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A potential kernel on the half-line related to a q-analogue of the Digamma function

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Potential kernels on a group G (resp. the half-line) are measures of the form

$$\kappa = \int_0^\infty \mu_t \, dt,$$

for a convolution semigroup $(\mu_t)_{t>0}$ of subprobabilities on G (resp. $[0,\infty[$), i.e.

$$\mu_t * \mu_s = \mu_{t+s}, \quad \mu_t([0,\infty[) \leq 1, \quad \lim_{t\to 0} \mu_t = \delta_0.$$

The most famous potential kernel for $G = \mathbb{R}^3$ is the Newtonian kernel

$$\frac{1}{4\pi||x||} = \int_0^\infty (4\pi t)^{-3/2} \exp(-||x||^2/4t) \, dt, \quad x \in \mathbb{R}^3,$$

where we integrate the Gaussian convolution semigroup of normal distributions on $\mathbb{R}^3$ with respect to t. This formula is the clue to the relationship between classical potential theory and Brownian motion.

A. Durán and the speaker have studied a transformation from normalized Hausdorff moment sequences (a_n) to normalized Hausdorff moment sequences $T(a_n)$ defined by

$$T(a_n)_n = (1 + a_1 + \cdots + a_n)^{-1}.$$

I will discuss joint work with Helle B. Petersen in which the iteration of T starting with the moment sequence (q^n), leads to a potential kernel on the half-line related to a q-analogue of the Digamma function, where $0 < q < 1$.

References

[BP11] C. Berg and H.B. Petersen. On an iteration leading to a q-analogue of the digamma function. *Arxiv preprint arXiv:1111.0250*, 2011.