```
> restart:
  # This is a Maple (version 5.3) worksheet for
  # Gábor Czédli and Ádám Kunos' paper
  # ON THE GEOMETRIC CONSTRUCTIBILITY OF CYCLIC
  # POLYGONS WITH EVEN NUMBER OF VERTICES
  # Version of July 20, 2013
  #
> #        CONTENT:
> #
  # **0** General comments
  #    Easy computations:
  # **1** Formulas for cos(k*x) and sin(k*x)
  # **2** Sine formula to exploit alpha+beta=Pi
  # **3** Cosine formula (2.11)
  #          to exploit alpha+beta+gamma=Pi
  # **4** Cosine formula (2.15)
  #          to exploit alpha+beta+gamma+delta=Pi
  # **5** Proof of Prop.1.3, n=3
  # **6** Proof of Prop.1.3, n=4
  # **7** Proof of Prop.1.3, n=5
  # **8** Proof of Thm.1.1,  n=6
  # **9** Proof of Prop.1.4, n=6
  #    Involved computations
  # **11** hpoly(x) from (3.2)
  # **12** Corollary 3.1, n=15, p=13
  # **13** Corollary 3.1, n=15, p=11
  # **14** Corollary 3.1, n=17, p=n-2
  # **15** Corollary 3.1, n=51, p=n-2
  # **16** Corollary 3.1, n=85, p=n-2
  # **17** Corollary 3.1, n=255, p=n-2
  # **18** Corollary 3.1, n=257, p=n-2
  # **20** Polynom for Proposition 3.2
  # *21** Remarks to Proposition 3.2 #
  # **22** Proposition 3.2, n=15
  # **23** Proposition 3.2, n=17
  # **24** Proposition 3.2, n=51
  # **25** Proposition 3.2, n=85
  # **26** Proposition 3.2, n=255
  # **27** Proposition 3.2, n=257
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  #
  #
  ######################################
  # **0** General comments              #
```

```
########################################
#
#
# Since different Maple version are not always
# compatible, we note the following:
# Comment lines begin with "#". (But in print there
# could be line breaks violating this rule.)
# Commands are separated by colon or semicolon;
# the difference is that colons supress the output.
#
#
#
#
#
################################################
#  *1** Formulas for cos(k*x) and sin(k*x)   #
################################################
#
#
# We need the following two functions above, because they
# allow larger k than their built-in counterparts
#
> coskx:=proc(k,x) sum('(-1)^j * binomial(k,2*j) * x^(k-2*j) *
(1-x*x)^j', 'j'=0..floor(k/2)) end;
# Purpose: for x=cos(alpha), we get
#     coskx(k,x) = cos(k*alpha).
# If you want to test the function coskx, uncomment the
# following command, possibly with other parameters.
# for i from 1 to 7 by 1 do x1:=cos(alpha): print(i,th_coskxTest
= expand(coskx(i,x1) - cos(i*alpha))); od:
```

coskx := **proc**(*k*, *x*)

sum('(-1)^*j**binomial(*k*, 2**j*)**x*^(*k* − 2**j*)*(1 − *x*^2)^*j*', '*j*' = 0 .. floor(1 / 2**k*))

end

```
> sinkx:=proc(k,x)  sum('(-1)^s * binomial(k,2*s+1)*
(1-x*x)^((k-2*s-1)/2) * x^(2*s+1) ', 's'=0..floor((k-1)/2))
end;
# Purpose: for  x=sin(alpha), we get sinkx(k,x)=sin(k*alpha).
# WARNING: it is a polynomial iff k is odd. For k even, it is
# only its square that is a polynomial!!!
# If you want to test the function sinkx, uncomment the
# following, possibly with other parameters.
# for k from 9 to 10 by 1 do a2:=
sort(expand(sinkx(k,sin(alpha))^2)); b1:=expand(expand(
sin(k*alpha))^2 );
# b2:=sort(expand(subs(cos(alpha)=sqrt(1-sin(alpha)^2),b1 )));
```

```
  sinkx_test_is_zero:=a2-b2;  od;
  #
  #
```

sinkx := **proc**(*k*, *x*)
sum('(-1)^*s*∗binomial(*k*, 2∗*s* + 1)∗(1 − *x*^2)^(1 / 2∗*k* − *s* − 1 / 2)∗*x*^(2∗*s* + 1)',
'*s*' = 0 .. floor(1 / 2∗*k* − 1 / 2))
end

```
> #
> #
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  #
  #
  ################################################
  # **2** Sine formula to exploit alpha+beta=Pi  #
  ################################################
  #
> #
>  sinform2:=proc(a5,b5) a5^2-b5^2 end;
  # Purpose: if a5=sin(alpha), b5=sin(beta), and alpha+beta=pi,
  # then sinform2 returns 0.
```

sinform2 := **proc**(*a5*, *b5*) *a5*^2 − *b5*^2 **end**

```
> # To test sinform2, uncomment this (possibly,
  # with other parameters):
  #for j from 100 by 1/10 to 101 do  print(angle=j,
  sinform2test=evalf(sinform2(sin(j), sin(Pi-j)))); od:
> #
  #
> #
  #
  #
  #
> #
> ############################################
> #   **3** Cosine formula (2.11)            #
  #          to exploit alpha+beta+gamma=Pi   #
  # ##########################################
  #
> #
> #
> # Preliminary calculations:
> f1:=expand(cos(alpha+beta)+cos(gamma));
> # This gives 0 if alpha+beta+gamma=Pi.
> tempor:=-sin(alpha)*sin(beta);
  f2:= sort(expand ((f1-tempor)^2- tempor^2));
  # f2=0, because f1=0
```

```
> f3:=sort(expand(subs(sin(alpha)=sqrt(1-cos(alpha)^2),
  sin(beta)=sqrt(1-cos(beta)^2), f2 )));
  f4:=sort(subs(cos(alpha)=a6, cos(beta)=b6, cos(gamma)=c6,f3));
```

$$f1 := \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) + \cos(\gamma)$$

$$tempor := -\sin(\alpha)\sin(\beta)$$

$$f2 := -\sin(\alpha)^2\sin(\beta)^2 + \cos(\alpha)^2\cos(\beta)^2 + 2\cos(\gamma)\cos(\alpha)\cos(\beta) + \cos(\gamma)^2$$

$$f3 := \cos(\alpha)^2 + 2\cos(\gamma)\cos(\alpha)\cos(\beta) + \cos(\beta)^2 - 1 + \cos(\gamma)^2$$

$$f4 := 2\,a6\,b6\,c6 + a6^2 + b6^2 + c6^2 - 1$$

```
> # Now, we simply copy f4 into the function cosform3
  cosform3:=proc(a6,b6,c6) 2*a6*c6*b6+a6^2+c6^2+b6^2-1 end;
  # Purpose: if a6=cos(alpha),b6=cos(beta9, c6=cos(gamma),
  # and alpha+beta+gamma=pi, then cosform3 gives 0.
  #
  #
```

$$cosform3 := \textbf{proc}(a6, b6, c6)\ 2*a6*c6*b6 + a6\text{^}2 + c6\text{^}2 + b6\text{^}2 - 1\ \textbf{end}$$

```
> # To test cosform3, uncomment this (possibly, with other
  parameters):
  # for j from 5 by 0.2  to 5.6 do  print(cosform3_test_angle1=j):
  for k from 10 by 0.3 to 10.6 do
  #     print(angle2=k, should_be_zero=
  evalf(cosform3(cos(j),cos(k),cos(Pi-j-k)))): od:od:
  #
  #
> #
> #
> #
> #
> #
> #
> ##############################################
> # **4** Cosine formula (2.15)                #
  #       to exploit alpha+beta+gamma+delta=Pi #
  ##############################################
> #
> #
  # Preliminary calculations:
> f1:=sort(expand(cos(alpha+beta)+cos(gamma+delta)));
```

$$f1 := -\sin(\alpha)\sin(\beta) + \cos(\alpha)\cos(\beta) - \sin(\gamma)\sin(\delta) + \cos(\gamma)\cos(\delta)$$

```
> # This is 0, and so are f2,f3, ...
> auxiliary:=-sin(alpha)*sin(beta)-sin(gamma)*sin(delta);
```

$$auxiliary := -\sin(\alpha)\sin(\beta) - \sin(\gamma)\sin(\delta)$$

```
> f2:=sort(expand((f1-auxiliary)^2 -auxiliary^2));
```

$$f2 := -\sin(\alpha)^2\sin(\beta)^2 + \cos(\alpha)^2\cos(\beta)^2 - 2 \ldots)\sin(\alpha)\sin(\beta)\sin(\delta)$$

$+ 2\cos(\gamma)\cos(\delta)\cos(\alpha)\cos(\beta) - \sin(\gamma)^2\sin(\delta)^2 + \cos(\gamma)^2\cos(\delta)^2$

```
> auxiliary:=-2*sin(gamma)*sin(alpha)*sin(beta)*sin(delta);
```

$$auxiliary := -2\sin(\gamma)\sin(\alpha)\sin(\beta)\sin(\delta)$$

```
> f3:=sort(expand((f2-auxiliary )^2-auxiliary^2));
```

$$\begin{aligned} f3 := {} & \sin(\alpha)^4\sin(\beta)^4 - 2\sin(\alpha)^2\sin(\beta)^2\cos(\alpha)^2\cos(\beta)^2 + \cos(\alpha)^4\cos(\beta)^4 \\ & - 4\cos(\gamma)\sin(\alpha)^2\sin(\beta)^2\cos(\delta)\cos(\alpha)\cos(\beta) + 4\cos(\gamma)\cos(\delta)\cos(\alpha)^3\cos(\beta)^3 \\ & - 2\sin(\gamma)^2\sin(\alpha)^2\sin(\beta)^2\sin(\delta)^2 - 2\cos(\gamma)^2\sin(\alpha)^2\sin(\beta)^2\cos(\delta)^2 \\ & - 2\sin(\gamma)^2\sin(\delta)^2\cos(\alpha)^2\cos(\beta)^2 + 6\cos(\gamma)^2\cos(\delta)^2\cos(\alpha)^2\cos(\beta)^2 \\ & - 4\cos(\gamma)\sin(\gamma)^2\sin(\delta)^2\cos(\delta)\cos(\alpha)\cos(\beta) + 4\cos(\gamma)^3\cos(\delta)^3\cos(\alpha)\cos(\beta) \\ & + \sin(\gamma)^4\sin(\delta)^4 - 2\sin(\gamma)^2\cos(\gamma)^2\sin(\delta)^2\cos(\delta)^2 + \cos(\gamma)^4\cos(\delta)^4 \end{aligned}$$

```
> f4:=expand(subs(sin(alpha)=sqrt(1-cos(alpha)^2),sin(beta)=sqrt(1
  -cos(beta)^2),sin(gamma)=sqrt(1-cos(gamma)^2),sin(delta)=sqrt(1-
  cos(delta)^2),   f3)) ;
```

$$\begin{aligned} f4 := {} & -2\cos(\alpha)^2\cos(\beta)^2 + 4\cos(\alpha)^2\cos(\beta)^2\cos(\delta)^2 - 2\cos(\gamma)^2\cos(\delta)^2 \\ & + 4\cos(\gamma)^2\cos(\alpha)^2\cos(\beta)^2 - 8\cos(\gamma)\cos(\delta)\cos(\alpha)\cos(\beta) + \cos(\delta)^4 \\ & + 4\cos(\gamma)^3\cos(\delta)\cos(\alpha)\cos(\beta) + 4\cos(\gamma)\cos(\delta)^3\cos(\alpha)\cos(\beta) \\ & + 4\cos(\gamma)^2\cos(\beta)^2\cos(\delta)^2 + 4\cos(\gamma)^2\cos(\alpha)^2\cos(\delta)^2 + \cos(\gamma)^4 \\ & + 4\cos(\gamma)\cos(\delta)\cos(\alpha)\cos(\beta)^3 + 4\cos(\gamma)\cos(\delta)\cos(\alpha)^3\cos(\beta) - 2\cos(\gamma)^2\cos(\alpha)^2 \\ & - 2\cos(\gamma)^2\cos(\beta)^2 - 2\cos(\beta)^2\cos(\delta)^2 - 2\cos(\alpha)^2\cos(\delta)^2 + \cos(\alpha)^4 + \cos(\beta)^4 \end{aligned}$$

```
> f5:=sort(subs(cos(alpha)=x,cos(beta)=y,cos(gamma)=z,cos(delta)=t
  ,f4));
```

$$\begin{aligned} f5 := {} & 4x^3zyt + 4x^2z^2y^2 + 4x^2z^2t^2 + 4x^2y^2t^2 + 4xz^3yt + 4xzy^3t + 4xzyt^3 + 4z^2y^2t^2 \\ & + x^4 - 2x^2z^2 - 2x^2y^2 - 2x^2t^2 - 8xzyt + z^4 - 2z^2y^2 - 2z^2t^2 + y^4 - 2y^2t^2 + t^4 \end{aligned}$$

```
> # Now, we simply copy the expression f5 here:
> cosform4:=proc(x,y,z,t)
  4*x^3*t*y*z+4*x^2*t^2*y^2+4*x^2*t^2*z^2+4*x^2*y^2*z^2+4*x*t^3*y*
  z+4*x*t*y^3*z+4*x*t*y*z^3+4*t^2*y^2*z^2+x^4-2*x^2*t^2-2*x^2*y^2-
  2*x^2*z^2-8*x*t*y*z+t^4-2*t^2*y^2-2*t^2*z^2+y^4-2*y^2*z^2+z^4
  end;
```

cosform4 := **proc**(*x*, *y*, *z*, *t*)

4*x^3*t*y*z + 4*x^2*t^2*y^2 + 4*x^2*t^2*z^2 + 4*x^2*y^2*z^2 + 4*x*t^3*y*z + 4*x*t*y^3*z + 4*x*t*y*z^3 + 4*t^2*y^2*z^2 + x^4 − 2*x^2*t^2 − 2*x^2*y^2 − 2*x^2*z^2 − 8*x*t*y*z + t^4 − 2*t^2*y^2 − 2*t^2*z^2 + y^4 − 2*y^2*z^2 + z^4

end

```
> # Next, to check that this is the same as given
> # in the paper, we type that formula here:
> cosform4_paper:=proc(x1,x2,x3,x4) (x1^4+x2^4+x3^4+x4^4)
     - 2*(x1^2*x2^2 + x1^2*x3^2 + x1^2*x4^2 + x2^2*x3^2 + x2^2*x4^2
  + x3^2*x4^2)
```

```
   + 4*x1*x2*x3*x4* (-2 + (x1^2+x2^2+x3^2+x4^2) )
   + 4* (x1^2*x2^2*x3^2 + x1^2*x2^2*x4^2 + x1^2*x3^2*x4^2 +
 x2^2*x3^2*x4^2 ) end;
> # And now we can check the formula:
 expand( cosform4(x,y,z,t)- cosform4_paper(x,y,z,t));
```

cosform4_paper := **proc**(*x1*, *x2*, *x3*, *x4*)

$x1\text{^}4 + x2\text{^}4 + x3\text{^}4 + x4\text{^}4 - 2*x1\text{^}2*x2\text{^}2 - 2*x1\text{^}2*x3\text{^}2 - 2*x1\text{^}2*x4\text{^}2$
$- 2*x2\text{^}2*x3\text{^}2 - 2*x4\text{^}2*x2\text{^}2 - 2*x4\text{^}2*x3\text{^}2$
$+ 4*x1*x2*x3*x4*(-2 + x1\text{^}2 + x2\text{^}2 + x3\text{^}2 + x4\text{^}2) + 4*x1\text{^}2*x2\text{^}2*x3\text{^}2$
$+ 4*x1\text{^}2*x4\text{^}2*x2\text{^}2 + 4*x1\text{^}2*x4\text{^}2*x3\text{^}2 + 4*x4\text{^}2*x2\text{^}2*x3\text{^}2$

end

$$0$$

```
> # Some further checks that you can uncomment:
 #cosform4_paper(sqrt(2)/2,sqrt(2)/2,sqrt(2)/2,sqrt(2)/2);
 #cosform4_paper(sqrt(3)/2,1/2,sqrt(2)/2,sqrt(2)/2);
 #evalf(cosform4_paper(
 cos(12*Pi/100),cos(22*Pi/100),cos(30*Pi/100),cos(36*Pi/100)));
```

```
> #
 #
 #
 #
 #
 #################################
 # **5** Proof of Prop.1.3, n=3  #
 #################################
 #
 #
 f1:=sort(subs(u=x,cosform3(1*u,2*u,3*u)));
 f2:=subs(x=y/2,2*f1);
 irreduc(f1); irreduc(f2);
```

$$f1 := 12\,x^3 + 14\,x^2 - 1$$

$$f2 := 3\,y^3 + 7\,y^2 - 2$$

true

true

```
> #
 #
 #
 #
 #
 #
 #################################
```

```
# **6** Proof of Prop.1.3, n=4  #
##################################
#
# We check that c_2x^2 + c_0 mentioned
# in the paper is not the zero polynomial:
a1:=sort(collect(expand(cosform4_paper(d1*u,d2*u,d3*u,d4*u)/(u^4
)),u),u);
a2:=op(1,a1)/u^2;
c2:=sort(factor(a2));
```

$$a1 := (4\, d1^3\, d2\, d3\, d4 + 4\, d1\, d2^3\, d3\, d4 + 4\, d1\, d2\, d3^3\, d4 + 4\, d1\, d2\, d3\, d4^3 + 4\, d1^2\, d2^2\, d3^2 + 4\, d1^2\, d4^2\, d2^2 + 4\, d1^2\, d4^2\, d3^2 + 4\, d4^2\, d2^2\, d3^2)\, u^2 + d1^4 + d2^4 + d3^4 + d4^4 - 2\, d1^2\, d2^2 - 2\, d1^2\, d3^2 - 2\, d1^2\, d4^2 - 2\, d2^2\, d3^2 - 2\, d4^2\, d2^2 - 2\, d4^2\, d3^2 - 8\, d1\, d2\, d3\, d4$$

$$a2 := 4\, d1^3\, d2\, d3\, d4 + 4\, d1\, d2^3\, d3\, d4 + 4\, d1\, d2\, d3^3\, d4 + 4\, d1\, d2\, d3\, d4^3 + 4\, d1^2\, d2^2\, d3^2 + 4\, d1^2\, d4^2\, d2^2 + 4\, d1^2\, d4^2\, d3^2 + 4\, d4^2\, d2^2\, d3^2$$

$$c2 := 4\, (d3\, d4 + d2\, d1)\, (d3\, d2 + d4\, d1)\, (d3\, d1 + d4\, d2)$$

```
> #
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#
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#
##################################
# **7** Proof of Prop.1.3, n=5  #
##################################
#
#
f1:=cosform3(2*u^2-1, 2*(2*u)^2-1, 3*u  );
f2:=sort(expand(subs(u=x,f1)));
irreduc(f2);
#
#
```

$$f1 := 6\, (2\, u^2 - 1)\, u\, (8\, u^2 - 1) + (2\, u^2 - 1)^2 + 9\, u^2 + (8\, u^2 - 1)^2 - 1$$

$$f2 := 96\, x^5 + 68\, x^4 - 60\, x^3 - 11\, x^2 + 6\, x + 1$$

true

```
> #
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> #
> #
>
> ###################################
> # **8** Proof of Thm.1.1,  n=6   #
> ###################################
> #
> #
```

```
> f1:=cosform3(1-4*u, 1-6*u, 1-10*u );
> f2:=sort(expand(subs(u=x,f1/(-4))));
  irreduc(f2);
```

$$f1 := 2\,(1-4\,u)\,(1-10\,u)\,(1-6\,u)+(1-4\,u)^2+(1-10\,u)^2+(1-6\,u)^2-1$$

$$f2 := 120\,x^3-100\,x^2+20\,x-1$$

true

```
> #
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> #
> ###################################
> # **9** Proof of Prop.1.4, n=6    #
  ###################################
> #
> #
> f1:=sort(collect(cosform3(coskx(4,1*u), 2*u, 3*u),u),u);
  f2:=sort(factor(expand(f1)));
  f3:=subs(u=x,f2/u^2);
  irreduc(f3);
  f4:=sort(expand(subs( x=sqrt(y+2)/2,f3) ));
  irreduc(f4);
  # Now, we check the degree:
  f1param:= sort(collect(cosform3(coskx(4,1*u), d5*u,
  d6*u),u),u);
  # And we see that the leading coefficient
  # of f1 equals that of f1param.
```

$$f1 := 64\,u^8-32\,u^6-16\,u^4+9\,u^2$$

$$f2 := (64\,u^6-32\,u^4-16\,u^2+9)\,u^2$$

$$f3 := 64\,x^6-32\,x^4-16\,x^2+9$$

true

$$f4 := y^3+4\,y^2+1$$

true

$$f1param := 64\,u^8+(16\,d6\,d5-128)\,u^6+(-16\,d6\,d5+80)\,u^4+(2\,d6\,d5+d5^2-16+d6^2)\,u^2$$

```
> #
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```

```
> #
> ###############################
> # **11** hpoly(x) from (3.2) #
  ###############################
> #
> #
> hpoly:=proc(n,p,a,b,  x) sort(collect(expand(
         sinform2( sinkx(p,a*x)  ,sinkx(n-p,b*x ) )
  ),x),x); end;
> # The idea of hpoly should be clear. Namely,
  # n and p are odd, 0<p<n, and  need not be a prime.
  # p of the sidelength equal a, the rest n-p equal b.
  # As in the paper, we have sin(alpha)=a*x and sin(beta)=b*x,
  # where u=1/(2*r). We know that  p*alpha+(n-p)*beta=Pi.
  # Hence, using our functions, it follows that u=1/(2*r)
  # is a root of the polynomial hpoly(n,p,a,b, x),
  # in which the variable is x.
```

hpoly := **proc**(*n*, *p*, *a*, *b*, *x*)

sort(collect(expand(sinform2(sinkx(p, $a*x$), sinkx($n-p$, $b*x$))), x), x)

end

```
> #
> #
  #
> #
> #
  #####################################
  # **12** Corollary 3.1, n=15, p=13 #
  #####################################
  #
> #
> n:=15; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2, x))/y)):
  degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f); if
  degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 15$$

$$degree\_of\_f := 12$$

$$is\_f\_irreducible := true$$

$$f\_is\_this = 16777216\,y^{12} - 109051904\,y^{11} + 313524224\,y^{10} - 524812288\,y^{9} + 566558720\,y^{8} - 412778496\,y^{7} + 206389248\,y^{6} - 70606848\,y^{5} + 16180736\,y^{4} - 2379520\,y^{3} + 208208\,y^{2} - 9400\,y + 153$$

```
> #
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  #
```

```
####################################
# **13** Corollary 3.1, n=15, p=11 #
####################################
#
#
n:=15; f:=sort(expand(subs(x=sqrt(y), hpoly(n,11,1,2,x))/y)):
degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f); if
degree_of_f<40 then print(f_is_this=f) fi;
# p=11 gives a smaller degree:
```

$$n := 15$$

$$degree\_of\_f := 10$$

$$is\_f\_irreducible := true$$

$$f\_is\_this = 1048576\,y^{10} - 5767168\,y^{9} + 13697024\,y^{8} - 18382848\,y^{7} + 15319040\,y^{6} - 8200192\,y^{5} + 2818816\,y^{4} - 587648\,y^{3} + 67312\,y^{2} - 3560\,y + 57$$

```
> #
#
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#
#
#
#####################################
# **14** Corollary 3.1, n=17, p=n-2 #
#####################################
#
#
#
n:=17; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2,x))/y)):
degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f); if
degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 17$$

$$degree\_of\_f := 14$$

$$is\_f\_irreducible := true$$

$$f\_is\_this = 268435456\,y^{14} - 2013265920\,y^{13} + 6794772480\,y^{12} - 13631488000\,y^{11} + 18087936000\,y^{10} - 16713252864\,y^{9} + 11026104320\,y^{8} - 5239111680\,y^{7} + 1786060800\,y^{6} - 429977600\,y^{5} + 70946304\,y^{4} - 7637760\,y^{3} + 495040\,y^{2} - 16736\,y + 209$$

```
> #
#
#
#
#####################################
# **15** Corollary 3.1, n=51, p=n-2 #
#####################################
#
#
```

```
n:=3*17; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2,
x))/y)): degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f);
if degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 51$$
$$degree\_of\_f := 48$$
$$is\_f\_irreducible := true$$

```
> #
#
#
#
#
#
######################################
# **16** Corollary 3.1, n=85, p=n-2 #
######################################
#
##
#
n:=5*17; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2,
x))/y)): degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f);
if degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 85$$
$$degree\_of\_f := 82$$
$$is\_f\_irreducible := true$$

```
> #
#
#
#
#
######################################
# **17** Corollary 3.1, n=255, p=n-2 #
######################################
#
#
n:=15*17; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2,
x))/y)): degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f);
if degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 255$$
$$degree\_of\_f := 252$$
$$is\_f\_irreducible := true$$

```
> #
#
#
#
```

```
#
#
#####################################
# **18** Corollary 3.1, n=257, p=n-2 #
#####################################
#
#
n:=257; f:=sort(expand(subs(x=sqrt(y), hpoly(n,n-2,1,2, x))/y)):

# Should you want to see the astronomically
# long f(x)=h(x)/x, change : to ; above
degree_of_f:=degree(f,y); is_f_irreducible:=irreduc(f); if
degree_of_f<40 then print(f_is_this=f) fi;
```

$$n := 257$$

$$degree\_of\_f := 254$$

$$is\_f\_irreducible := true$$

```
> #
#
#
#
#
#
#
#
#
#
#
#
#######################################
# **20** Polynom for Proposition 3.2  #
#######################################
#
#
# We assume that p of the distances is a
# and n-p of them is b. For the half central
# angles cos(alpha)=a/r=a*u, cos(beta)=b*u.
# Therefore, since p*alpha+(n-p)*beta=Pi,
# u is a root of the following polynomial
dpoly:=proc(n,p,a,b, x)  coskx(p,a*x) + coskx(n-p,b*x) end;
```

$$dpoly := \textbf{proc}(n, p, a, b, x)\ \text{coskx}(p, a*x) + \text{coskx}(n-p, b*x)\ \textbf{end}$$

```
> #
#
#
#
#
#
```

```
#
######################################
# **21** Remarks to Proposition 3.2  #
######################################
#
> #
# With given distinct integers a and b,
# the polygon is rarely exists. Hence,
# we have to check if the degree is the
# same as if a and b were parameters.
# Why do not we deal with the irreducibility
# of parametric polynomials? Because this
# would exhaust the capacity of Maple sooner.
#
#
#
#
#
######################################
# **22** Proposition 3.2, n=15       #
######################################
#
> #
> n:=15; p:= n-4; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 15$$
$$p := 11$$
$$degree\_of\_f := 11$$
$$is\_f\_irreducible := true$$
$$degree\_check := 0$$
$$f\_is\_this = 1024\,x^{11} - 2816\,x^9 + 2816\,x^7 - 1232\,x^5 + 128\,x^4 + 220\,x^3 - 32\,x^2 - 11\,x + 1$$
$$fparam\_is = 1024\,x^{11} - 2816\,x^9 + 2816\,x^7 - 1232\,x^5 + 8\,b^4\,x^4 + 220\,x^3 - 8\,b^2\,x^2 - 11\,x + 1$$

```
> #
#
#
#
#
#
######################################
# **23** Proposition 3.2, n=17       #
```

```
#########################################
#
> #
> n:=17; p:= n-4; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 17$$
$$p := 13$$
$$degree\_of\_f := 13$$
$$is\_f\_irreducible := true$$
$$degree\_check := 0$$

$f\_is\_this =$

$$4096\,x^{13} - 13312\,x^{11} + 16640\,x^{9} - 9984\,x^{7} + 2912\,x^{5} + 128\,x^{4} - 364\,x^{3} - 32\,x^{2} + 13\,x + 1$$

$fparam\_is =$

$$4096\,x^{13} - 13312\,x^{11} + 16640\,x^{9} - 9984\,x^{7} + 2912\,x^{5} + 8\,b^{4}\,x^{4} - 364\,x^{3} - 8\,b^{2}\,x^{2} + 13\,x + 1$$

```
> #
#
#
#
#
#
#########################################
# **24** Proposition 3.2, n=51       #
#########################################
#
> #
> n:=51; p:= n-4; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 51$$
$$p := 47$$
$$degree\_of\_f := 47$$
$$is\_f\_irreducible := true$$
$$degree\_check := 0$$

```
> #
#
```

```
#
#
#
#
#
######################################
# **25** Proposition 3.2, n=85       #
######################################
#
> #
> n:=85; p:= n-12; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 85$$

$$p := 73$$

$$degree\_of\_f := 73$$

$$is\_f\_irreducible := true$$

$$degree\_check := 0$$

```
> #
#
#
#
#
#
#
######################################
# **26** Proposition 3.2, n=255       #
######################################
#
> #
> n:=255; p:= n-124; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 255$$

$$p := 131$$

$$degree\_of\_f := 131$$

$$is\_f\_irreducible := true$$

$$degree\_check := 0$$

```
> #
```

```
#
#
#
#
#
#
######################################
# **27** Proposition 3.2, n=257      #
######################################
#
> #
> n:=257; p:= n-126; f:=sort(expand( dpoly(n,p,1,2, x))):
degree_of_f:=degree(f,x); is_f_irreducible:=irreduc(f);
fparam:=sort(collect(expand( dpoly(n,p,1,b, x)),x),x):
degree_check:=degree(fparam,x)-degree_of_f;
if degree_of_f<40 then print(f_is_this=f) fi;
if degree_of_f<40 then print(fparam_is=fparam) fi;
```

$$n := 257$$

$$p := 131$$

$$degree\_of\_f := 131$$

$$is\_f\_irreducible := true$$

$$degree\_check := 0$$

\>