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Asymptotics of Carleman polynomials for level curves of the inverse of a shifted Zhukovsky transformation

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This work complements the recent investigation of [DMD10] on the asymptotic behavior of polynomials orthogonal over the interior of an analytic Jordan curve L. We study the specific case of $L = \{z = w - 1 + (w - 1)^{-1}, |w| =$ R, for some R > 2, providing an example that exhibits the new features discovered in [DMD10], and for which the asymptotic behavior of the corresponding orthonormal polynomials $\{p_n(z)\}_{n=0}^{\infty}$ is established over the entire domain of orthogonality. Surprisingly, this variation of the classical example of the ellipse turns out to be quite sophisticated. For instance, we find that after properly normalizing the p_n 's, and on certain critical subregion of the orthogonality domain, a subsequence $\{p_{n_k}\}$ converges if and only if $\log_{\mu}(n_k)$ converges modulo 1 (μ being an important quantity associated to L). As a consequence, the limiting points of the sequence $\{p_n\}$ form a one parameter family of functions, the parameter's range being the interval [0, 1]. The behavior of the polynomials is much influenced by certain integrand function, the explained behavior being the result of this integrand having a non-isolated singularity that is a cluster point of poles. The nature of this singularity sparks purely from geometric considerations, as opposed to the more common situation where the critical singularities come from the orthogonality weight.

References

[DMD10] Peter Dragnev and Erwin Miña-Díaz. Asymptotic behavior and zero distribution of Carleman orthogonal polynomials. J. Approx. Theory, 162(11):1982-2003, 2010. CODEN JAXTAZ. ISSN 0021-9045. URL http://dx.doi.org/10.1016/j.jat.2010.05.006.