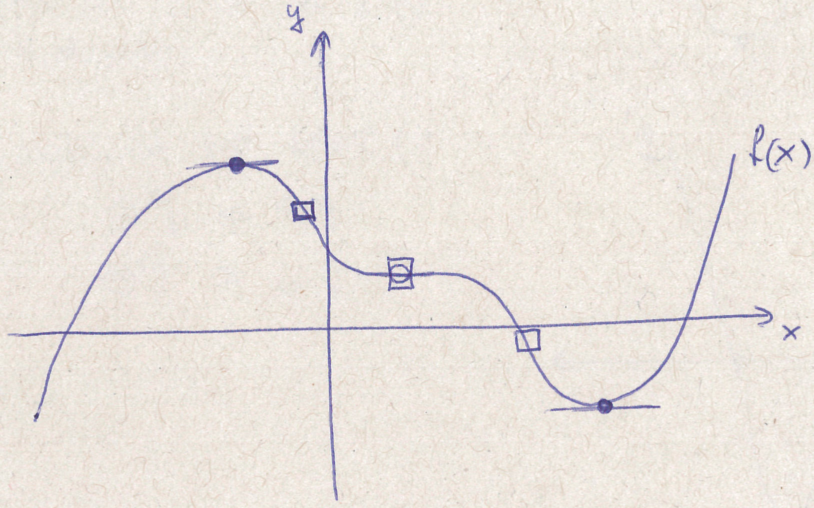
Practice 5

Graphical derivative

$f'(x_0)$ means the slope of the tangent line to the f at x_0



Critical points:

1.) Extremal points (•)
$f'=0$.

2.) Flat part (o)
$f'=0$.

3.) Inflection point: (□)
f' has extremum

Inflection point: At x_0 the function f has an inflection point, if $f(x)$ changes its convexity around x_0.

Rules: 1.) f has extremum $\Rightarrow f'=0$

$\nLeftarrow$

2.) $f \nearrow \Leftrightarrow f' = \oplus$

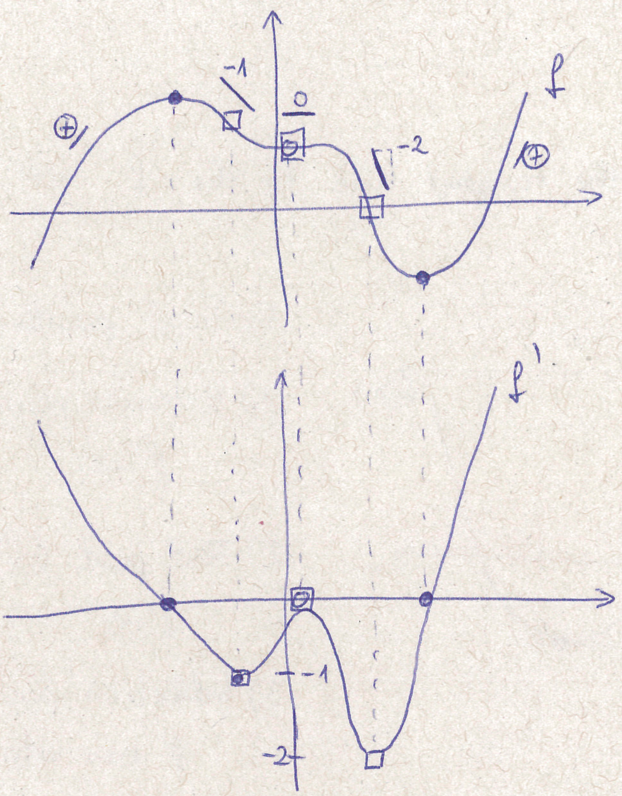
$f \searrow \Leftrightarrow f' = \ominus$

3.) $f \cup$ (convex) $\Leftrightarrow f'' = \oplus$

$f \cap$ (concave) $\Leftrightarrow f'' = \ominus$

4.) f has inflection point $\Leftrightarrow$ f' has extremum $\Rightarrow f''=0$

Problem 1. Sketch the derivative of the following function:



1.) f extrema ⇒ $f' = 0$ (•)

2.) f flat part ⇒ $f' \approx 0$ (o)

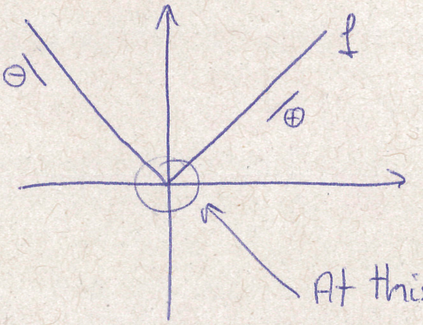
3.) f IP ⇒ f' has extremum (□)
(estimate it)

4.) Sketch the derivative

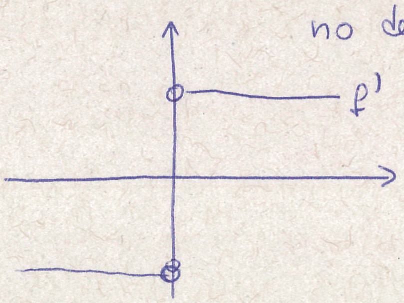
$f \nearrow \Rightarrow f' \oplus$

$f \searrow \Rightarrow f' \ominus$

Problem 2. What about the $abs(x) = |x|$ function?



At this peak point, the function has no derivative.



Derivatives of basic functions

$$c' = 0$$
$$x' = 1$$
$$\left[x^{\alpha}\right]' = \alpha x^{\alpha-1}$$

$$\left[\ln x\right]' = \frac{1}{x}$$
$$\left[\log_a x\right]' = \frac{1}{x \cdot \ln a}$$
$$\left[\lg x\right]' = \frac{1}{x \cdot \ln 10}$$

$$\left[e^x\right]' = e^x$$
$$\left[a^x\right]' = a^x \cdot \ln a$$

$$\left[\sin x\right]' = \cos x$$
$$\left[\cos x\right]' = -\sin x$$
$$\left[\tan x\right]' = \frac{1}{\cos^2 x}$$

Rules of derivatives

$$\left[f + g\right]' = f' + g'$$
$$\left[f - g\right]' = f' - g'$$
$$\left[c \cdot f\right]' = c \cdot f'$$

$$\left[f \cdot g\right]' = f'g + fg'$$
$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)$$

Problem 3. Give the derivative of the following functions:

1.) $$\left[x^2 - 3x + 5\sqrt{x}\right]' = \left[x^2 - 3x + 5 \cdot x^{1/2}\right]' = 2x - 3 + 5 \cdot \frac{1}{2} x^{-1/2}$$

2.) $$\left[\sqrt[3]{x} \cdot \sin x\right]' = \left[x^{1/3} \cdot \sin x\right]' = \frac{1}{3} x^{-2/3} \cdot \sin x + x^{1/3} \cdot \cos x$$

3.) $$\left[\frac{\ln x}{e^x + \frac{1}{x}}\right]' = \left[\frac{\ln x}{e^x + x^{-1}}\right]' = \frac{\frac{1}{x} \cdot (e^x + x^{-1}) - \ln x \cdot (e^x - x^{-2})}{(e^x + x^{-1})^2}$$

4.) $$\Big[\underbrace{x^2}_{f} \cdot \underbrace{2^x \cdot \log_2 x}_{g}\Big]' = x^2 \cdot \left[2^x \cdot \log_2 x\right]' + 2x \cdot \left(2^x \cdot \log_2 x\right) = x^2\left(2^x \cdot \ln 2 \cdot \log_2 x + 2^x \frac{1}{x \ln 2}\right)$$
$$+ 2x\left(2^x \cdot \log_2 x\right)$$

5.) $\left[\sin(x^2)\right]' = \underline{\underline{\cos x^2 \cdot (2x)}}$

$f = \sin x \rightarrow f' = \cos x$

$g = x^2 \rightarrow g' = 2x$

6.) $\left[e^{-x^2}\right]' = \underline{\underline{e^{-x^2} \cdot (-2x)}}$

$f = e^x \rightarrow f' = e^x$

$g = -x^2 \rightarrow g' = -2x$

7.) $\left[\ln(x \cdot \cos x)\right]' = \underline{\underline{\frac{1}{x \cdot \cos x} \cdot (\cos x - x \sin x)}}$

$f = \ln x \rightarrow f' = \frac{1}{x}$

$g = x \cdot \cos x \rightarrow g' = \cos x + x(-\sin x)$

8.) $\left[\frac{1}{x \cdot e^{x^3}}\right]' = \underline{\underline{-\frac{1}{(x \cdot e^{x^3})^2} \cdot \left(e^{x^3} + 3x^3 e^{x^3}\right)}}$

$f = \frac{1}{x} \rightarrow f' = -\frac{1}{x^2} \quad \left(= -x^{-2}\right)$

$g = x \cdot e^{x^3} \rightarrow g' = e^{x^3} + x \cdot e^{x^3} \cdot 3x^2$

9.) $\left[\sqrt[3]{x^2 - \sin(2x)}\right]' = \underline{\underline{\frac{1}{3} \cdot (x^2 - \sin 2x)^{-2/3} \cdot (2x - 2\cos 2x)}}$

$f = \sqrt[3]{x} = x^{1/3} \rightarrow f' = \frac{1}{3} \cdot x^{-2/3}$

$g = x^2 - \sin 2x \rightarrow g' = 2x - \cos 2x \cdot 2$

10.) $\left[\cos(\ln x)\right]' = \underline{\underline{-\sin(\ln x) \cdot \frac{1}{x}}}$

$f = \cos x \rightarrow f' = -\sin x$

$g = \ln x \rightarrow g' = \frac{1}{x}$