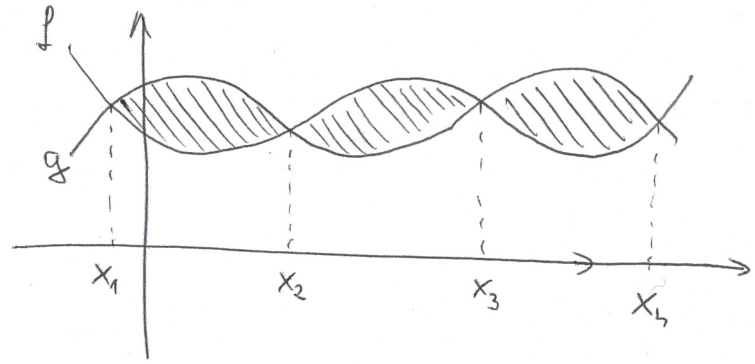
Practice 11

Applications of definite integral

(4.) Area between two functions.



$$\boxed{T = \int_{x_1}^{x_4} |f(x) - g(x)|\, dx}$$

Not so easy to calculate!

Alternative method:

1.) $f(x) = g(x)$: solutions : $x_1 < x_2 < x_3 < x_4$

2.) $$T = \left| \int_{x_1}^{x_2} f(x) - g(x)\, dx \right| + \left| \int_{x_2}^{x_3} f(x) - g(x)\, dx \right| + \left| \int_{x_3}^{x_4} f(x) - g(x)\, dx \right|$$

Problem 1. Calculate the absolute area enclosed by the functions $f(x) = x^3$ and $g(x) = 3x^2 - 2x$.

1.) $x^3 = 3x^2 - 2x$

$x^3 - 3x^2 + 2x = 0$

$x(x^2 - 3x + 2) = 0$

$\underline{x = 0}$ or $x^2 - 3x + 2 = 0$

$\underline{x_1 = 1}$ $\quad$ $\underline{x_2 = 2}$

2.) $\int f(x) - g(x)\, dx =$

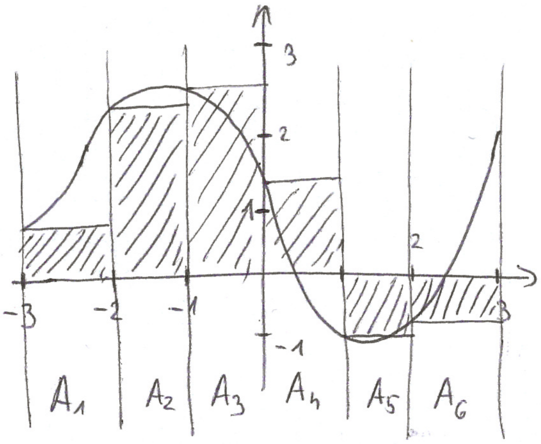
$\int (x^3 - 3x^2 + 2x)\, dx$

$= \frac{x^4}{4} - x^3 + x^2 + C$

$$T = \left| \int_0^1 f - g\, dx \right| + \left| \int_1^2 f - g\, dx \right| = \left| \underbrace{\left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1}_{1/4} \right| + \left| \underbrace{\left[\frac{x^4}{4} - x^3 + x^2 \right]_1^2}_{-1/4} \right| = \frac{1}{4} + \frac{1}{4} = \boxed{\frac{1}{2}}$$

5. Approximation of the definite integral

5/1 Left side approximation

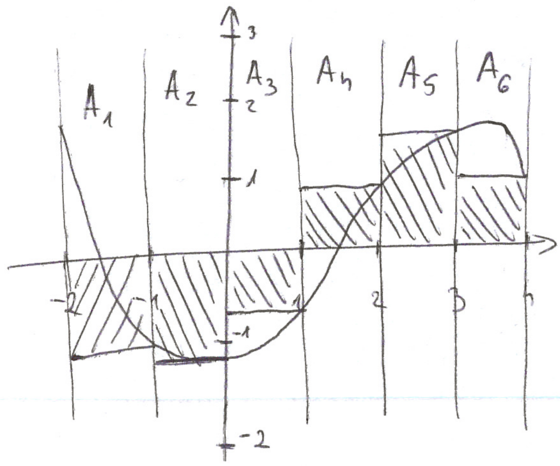


$$A_1 \approx 1 \cdot 0{,}9 = 0{,}9$$
$$A_2 \approx 1 \cdot 2{,}4 = 2{,}4$$
$$A_3 \approx 1 \cdot 2{,}6 = 2{,}6$$
$$A_4 \approx 1 \cdot 1{,}4 = 1{,}4$$
$$A_5 \approx 1 \cdot (-1) = -1$$
$$A_6 \approx 1 \cdot (-0{,}8) = -0{,}8$$

$$A_1 + A_2 + \dots + A_6 = \underline{\underline{5{,}5}}$$

The heigt of the rectangles are determined by the left sides of the function in each interval.

5/2 Righ side approximation

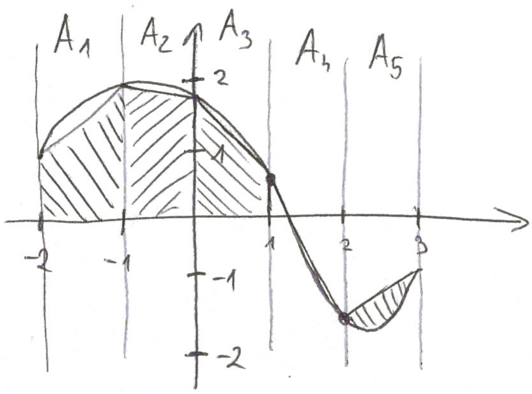


$$A_1 \approx 1 \cdot (-1) = -1$$
$$A_2 \approx 1 \cdot (-1{,}2) = -1{,}2$$
$$A_3 \approx 1 \cdot (-0{,}7) = -0{,}7$$
$$A_4 \approx 1 \cdot 0{,}9 = 0{,}9$$
$$A_5 \approx 1 \cdot 1{,}6 = 1{,}6$$
$$A_6 \approx 1 \cdot 1 = 1$$

$$A_1 + A_2 + \dots + A_6 = \underline{\underline{0{,}6}}$$

The heigh of the rectangles are determined by the right side of the function in each interval.

5/3 Trapezoid approximation



$$A_1 \approx \frac{(1 + 1{,}9) \cdot 1}{2} = 1{,}45$$
$$A_2 \approx \frac{(1{,}9 + 1{,}8) \cdot 1}{2} = 1{,}85$$
$$A_3 \approx \frac{(1{,}8 + 0{,}6) \cdot 1}{2} = 1{,}2$$
$$A_4 \approx \frac{(0{,}6 + (-1{,}5)) \cdot 1}{2} = -0{,}45$$
$$A_5 \approx \frac{(-1{,}5 + (-1)) \cdot 1}{2} = -1{,}25$$

$$A_1 + \dots + A_5 = \underline{\underline{2{,}8}}$$