

# Semimodular lattices determined by permutations\*

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Motivation: **Jordan-Hölder Theorem**, 1870.

$L, [a, b] \searrow [c, d]$  iff  $[a, b] \nearrow [x, y]$  and  $[x, y] \searrow [c, d]$

G. Grätzer and J.B. Nation.

**Uniqueness:** G.Cz. & E.T. Schmidt, AU 2011; Malenovice 2010.

Main tool: **slim semimodular** lattices. 15 minutes  $\Rightarrow$  diagrams!

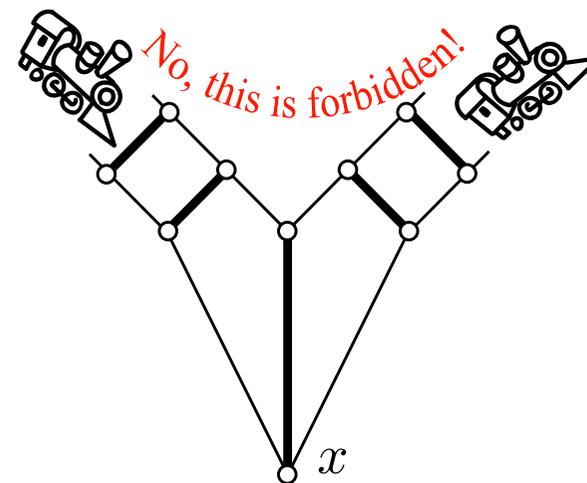
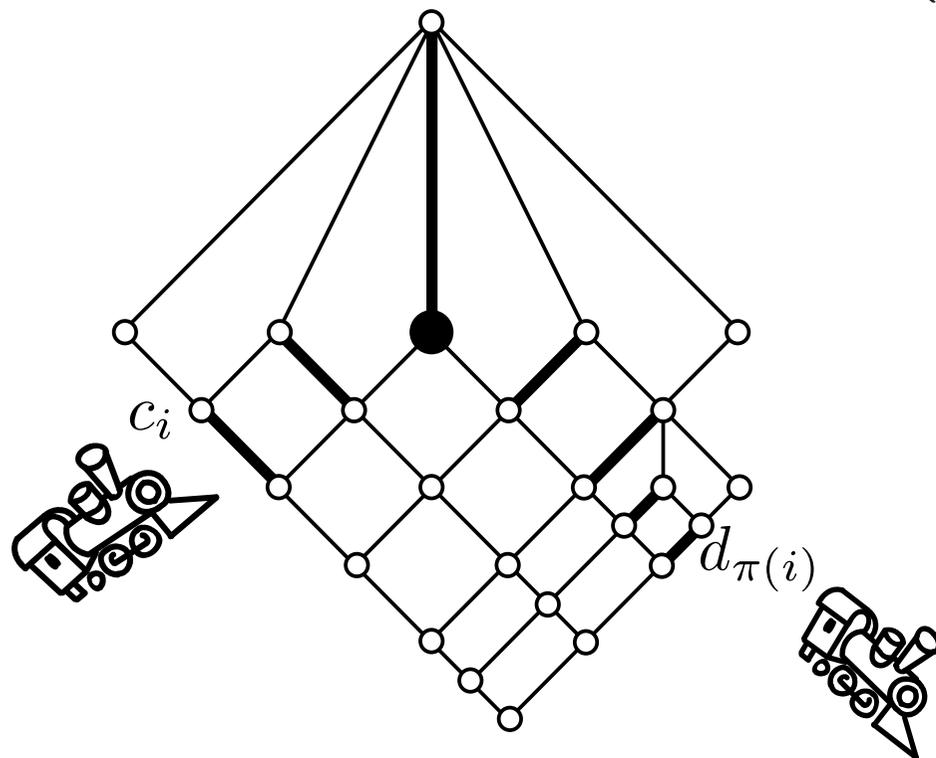
**Main Theorem**  $D \mapsto \pi_D$  and  $\pi \mapsto D_\pi$  are reciprocal bijections between {diagrams of length n} and  $S_n$ .

Several approaches.

**Applications:**

- Counting  $L$ 's of a given length  $n$ : AAA83 Novi Sad 2012 (with: László Ozsvárt and Balázs Uvari); up to  $n \leq 100$  and, also, asymptotically.
- Counting  $L$ 's and  $D$ 's of a given size  $\leq 50$ , Szeged, June 21–25, 2012 (with: Tamás Dékány, László Ozsvárt, Nóra Szakács, and Balázs Uvari). Also,  $D$  asymptotically (G.Cz).
- Representing slim semimodular lattices with subgroups of finite Abelian groups (G.Cz–E.T.Sch.; G.Cz., L.O., and B.U.).

Opposite sides, trajectories (railways,  $\rightarrow$ ), properties.

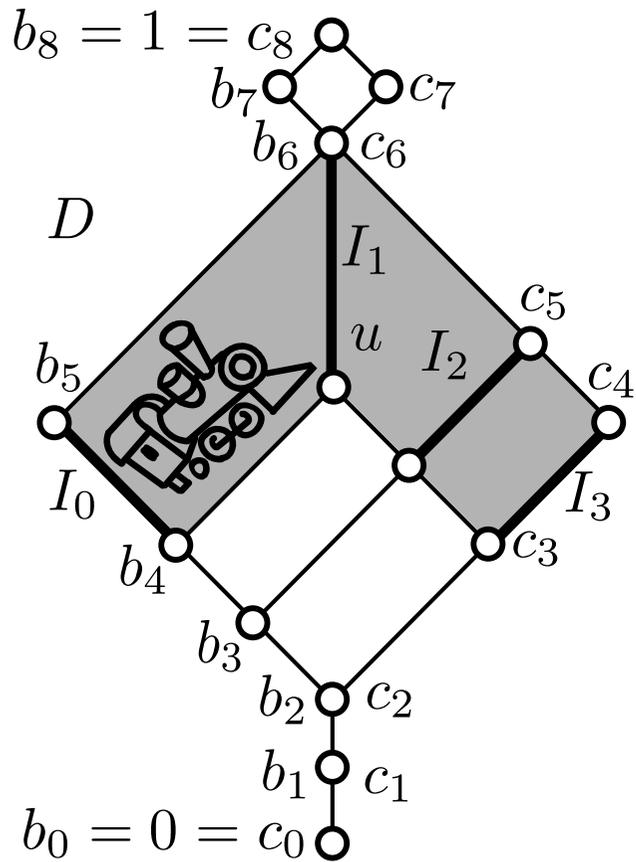


Def<sub>1</sub>:  $\pi_D$

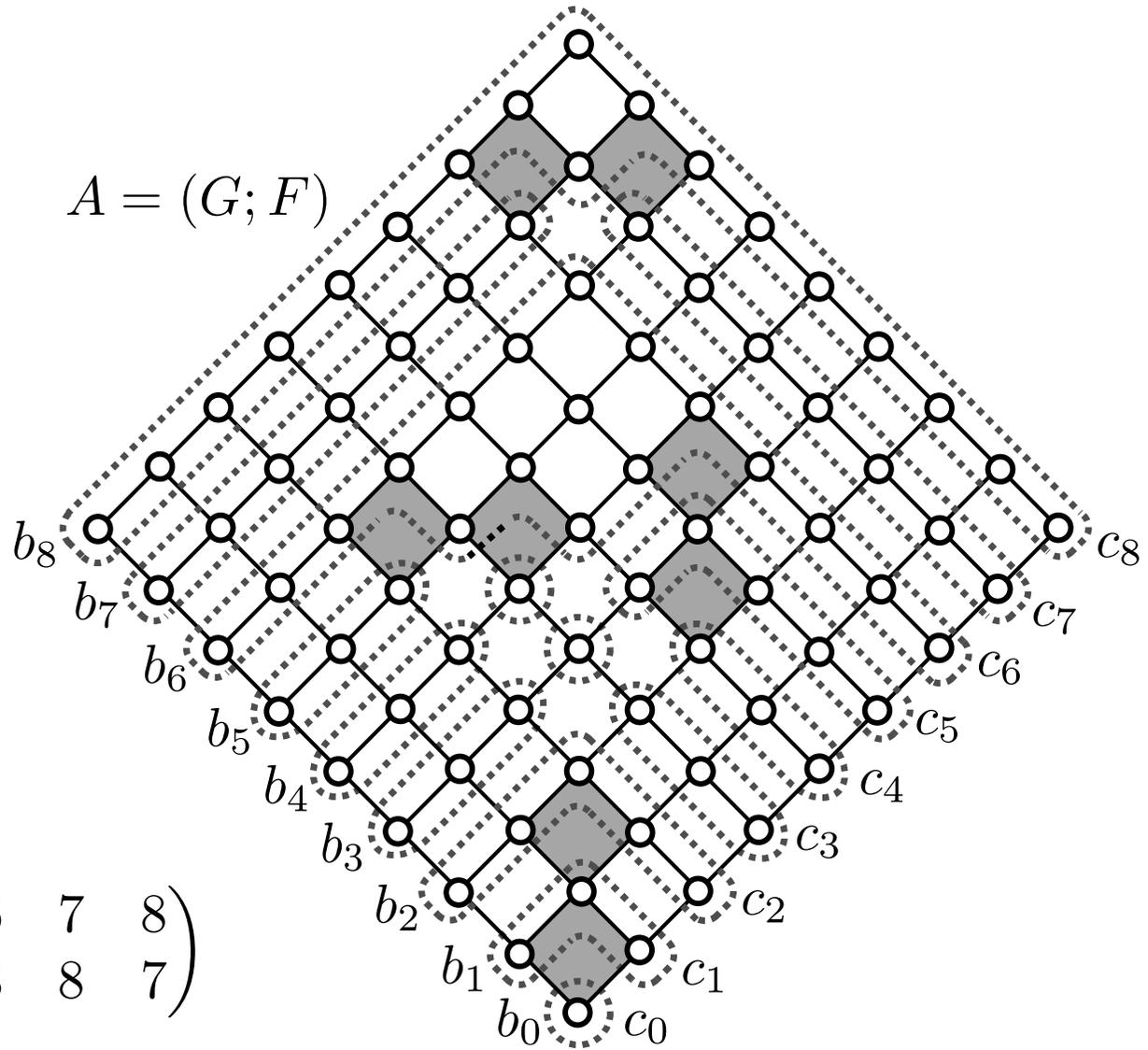
Def<sub>2</sub>:  $\pi = \{(i_u, j_u) : u \in M(L)\}$ . !

Same  $\pi$ ?

Does  $\pi_D$  determine  $D$  (and how)?



$$A = (G; F)$$



$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 5 & 6 & 4 & 3 & 8 & 7 \end{pmatrix}$$

$\varphi: (b_i, c_j) \mapsto b_i \vee c_j$ , diagram homomorphism, source cell,

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