

The asymptotic number of ways to intersect two composition series

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Assume that $\vec{H} = \{1 = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_n = G\}$ and $\vec{K} = \{1 = K_0 \triangleleft K_1 \triangleleft \cdots \triangleleft K_n = G\}$ are composition series of a group G . Let $(\{H_i \cap K_j : i, j \in \{0, \dots, n\}\}; \subseteq)$ be denoted by $\text{CSL}_G(\vec{H}, \vec{K})$. It is a partially ordered set. Actually, $\text{CSL}_G(\vec{H}, \vec{K})$ is a lower semimodular lattice of length n . We call it a *composition series lattice*; this is where the notation CSL comes from. The number of (isomorphism classes of) lattices $\text{CSL}_G(\vec{H}, \vec{K})$ of length n is denoted by $f(n)$. Our goal is to determine its asymptotic behavior as follows.

THEOREM. $f(n)$ is asymptotically $n!/2$. That is, $f(n)/n! \rightarrow 1/2$ as $n \rightarrow \infty$.

The proof is based on three different areas. From group theory, we need H. Wielandt's 1939 result implying that $\text{CSL}_G(\vec{H}, \vec{K})$ is really a lattice. From lattice theory, we need a recent description of these lattices by permutations, due to G. Czédli and E. T. Schmidt. Finally, since different permutations may determine isomorphic lattices, we need a combinatorial argument to conclude the proof; this part is due to G. Czédli, L. Ozsvárt, and B. Udvari.

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