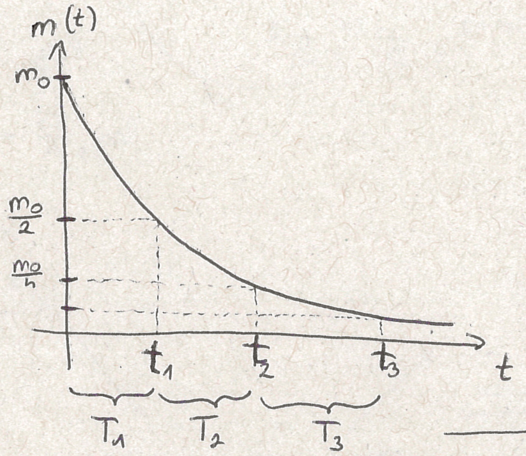
Practice 3.

Application of inverse : half-life, doubling time

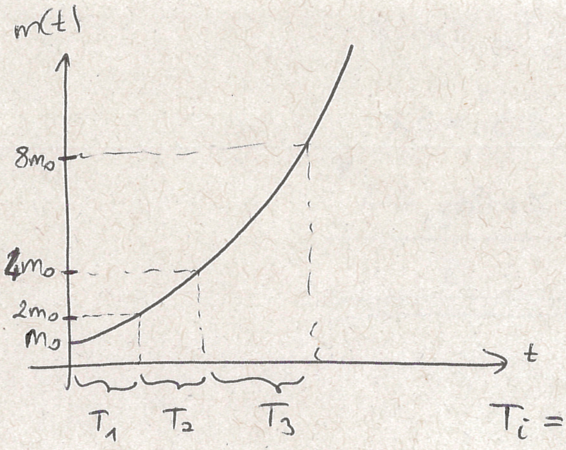
• Graphical approach :



⟶ In general $T_i \neq T_j$.

$T = T_1 = T_2 = T_3 = \dots$ if and only if $m(t)$ is exponential function.

In this case : T is the **half-life** of the function $m(t)$.



Doubling time

$T_i = T_j \iff m(t)$ is exponential

Increasing exp. function ⟺ doubling time
Decreasing exp. function ⟺ half-life

Eg. Give the half-life of the function $f(x) = 5^{-3x+2}$.

1.) $f(0) = 5^{-3\cdot 0+2} = 25$

2.) $\frac{f(0)}{2} = f(x)$

$\frac{25}{2} = 5^{-3x+2} = 5^{-3x} \cdot 5^2$

$\frac{1}{2} = 5^{-3x}$

$\log_5 \frac{1}{2} = -3x$

$$x = \frac{\log_5 1/2}{-3}$$

OR: $\frac{25}{2} = 5^{-3x+2}$

$\log_5 \frac{25}{2} = -3x+2$

$\log_5 \frac{25}{2} - 2 = -3x$

$$x = \frac{\log_5 \frac{25}{2} - 2}{-3}$$

These expressions are numbers, so we are ready.

Eg. Determine the doubling time of $f(x) = 3^{5x-7}$

1.) $f(0) = 3^{-7}$

2.) $2 f(0) = f(x)$

$2 \cdot 3^{-7} = 3^{5x-7}$

$2 = 3^{5x}$

$\log_3 2 = 5x$

$$x = \frac{\log_3 2}{5}$$

or $\longrightarrow$ $2 \cdot 3^{-7} = 3^{5x-7}$

$\log_3 2 \cdot 3^{-7} = 5x - 7$

$7 + \log_3 2 \cdot 3^{-7} = 5x$

$$x = \frac{7 + \log_3 2 \cdot 3^{-7}}{5}$$

Basic transformations of functions

We have a basic f function and a complicate function g derived from f.

Eg: $f(x) = x^2$ $\quad g(x) = -2(2x+3)^2 + 1$

In general: $f(x)$ $\quad g(x) = a\,f(bx+c) + d$

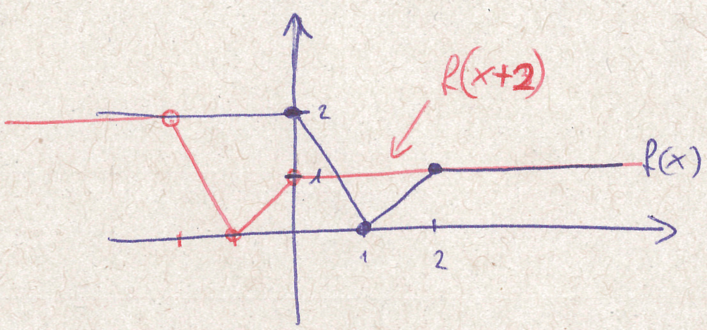
How can we derive the graph of $g(x)$ from graph of $f(x)$?

Step 1. Plot $f(x)$.

Step 2. Plot $f(x+c)$.

$c>0 \Rightarrow$ Shift left

$c<0 \Rightarrow$ Shift right



Step 3. Plot $f(bx+c)$.

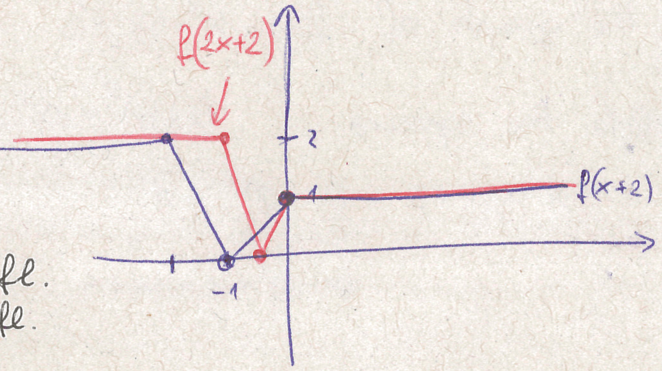
$b>1 \Rightarrow$ horizontal compress

$0<b<1 \Rightarrow$ — " — strech

$b=-1 \Rightarrow$ reflection to the vertical axis

$b<-1 \Rightarrow$ horizontal compress + refl.

$-1<b<0 \Rightarrow$ — " — strech + refl.



Step 4. Plot $a \cdot f(bx+c)$

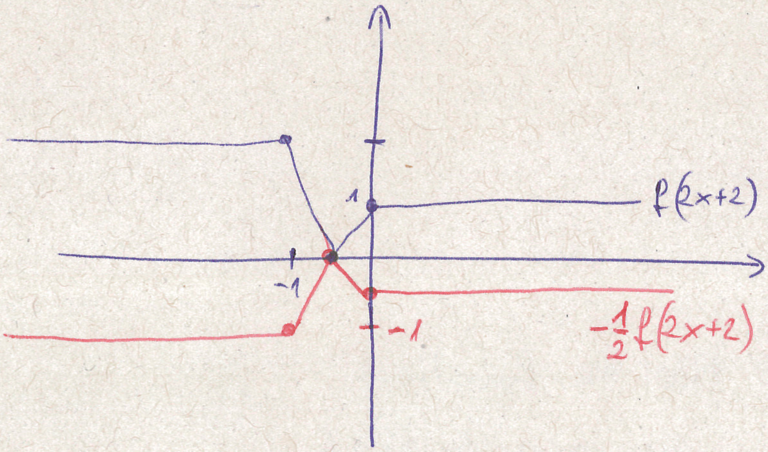
$a>1 \Rightarrow$ stretch vertically

$0<a<1 \Rightarrow$ compress vertically

$a=-1 \Rightarrow$ reflection to the horizontal axis

$a<-1 \Rightarrow$ strech + refl

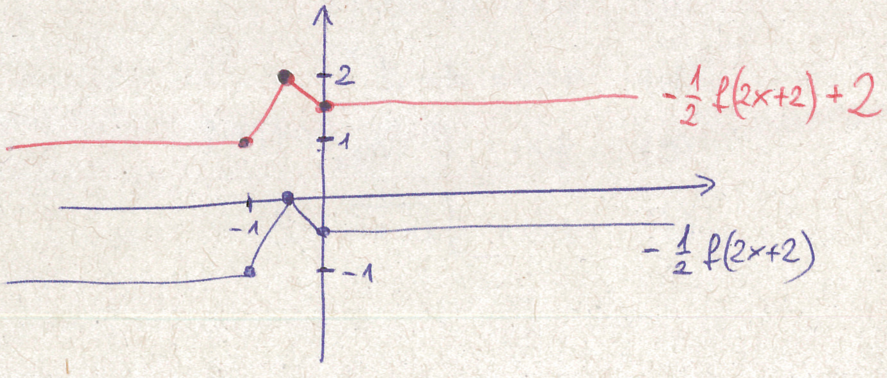
$-1<a<0 \Rightarrow$ compress + refl.



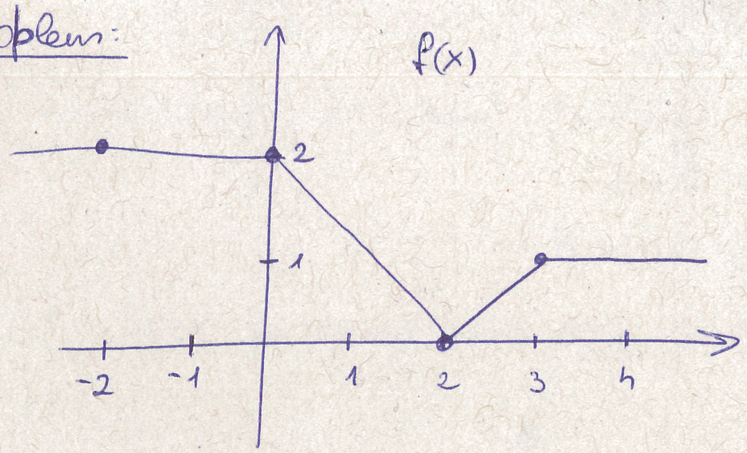
Step 5. Plot $a \cdot f(bx+c) + d$

$d>0 \Rightarrow$ Shift upward

$d<0 \Rightarrow$ Shift downward



Problem:



Draw the graphs of the following functions.

(a) $f(2x)$

(b) $-f(x+1)$

(c) $2f(x-1)+2$

(d) $1-f(-x)$

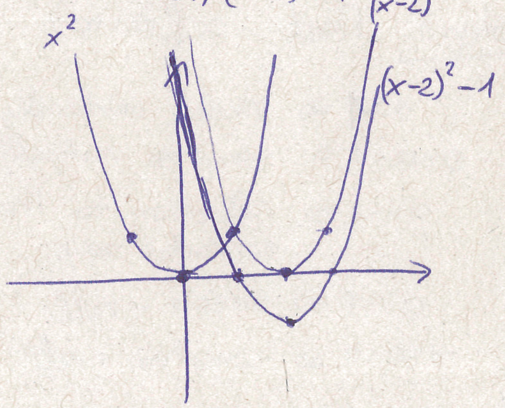
1.) $f(x)$
2.) $-$
3.) $f(-x)$
4.) $-f(-x)$
5.) $-f(-x)+1$

Eg. Draw the graph of $f(x) = x^2-4x+3$ using transformations.

Complete the square: $(a\pm b)^2 = a^2 \pm 2ab + b$

$$x^2-4x+3 = \underbrace{(x-2)^2}_{(x-2)^2 = x^2-4x+4} - 1$$

Plotting:
1.) x^2
2.) $(x-2)^2$
3.) $-$
4.) $-$
5.) $(x-2)^2-1$



Eg. Plot the functions.

(a) $g(x) = (x-3)^3+1$

(b) $g(x) = 1-2^{-x}$

(c) $g(x) = 2\cdot \log_2(x-1)+1$

(d) $g(x) = \frac{1}{2}\sqrt{x+1} - 2$