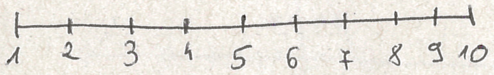
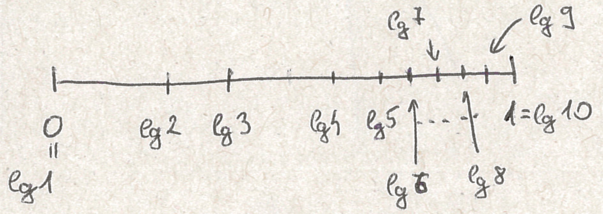
Practice 4

Logarithmic transformation

Linear scale :



Log. scale :



$\lg 1 = 0$

$\lg 2 = 0{,}3$

$\lg 3 = 0{,}4$

$\lg 4 = 2\lg 2 = 0{,}6$

$\lg 5 = \lg \frac{10}{2} = 1 - \lg 2 = 0{,}7$

$\lg 6 = \lg 2 + \lg 3 = 0{,}7$

$\lg 8 = 3 \lg 2 = 0{,}9$

$\lg 9 = 2 \cdot \lg 3$

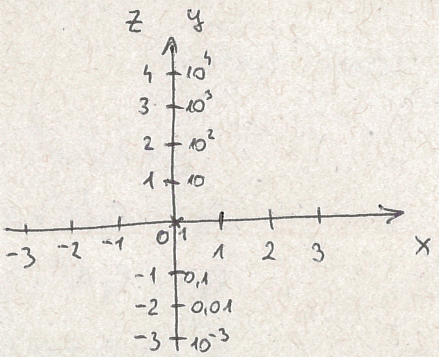
$\lg 10 = 1$

Advantage of the transformation: we can compress a too large axis.

$$[1, 10^6] \xrightarrow[\lg(\ )]{} [0, 6]$$

If we use this technique, we can plot exp. and log. functions with smaller numbers, but we do not lose information.

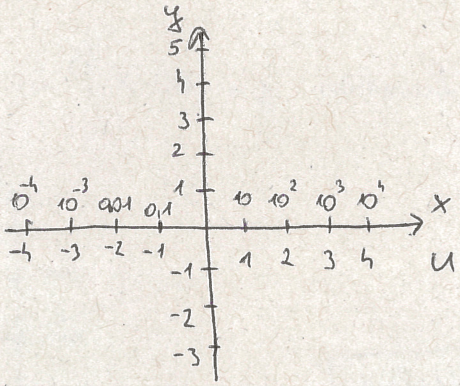
Logarithmic coordinate systems



lin - log coord. sys.

$(x - z)$

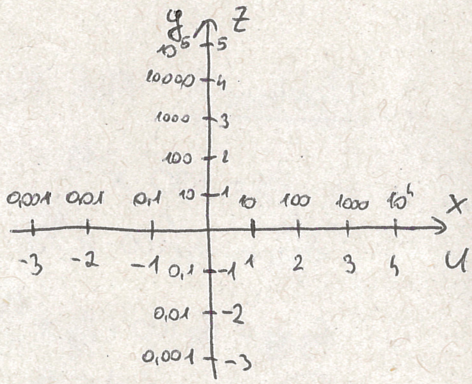
$\boxed{z = \lg y}$



log - lin coord. sys.

$(u - y)$

$\boxed{u = \lg x}$



log - log coord sys.

$(u - z)$

$\boxed{\begin{array}{l} u = \lg x \\ z = \lg y \end{array}}$

Problem 1 Plot the function $y = 10^{2x+3}$ in log. coord. system.

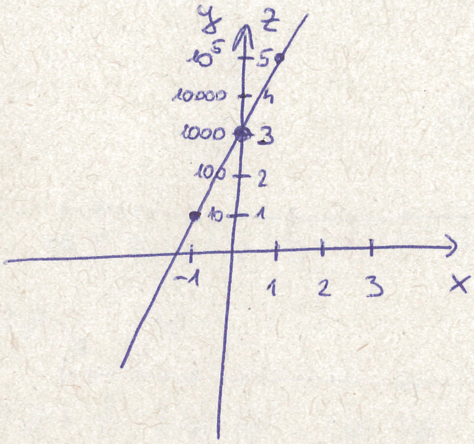
$y = 10^{2x+3}$

$\lg y = 2x+3 \quad \boxed{z = \lg y}$

$\underline{\underline{z = 2x+3}}$

$\uparrow$

This is an equation of a line in the (x, z) coord. system.



$y = 10^{ax+b}$

$\lg(y) = ax+b$

$z = ax+b$

$\boxed{\text{Exponential function} \Downarrow \text{lin-log coord. sys.}}$

Problem 2. Plot the function $y = \lg(0{,}1 \cdot x^{2/3})$ in log. coord. system.

$y = \lg(0{,}1 \cdot x^{2/3})$

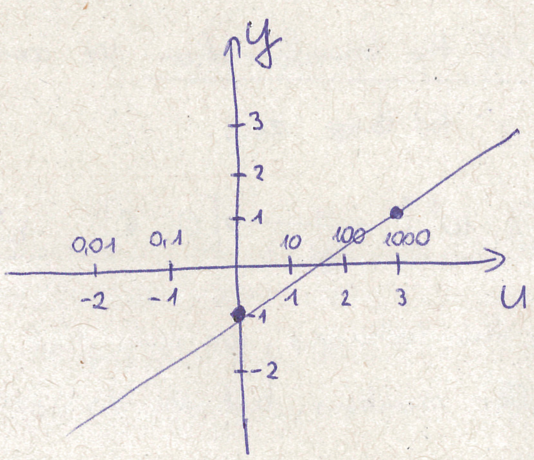
$y = \lg 0{,}1 + \lg x^{2/3}$

$y = -1 + \frac{2}{3}\lg x \quad \boxed{u = \lg x}$

$\underline{\underline{y = \frac{2}{3}u - 1}}$

$\uparrow$

This is an equation of a line in the (u, y) coord. system.



$y = \lg(10^b \cdot x^a)$

$y = \lg 10^b + \lg x^a$

$y = b + a \lg x$

$y = a \cdot u + b$

$\boxed{\text{Log. function} \Downarrow \text{log-log coord. s.}}$

Problem 3. Plot the function $y = 100 \cdot x^{-1/3}$ in log. coord. system.

$y = 100\, x^{-1/3}$

$\lg y = \lg(100 \cdot x^{-1/3})$

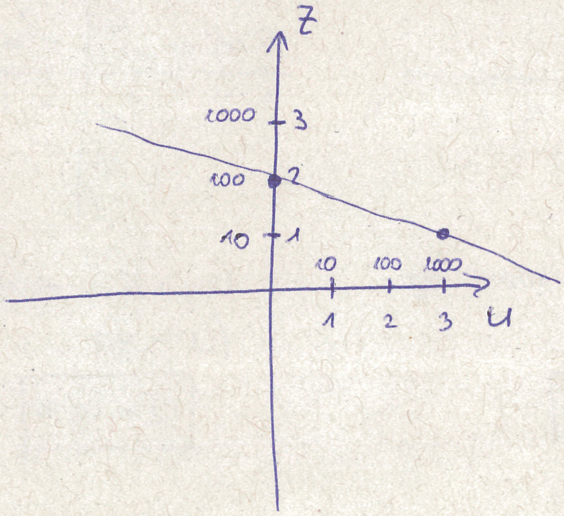
$\lg y = \lg 100 + \lg x^{-1/3}$

$\lg y = 2 + \left(-\frac{1}{3}\right)\lg x \quad \boxed{\begin{array}{c} u = \lg x \\ \hline z = \lg y \end{array}}$

$\underline{\underline{z = -\frac{1}{3}u + 2}}$

$\uparrow$

This is an equation of a line in the (u, z) coordinate system.



$y = 10^b \cdot x^a$

$\lg y = \lg(10^b \cdot x^a)$

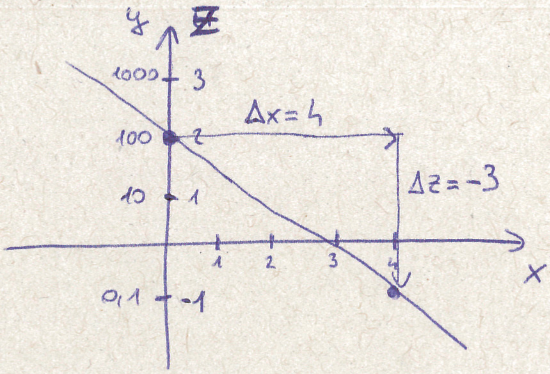
$\lg y = \lg 10^b + \lg x^a$

$\lg y = b + a \lg x$

$z = au + b$

$\boxed{\text{Power function} \Downarrow \text{log-log. coord. s.}}$

Problem 4. Give the equation of the following line.



lin-log $\Rightarrow$ x-z $\boxed{z = \lg y}$

Equation of the line: $z = ax + b$

$b = 2$ (intersection with vert. axis)

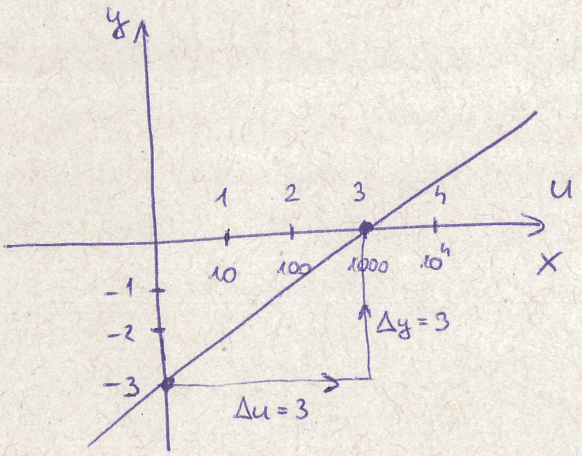
$a = \frac{\Delta z}{\Delta x} = -\frac{3}{4}$

$$z = -\frac{3}{4}x + 2$$

$$\lg y = -\frac{3}{4}x + 2$$

$$\underline{\underline{y = 10^{-\frac{3}{4}x + 2}}}$$

Problem 5. Give the equation of the following line.



log-lin $\Rightarrow$ u-y $\boxed{u = \lg x}$

$y = ax + b$

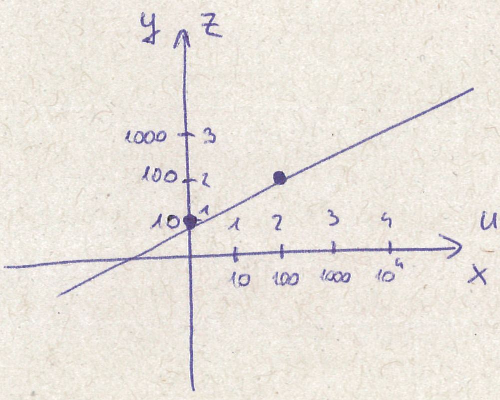
$b = -3$

$a = \frac{\Delta y}{\Delta u} = \frac{3}{3} = 1$

$y = u - 3$

$$\underline{\underline{y = \lg x - 3}}$$

Problem 6. Give the equation of the following line.



log-log $\Rightarrow$ u-z $\boxed{u = \lg x,\ z = \lg y}$

$z = au + b$

$b = 1$

$a = \frac{1}{2}$

$z = \frac{1}{2}u + 1$

$\lg y = \frac{1}{2}\lg x + 1$

$$y = 10^{\frac{1}{2}\lg x + 1} = 10^{\lg x^{1/2}} \cdot 10 = \underline{\underline{10\,x^{1/2}}}$$