THE RADON TRANSFORM ON HYPERBOLIC SPACE

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The original Radon transform, that integrates a function in $\mathbb{R}^n$ over the hyperplanes, can be defined on the hyperbolic space $\mathbb{H}^n$ on two natural ways according to how one regards the hyperplanes in $\mathbb{R}^n$.

If one think about the hyperplanes in $\mathbb{R}^n$ as a sphere with infinite radius and center in infinity, then one gets the Radon transform on $\mathbb{H}^n$ that integrates on horospheres. This Radon transform is completely described in the literature since one can introduce a natural Fourier transform that makes very easy the calculations [P.D.Lax & R.S.Philips: Translation representations for the solution of the non–Euclidean wave equation, Comm.Pure Appl.Math. 32(1979),617–667].

If one think about the hypeplanes in $\mathbb{R}^n$ as a totally geodesic submanifolds with codimension 1, then the Radon transform on $\mathbb{H}^n$ is obtained that integrates over the totally geodesic submanifolds with codimension 1 (we call these hyperplanes). In this case until now there was no found any really good working analogy with the Euclidean case as it was the Fourier transform in the previous case [S.Helgason:The Radon transform, Birkhäuser,1980]

In my lecture I posed a new method to investigate this latter case. This method is based on the theory of spherical harmonics. The use of this theory is very natural because the Poincarte's sphere model is conform with the underlying Euclidean space.

It was shown in the talk that this is a good working analogy to the Euclidean Radon transform. This depends on that if the function f is expanded into spherical harmonics as $f = \Sigma f_{l,m} Y_{l,m}$ and similarly $Rf = \Sigma (Rf)_{l,m} Y_{l,m}$ then $(Rf)_{l,m}$ is a special one dimensional Volterra–type integral transformation of $f_{l,m}$. Thus, inverting of this integral transformation gives an inversion formula for the Radon transform. We could write this in a closed formula too.

The closed inversion formula is different for even and odd dimension. While an inversion formula in odd dimension was known, in even dimension there was no closed inversion formula before. These formulas show that the invers Radon transform in odd dimension is local and in even dimension it is global (as in the Euclidean case).

Finally we note that after considering the coefficients in the spherical harmonic expansion we also obtained a description of the range and the null space of the Radon transform.

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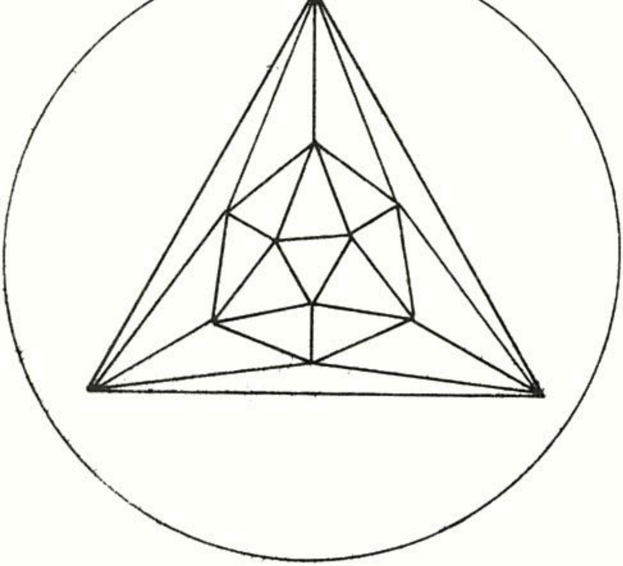

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