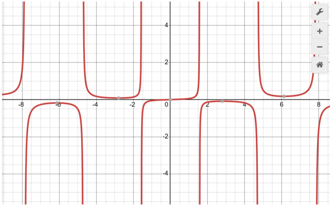


Reading the graph of a real function: Odd functions









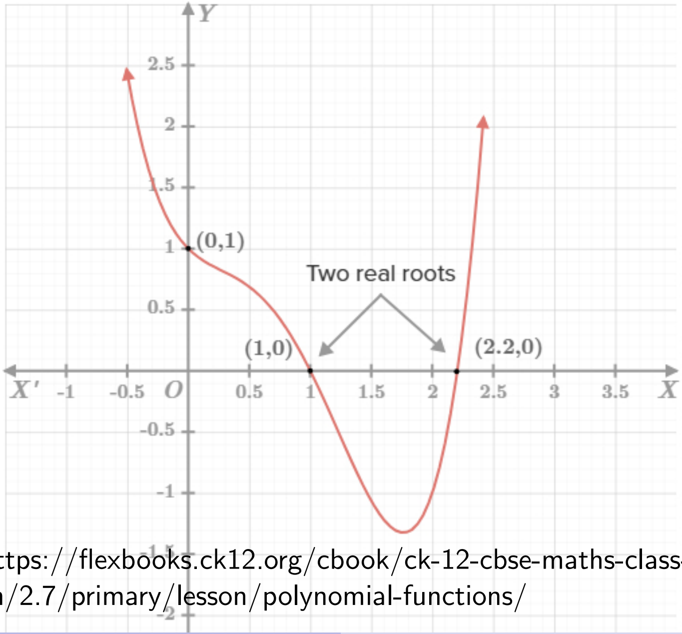
Further properties of functions

A function f is called

- **convex** iff the line segment connecting any two points on the graph of f lies above or on the graph itself,
- **concave** iff the line segment connecting any two points on the graph of f lies below or on the graph itself.

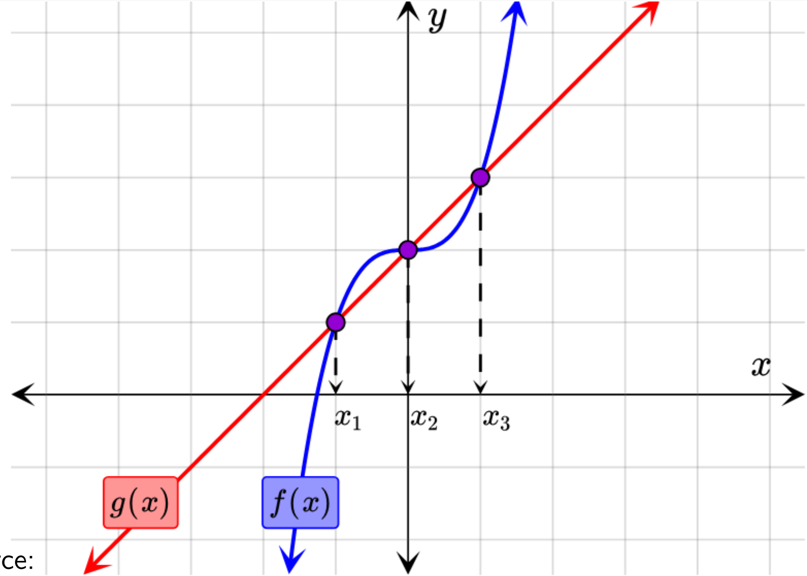
Note that f is concave iff $-f$ is convex.

Zeros of functions



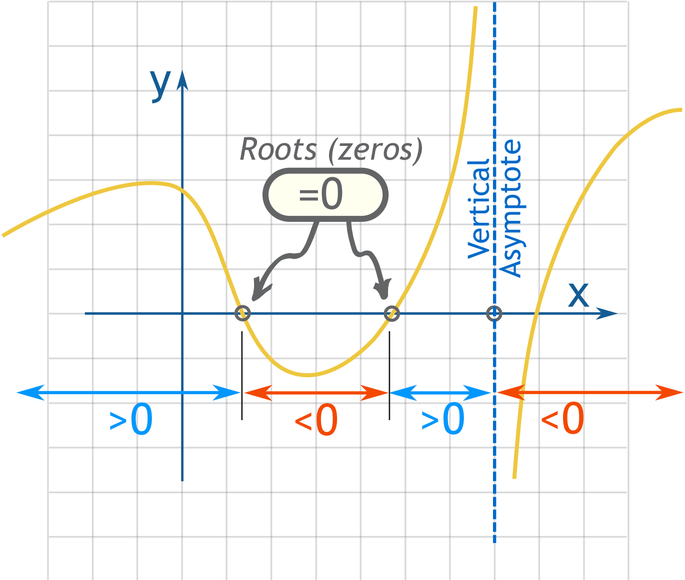
Source: https://flexbooks.ck12.org/cbook/ck-12-cbse-maths-class-11/section/2.7/primary/lesson/polynomial-functions/

Solving the equation $f(x) = g(x)$



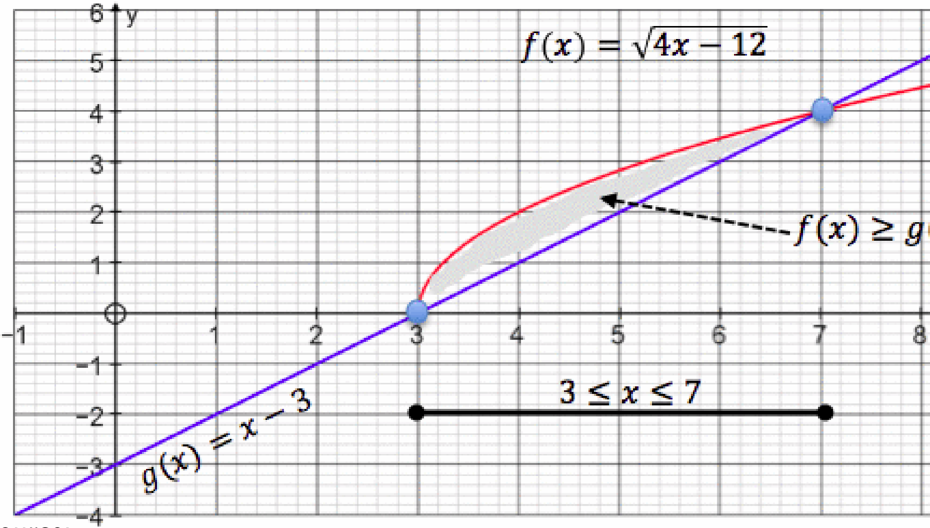
Source:
https://mathleaks.com/study/kb/method/solvinganequationgraphically

Sign of functions



Source:
https://www.mathsisfun.com/algebra/inequality-rational-solving.html

Inequalities



Source:
https://www.mathsisfun.com/algebra/inequality-rational-solving.html

Thank you for your attention!