Curriculum Plan of the Doctoral School of Mathematics and Computer Science at the University of Szeged

April 11, 2019

I. Introduction and Table of Contents

Doctoral training starts with *admission*, proceeds in *training programs*, through *doctoral courses* and announced *research topics*, paced according to our *credit table*, and supervised each semester by *student reports*. The goal is to obtain the PhD degree, requiring passing a *comprehensive exam* according to specific *rules* and exam syllabus, meeting *foreign language* and *publication requirements*. The entire process follows the Doctoral School *Operating Regulations* and *Quality Assurance Plan*, in harmony with *higher-level legislation*. A full overview of the plan includes all *italicized* components, though to varying degrees. According to point I.4 of the Quality Assurance Plan, it is unwise to publish the same document in multiple places as this would cause confusion due to amendments. Hence this document refers to some important components (instead of copying), and in other cases notes the current date as the document status, subject to change.

Except for the current research topics listed in the *National Doctoral Database*, the information here (and its later updated versions) can be found on the *Doctoral School website*, also accessible from the *SZTE Bolyai Institute website* and the Doctoral School leader's *personal website*.

Contents:

(External reference)	*Publication Requirements* (DI-MSZ, point 5.3)	-
(External reference)	*Doctoral School Operating Regulations*	-
(External reference)	*Higher-level (comprehensive) Regulations*	-
(External reference)	*Quality Assurance Plan*	-

II. Training Programs and Their Leaders

April 9, 2019

> (Current status is always visible on the *Doctoral School website*)

The DS primarily offers courses belonging to the following training programs:

- Algebra program, led by Dr. Zádori László, university professor, Doctor of Science;
- Analysis program, led by Dr. Molnár Lajos, head of department, university professor, Doctor of Science;
- Dynamical Systems program, led by Dr. Krisztin Tibor, head of department, university professor, member of the Hungarian Academy of Sciences;
- Stochastics program, led by Dr. Pap Gyula, university professor, Doctor of Science;
- Geometry, combinatorics and theoretical computer science program, led by Dr. Hajnal Péter, habilitated head of department, university associate professor, Candidate of Science.

Exceptional research topics not belonging to any of these programs may be announced. The above programs also include didactic courses and research topics.

- Coordinator responsible for didactic training and research: Dr. Kosztolányi József, associate professor, PhD.

III. Research Topics by Training Program

April 11, 2019

> (According to the *Doctoral Database*. The list changes yearly; current status is always visible on the *Doctoral Database*. Only topics announced in English are available for English training. Current supervisors do not necessarily announce new topics.)

Algebra Training Program

Gábor Czédli: **Lattices and categories**

Lattices and categories are modern mathematical tools that enable the avoidance of direct element calculations and reveal the true cause of a theorem. It is not surprising that there are strong connections between these two tools. The research topic aims at examining these connections and further developing previous results. The announced topic has two highlighted areas. One is based on Hutchinson's theorem, studying exact embedding functors between categories of submodule lattices (Abelian), which is equivalent to studying inclusion relations of quasi-varieties of these lattices; concrete tasks include proving lattice-theoretical results achieved with several categories and extending these results. The other area is based on (partly joint with the supervisor) results of Marcel Ernè, stating that categorical closure operators can be naturally and usefully defined even for algebraic (and continuous) lattices.

Eszter Katonáné Horváth: **Boolean functions and permutation groups**

Invariance groups, i.e. permutations of variables of Boolean functions that leave the function invariant. Galois connection between permutation groups and functions, closure operator. Generalizations, open problems.

Eszter Katonáné Horváth: **Applications of lattice theory**

Concept lattice, Galois connection, lattice-valued fuzzy sets. Applications of the topic in natural, social, technical sciences, and medicine. Methodological aspects.

Ágnes Szendrei: **Study of finite algebras with clone-theoretic tools**

If A is any algebra, its clone consists of all operations derived by composition from the basic operations and projections of A. Two algebras are equivalent if their clones are equal. Equivalent algebras share all algebraically essential structural properties; thus the clone, not basic operations, is the fundamental aspect. It is well known that if A is finite, its clone is determined by its compatible relations, including algebraically important ones such as subalgebras, congruences, and isomorphisms between factor algebras. The study of the structural properties of finite algebras and their clones is a wide research field. If the variety generated by A is congruence-modular, effective commutator theory tools can be used. An important open problem is how many algebras on a given finite set (up to equivalence) generate congruence-permutable varieties. Also largely unexplored is determining invariants (compatible relations) of clones of finite groups and other classical algebraic structures.

Ágnes Szendrei: **The structure of the clone lattice**

There are only countably many clones on a two-element set, and the clone lattice is fully known (Post, 1941). However, on larger finite sets, there is a continuum of clones (Janov–Mucsynik, 1954), and the clone lattice appears very complex. The situation is even more complicated on infinite sets, depending on set-theoretic assumptions (Goldstern–Shelah, 2002). For finite sets, the clone lattice can be studied from several directions. One approach is to describe the bottom and top of the clone lattice, focusing on minimal or almost minimal clones and submaximal clones (maximal clones are known, Rosenberg, 1970). Another approach studies monoid intervals — wide classes of transformation monoids whose intervals in the clone lattice are finite. Given a transformation monoid M, the monoid interval associated to M consists of those clones whose unary parts form M. Recently studies of an interesting order filter of the clone lattice began — the filter consists of clones with only finitely many non-equivalent operations on the base set. (Two operations are C-equivalent if they can be obtained from each other by substituting operations from C.) Many open questions remain in all these fields.

Mária Szendrei: **Regular semigroups and their generalizations**

McAlister's theory provides a useful connection for E-unitary inverse semigroups, factorizable inverse semigroups, and inverse semigroups constructed as semidirect products of semilattices and groups. Parts of this theory have been generalized to broader semigroup classes, but many open problems remain especially for classes like locally inverse semigroups, orthodox semigroups, and adequate semigroups.

Mária Szendrei: **Extensions of inverse semigroups**

Similar to group extensions, extensions of inverse semigroups can be studied. So far, two special types of extensions have been shown to generalize the Kaluzsnyin–Krasner theorem of group theory to inverse semigroups: idempotent-separating and idempotent-pure extensions. Many interesting open questions remain, e.g., whether other extension classes exist where the Kaluzsnyin–Krasner theorem holds, whether the semidirect product is the proper generalization of groups, and if there exist constructions that generally satisfy the theorem.

Tamás Waldhauser: **Functions and function classes on finite sets**

Study of multivariate functions on finite sets (e.g., Boolean functions and their generalizations) and their classes from algebraic and combinatorial perspectives: minors (subfunctions), arity gaps, function equations, composition of function classes, clones, partial clones.

Analysis Training Program

László Kérchy: **Operators on Hilbert spaces**

Study of operators on Hilbert and Banach spaces with emphasis on invariant sublattices, reflexivity, and cyclicity. Special attention to contractions and operators with power-bounded and regular norms. Also study semigroups of operators in both discrete and continuous cases.

László Leindler: **Orthogonal series, inequalities, function classes**

Focus on convergence and strong approximation of general orthogonal and Fourier series. Interested in infinite series related to inequalities. Investigates connections between function classes and approximation problems.

Lajos Molnár: **Preservation problems**

Preservation problems arise in many areas of mathematics and can be formulated as: For a given structure X, determine all transformations (or mappings satisfying additional conditions) that preserve:

- Some quantity assigned to elements of X,
- a predetermined subset of elements of X,
- a certain relation among elements of X.

From the algebraic viewpoint, these transformations are automorphisms or symmetries of the structure, and the task is to characterize them.

Studies of matrix space preservers remain an active research topic including infinite-dimensional operator-preserving transformations, with applications in quantum mechanics operator structures.

Lajos Molnár: **Isometries of function and operator spaces**

Determination of surjective linear isometries of various function and operator algebras is a classical problem in functional analysis, motivated by basic theorems of Banach and Stone and their extensions by Kadison. The doctoral research aims at studying nonlinear metric structures formed by functions and operators across mathematical fields and exploring their isometries.

Lajos Molnár: **Commutativity in operator algebras**

Commutativity of matrices and linear operators is a fundamental relation with numerous applications in and beyond mathematics (e.g., quantum mechanics). The doctoral work aims to process existing results and obtain new ones characterizing commutativity, centrality of elements, and commutability of operator pairs.

Lajos Molnár: **Local automorphisms**

A transformation of a mathematical structure with a binary operation is called a local automorphism (more precisely 2-local) if its values on any arbitrarily chosen two points coincide with the values of some automorphism (depending on the pair) at these two points. For structures formed by linear operators, local automorphisms often are actually (global) automorphisms; that is, automorphisms are completely characterized by their effect on 2-point subsets. The doctoral research aims to continue studies and obtain new results in this direction.

Ferenc Móricz: **Differentiability of sums of absolutely convergent multiple Fourier series**

Study of multivariate differentiable functions whose Fourier series converge absolutely. Seeking sufficient conditions on the magnitude of Fourier coefficients to ensure partial derivatives can be computed as sums of partial derivatives of terms in multiple Fourier series.

László Stachó: **Banach spaces and automorphism groups on topological vector spaces**

Research topics in nonlinear complex functional analysis: isomorphisms of Banach spaces with invariant metrics, Hille-Yosida type theory with holomorphic mappings, weighted grids and their applications in Jordan triples.

László Stachó: **Complex analysis in Banach spaces**

Strongly continuous one-parameter groups of holomorphic self-maps of bounded domains. Holomorphic automorphisms of bounded circular domains and natural Banach-Lie algebras (partial JB*-triples), relevant in finite dimensions.

Vilmos Totik: **Orthogonal polynomials, polynomial inequalities, and potential theory**

Research focused on orthogonal polynomials and approximation theory, particularly problems requiring complex analysis and potential theory methods. These areas form a key, intensively studied part of modern research.

Róbert Vajda: **Symbolic and Numeric Computation for Reachability Sets, Stability Regions, and Optimization**

Overview of current theories and tools for symbolic computation with semialgebraic sets. Suitable problem formulations to answer dynamical system questions. Case studies include stability regions of difference equations (predator-prey, economic models) and reachable sets of differential equations (e.g., mobile vehicle avoiding unsafe regions). Extensions include numerical techniques and machine learning approximations.

Dynamical Systems Training Program

Attila Dénes: **Dynamical Systems**

Applied dynamical systems: modeling, simulation, numerical and theoretical study of natural science problems including mathematical biology subjects.

László Hatvani: **Stability problems with applications**

Qualitative analysis of model solutions focusing on oscillations, stability, etc., employing Lyapunov direct method and its extensions considering cases with semidefinite derivatives. Investigations include dependence on system parameters and bifurcation, with focus on biological and mechanical models.

László Hatvani: **Non-autonomous second order differential equations**

Study of mechanical vibrations described by second order differential equations, including time-dependent coefficients (non-autonomous systems). Explores stability conditions, effect of friction coefficient changes, and special cases like Hill and Meissner equations.

Tibor Krisztin: **Dynamical Systems**

Qualitative theory of nonlinear dynamical systems covering various types of differential and difference equations; studies stability, bifurcations, and attractor structures.

Gergely Röst: **Nonlinear dynamics in mathematical epidemiology**

Modern epidemic models growing in complexity, including delay and threshold effects leading to functional differential equation models. Spatial spread modeled by partial differential equations; analysis employs tools from infinite dimensional dynamical systems theory like stability, bifurcations, periodic oscillations, invariant sets, and permanence. Applications include influenza, SARS, tuberculosis, host-parasite systems, West Nile virus, etc.

Gergely Röst: **Dynamical Systems**

Applied dynamical systems modeling with examples from natural sciences including mathematical biology.

Mónika Van Leeuwen-Polner: **Dynamical Systems**

Applied dynamical systems: modeling and simulation including mathematical biology, immunology, and cancer modeling.

Gabriella Vas: **Dynamical Systems**

Qualitative theory of nonlinear dynamical systems including stability, bifurcation, and attractors.

Geometry, Combinatorics, and Theoretical Computer Science Training Program

Béla Csaba: **Packing Problems**

The core layout problem involves deciding if graph H is a subgraph of graph G. This is a central and actively researched topic in graph theory. Packing problems have many applications in combinatorics, geometry, computer science, and elsewhere. The containment problem is often NP-complete, meaning we seek sufficient conditions that guarantee containment, leading to extremal graph theory questions. Important are searches for trees and bounded-degree subgraphs in dense or random graphs, with structural constraints. The goal is to study such problems, solve specific cases, and develop general methods if needed.

Béla Csaba: **Graph embedding problems**

What conditions guarantee that a graph H is a spanning or almost spanning subgraph of graph G? Special focus on developing new techniques to avoid reliance on the Regularity Lemma.

Tibor Csendes: **Further development of the stochastic global optimization method**

The task is to enhance the GLOBAL stochastic global optimization method by:

- Increasing the dimension of solvable problems, modifying internal algorithm structures accordingly.
- Testing Matlab implementations on standard and additional test sets and documenting efficiency improvements.
- Developing an optimization server for solving network-submitted tasks with proper reporting.
- Comparing and replacing local search methods with modern alternatives.
- Creating interfaces for standard modeling systems (AMPL, GAMMS).
- Conducting theoretical studies on the correctness and efficiency of the modified algorithm.

Software and detailed algorithm description available at the author’s website.

Two important reference books:

- Bazara, M.S., H.N. Sherali and C.M. Shetty: Nonlinear Programming, John Wiley & Sons, 1993
- Horst, R. and P.M. Pardalos (eds.): Handbook of Global Optimization, Kluwer, 1995

Ferenc Fodor: **Stochastic Geometry**

The research topic: study of stochastic geometric structures focusing mainly on random polytopes, investigating properties like volume, surface, mixed volumes, variances, laws of large numbers, and central limit theorems.

Péter Hajnal: **Extremal problems of combinatorial and geometric structures**

Studies on extremal problems related to graphs, finite geometries, finite point sets, and set systems.

Péter Hajnal: **Combinatorial complexity theory**

Investigation of questions related to combinatorial models of complexity: decision trees, communication complexity, formulas and networks, relation between deterministic and probabilistic computational complexity.

Zoltán Kató: **Efficient Markov Chain Monte Carlo Samplers for Bayesian Image**

Formulating image processing tasks probabilistically; Bayesian modeling for inverse problems in image analysis; MCMC methods for sampling where gradient-based methods fail due to complexity; challenges due to neighborhood dependencies and parametric variation.

Goals:

- Novel probabilistic image model
- New MCMC sampler for optimization
- Analyze algorithm convergence
- Apply to real-world problems like segmentation and object detection.

Zoltán Kató: **Segmentation of 3D Structures in Volume Images Using Higher Order Active Contours**

Incorporates higher level knowledge (shape priors) into bottom-up segmentation methods in image processing to improve segmentation of multiple objects. Extends HOAC models to 3D volume images.

Zoltán Kató: **Estimation of Multi-Dimensional Homeomorphisms for Automated Image Registration**

Addresses fundamental problem of reliably estimating transformations between images for registration; proposes parametric continuous 2D transformation model and linear parameter estimation.

Applications include medical imaging deformation detection, security verification, low bit-rate coding, and remote sensing.

Árpád Kurusa: **Geometric tomography**

Using classical and modern integral geometry, convex geometry, and integral transforms to determine properties of geometric objects or functions representing various observations; includes applications such as Hammer and Aleksandrov theorems.

Árpád Kurusa: **Metrics on projective spaces**

Classification of metric projective geometries starting from integral geometric constructions and studies of relevant problems, tracing back to Hilbert's fourth problem.

Judit Nagy-György: **Analysis of online algorithms**

Explores competitive analysis of online algorithms, which must make decisions without knowledge of the future; studies deterministic and randomized models; includes advice complexity.

András Pluhár: **Extremal and algorithmic graph theory problems**

Studies interval representability of graphs and related open problems, including algorithmic challenges in partitioning planar graphs and exploring bandwidth and chromatic number connections.

Viktor Vígh: **Stochastic and convex geometry**

Research on geometric properties of mainly random structures, polytopes, and approximations, using real analysis, convex geometry, and probability theory.

Stochastics Training Program

Péter Kevei: **Asymptotics of branching processes**

Models populations where lifespans and offspring numbers are independent; theory developed since 1873 Galton-Watson process covers more complicated systems; covers topics connecting generating functions to stochastic differential equations.

Gyula Pap: **Branching processes**

Studies discrete and continuous time, discrete and continuous state-space branching processes; statistical analysis and parameter estimation; approximations; subcritical, critical and supercritical models; applications.

Research topics including Didactics

Eszter Katonáné Horváth: **Islands from lattice and methodological perspectives**

Studies of islands and island systems on various grids; maximal island systems, related counting problems; combinatorial and lattice-theoretic approaches; conditions for lattice independence and teaching elementary island problems.

István Szalay: **Cube model of Euclidean geometry**

1. Mainly a higher education mathematics didactics topic, showing a finite model where Euclidean geometry elements differ from everyday intuition (e.g., “lines” may be curved, “planes” curved surfaces, “space” finite but satisfies Hilbert axioms with suitable definitions). Not for high school teaching but useful for teacher candidates’ spatial intuition.
2. Tools are from analytic geometry and classic calculus.
3. Geometrically interesting is the differential geometry of space elements and related curves and surfaces.
4. Visual computer representation is important, requiring computing and graphics skills.

List of doctoral courses

(grouped by educational programs)
2 + 2 year system, from September 2016

Doctoral School of **Mathematics** and Computer Science
University of Szeged

February 2, 2017; reconsidered April 9, 2019

There are three types of doctoral courses.

a) **General courses**: their purpose is to give general mathematical education needed by a PhD degree. (Depending on the previous studies of a student, the Council of the Doctoral School may make one or two such courses obligatory. Except for students with research aiming at the didactics of mathematics, a general course can be taken ONLY with the permission of the Council and only once; see the Credit Table for further information.

b) **Basic courses**: they give basic knowledge in the educational program the student belongs to.

c) **Special courses**: they give special knowledge related to the research field chosen.

If a course is taken by at least five students, then the course is a classical blackboard+chalk course in a classroom, **2+0 hours** per week. Otherwise, for less then five students, the course is a so-called reading course: the PhD student reads the literature and comes for consultations to the teacher when this is needed.

Each course is worth 5 credits. Although the way the PhD student can select a course allows a lot of freedom, his/her choice is not quite arbitrary: the students should follow the Credit Table. The minimum amount of credits the PhD student has to fulfill in each semester is prescribed by the Doctoral Regulations of the University of Szeged.

(At present [February 2, 2017], at least 20 credits, but at most 45, have to be fulfilled in each semester; see Appendix 4 of the Regulations. Note that usually most of the credits come NOT from courses. Actually, the abovementioned Appendix 4 requires at least 130 ***research credits*** during the four years of PhD training.)

This document is only on the courses. For more information on PhD studies, see the Operational regulations of the Doctoral School (in short, Regulations) with emphasis on its Section 6 and the appendices, including the Credit Table.

Only the permanent courses are listed below, which have already been approved by the Council of the Doctoral School. In every semester, the electronic system of study records shows only those courses that the Administrator of the Doctoral School uploads upon actual interest. Occasionally, some additional courses not in the list below are also available provided the Council approves them. (Hence, the arrangement of an occasional course needs more time than a permanent course, because the Council also need time to decide.)

Technical comments: Besides listing the courses, we explain their codes (in the electronic system ETR). Note that the list of courses as well as the teaching staff change slowly, this is why there can be gaps in the code numbers.

1. General courses (ETR-codes: MDPT1n)

- MDPT11. Algebra
- MDPT12. Theory of measure and integration
- MDPT13. Topology
- MDPT14. Discrete mathematics
- MDPT15. Probability theory

2. Basic courses (ETR-codes: MDPT2mn)

Algebra program (ETR-codes: MDPT21n)

- MDPT211. Semigroup theory
- MDPT212. Lattice Theory
- MDPT213. Universal algebra
- MDPT214. Group theory

Analysis program (ETR-codes: MDPT22n)

- MDPT221. Chapters from the theory of complex-valued functions
- MDPT223. Introduction to the approximation theory
- MDPT224. Fourier series I
- MDPT225. Functional analysis

Dynamical systems program (ETR-codes: MDPT23n)

- MDPT231. Ordinary differential equations I
- MDPT232. Ordinary differential equations II
- MDPT233. Partial differential equations I
- MDPT234. Dynamical systems I
- MDPT235. Dynamical systems II

Program on geometry, combinatorics, and theoretical computer science (ETR-codes: MDPT24n)

- MDPT241. Combinatorial methods in geometry
- MDPT242. Riemannian geometry
- MDPT243. Convex bodies an classical integral geometry
- MDPT244. Computational geometry
- MDPT245. Geometric algebra
- MDPT246. Algebraic topology

Stochastics program (ETR-codes: MDPT25n)

- MDPT251. Probability theory I
- MDPT252. Probability theory II
- MDPT253. Mathematical statistics I
- MDPT254. Mathematical statistics II
- MDPT257. Stochastic processes I
- MDPT258. Stochastic processes II

Basic courses on the Didactics of mathematics: NOT offered in English.

3. Special courses (ETR-codes: MDPT3mnk)

Algebra program (ETR-codes: MDPT31nk)

- MDPT3100. Regular semigroups
- MDPT3101. Classes of semigroups from a universal algebraic point of view

- MDPT3102. Congruence varieties
- MDPT3103. Coordinatizaton theory of lattices
- MDPT3104. Finite ordered sets
- MDPT3105. Clones
- MDPT3106. Finite algebra
- MDPT3109. Field theory and Galois-theory
- MDPT3110. Rings and modules
- MDPT3112. Linear algebra
- MDPT3117. Free lattices
- MDPT3118. The algorithmic complexity of the graph homomorphism problem

Analysis program (ETR-codes: MDPT32nk)

- MDPT3200. Hilbert spaces, Banach spaces and their operators I
- MDPT3201. Hilbert spaces, Banach spaces and their operators II
- MDPT3202. Contractions in Hilbert spaces I
- MDPT3203. Contractions in Hilbert spaces II
- MDPT3204. Strong summation and approximation I
- MDPT3205. Strong summation and approximation II
- MDPT3210. Inequalities and numerical approximation
- MDPT3211. Fourier series II
- MDPT3214. Complex harmoic analysis I
- MDPT3215. Complex harmonic analysis II
- MDPT3216. Real harmonic analysis I
- MDPT3217. Real harmonic analysis II
- MDPT3218. Numericl analysis
- MDPT3219. Ortogonal polynomials I
- MDPT3220. Ortogonal polynomials II
- MDPT3221. Chapters from the Approximation theory I
- MDPT3222. Chapters from the Approximation theory II
- MDPT3224. Approximation by operators
- MDPT3225. Approximation by polynomials
- MDPT3226. Fractals and wavelets
- MDPT3227. Special functions
- MDPT3228. Potential theory and its applications
- MDPT3231. Orthogonal polynomials I
- MDPT3232. Fourier integrals

Dynamical systems program (ETR-codes: MDPT33nk)

- MDPT3300. Functional differential equations I
- MDPT3301. Functional differential equations II
- MDPT3302. Partial differential equations II
- MDPT3303. Stability theory I
- MDPT3304. Stability theory II
- MDPT3305. Bifurcation theory, chaos I
- MDPT3306. Bifurcation theory, chaos II
- MDPT3307. Introduction to the theory of control
- MDPT3308. Applications of differential equations
- MDPT3309. Numerical methods of differential equations
- MDPT3310. Difference equations
- MDPT3311. Differential and integral inequalities
- MDPT3312. Classical mechanics
- MDPT3314. Dynamical models in biology

Program on geometry, combinatorics, and theoretical computer science (ETR-codes: MDPT34nk)

- MDPT3400. Integral geometry of Gelfand type
- MDPT3401. Geometric analysis
- MDPT3402. Graph theory
- MDPT3403. Convex geometry
- MDPT3405. Integrable Systems
- MDPT3407. Combinatorics of polytopes
- MDPT3408. Set systems
- MDPT3409. Theory of connections and holonomy groups
- MDPT3410. Symmetric spaces
- MDPT3411. Enumeration problems
- MDPT3412. Special graph classes
- MDPT3413. Combinatorial optimization
- MDPT3414. Special classes of set systems
- MDPT3415. Designs and codes
- MDPT3416. Theory of matroids
- MDPT3417. The probabilistic method in combinatorics
- MDPT3418. Combinatorial methods in complexity I.

- MDPT3419. Combinatorial methods in complexity II.
- MDPT3421. Elementary combinatorics
- MDPT3422. Elementary complexity theory
- MDPT3423. Coxeter groups
- MDPT3424. Discrete geometry
- MDPT3425. Stochastic geometry
- MDPT3426. Groups and geometries
- MDPT3427. Combinatorics of finite point sets

Stochastics program (ETR-codes: MDPT35nk)

- MDPT3500. Classical limit theorems
- MDPT3501. Convergence of probability measures
- MDPT3502. Gaussian approximations in stochastics
- MDPT3503. Empirical and quantile processes
- MDPT3506. Extreme value theory
- MDPT3508. Elements of stochastic processes
- MDPT3510. Branching processes
- MDPT3511. Martingales
- MDPT3513. Stochastic analysis
- MDPT3516. Mathematical methods in statistical physics
- MDPT3517. Ergodic theory
- MDPT3518. Multivariate statistical analysis
- MDPT3519. Linear statistical models
- MDPT3520. Time series analysis
- MDPT3521. Statistics of stochastic processes
- MDPT3522. Nonparametric statistics
- MDPT3524. Resampling and simulation methods in statistics
- MDPT3525. Asymptotic methods in mathematical statistics
- MDPT3529. Graph limits
- MDPT35xx. Information theory
- MDPT35xx. Markov processes
- MDPT35xx. Diffusion processes
- MDPT35xx. Stochastic methods in mathematical finance
- MDPT35xx. Renewal theory
- MDPT35xx. Queueing theory

- MDPT35xx. Bayesian statistics
- MDPT35xx. Survival analysis
- MDPT35xx. Data mining

Special courses on the Didactics of mathematics: **NOT offered in English.**

Syllabi of doctoral courses
(grouped by educational programs)
2 + 2 year system, from September 2016

Doctoral School of **Mathematics** and Computer Science
University of Szeged

February 2, 2017; revised April 10, 2019

Algebra program:

MDPT11. Algebra
General introduction to the theory of classical algebraic structures, such as groups, rings, fields and modules. Fundamental algebraic concepts, constructions. Characterization and decomposition theorems. Lattices, important lattice classes. Basic concepts in category theory. Algebra classes as categories.
Literature:
D. S. Dummit, R. M. Foote: Abstract Algebra
T. W. Hungerford: Algebra

MDPT211. Semigroup theory
Transformation semigroup, free semigroup. Ideal and Rees congruence, semilattice and group congruences. Green relations, the structure of D-classes. Regular element, inverse of an element, regular D-classes. Simple semigroups, principal factors, the Rees theorem on completely 0-simple semigroups. Completely regular semigroups, semilattices of groups. Inverse semigroups, the Wagner-Preston representation, natural partial order. Fundamental inverse semigroups, the Munn representation. Commutative semigroups.
References:
Grillet, P. A.: Semigroups: An Introduction to the Structure Theory, Marcel Dekker, 1995.
Howie, J. M.: Fundamentals of Semigroup Theory, Clarendon Press, 1995.

MDPT212. Lattice theory

Basic concepts in lattice theory, duality, complete lattices. Algebraic lattices, subalgebra lattices. Distributive lattices: Birkhoff and Stone's theorems, the structure of finite distributive lattices. Birkhoff' criterion, Dedekind's criterion. Free modular and distributive lattices on three generators. Congruences of lattices. Modular lattices: decomposition of intervals and elements. Geometric lattices and complemented modular lattices. Projective geometries as modular lattices. Varieties of lattices.
Literature:
Czédli G.: Lattice Theory (in Hungarian)
G. Grätzer: General Lattice Theory

MDPT213. Universal algebra
Algebras, terms, polynomials. Subalgebra. Isomorphism, homomorphism, general isomorphism theorems. Direct product and other types of product. Subdirect decomposition, Birkhoff's theorem. Closure operators, closure systems. Congruence lattice. Free algebra. Varieties. Birkhoff's theorem for varieties, Birkhoff's completeness theorem. Equivalence of varieties. Properties characterized by identities for varieties. The theorems of Malcev and Pixley. Magari's theorem. Minimal varieties. Ultraproduct, congruence distributive varieties. Varieties generated by primal algebras. Quasi-primal algebras, discriminator varieties. Finite basis theorems.
Literature:
Burris–Sankappanavar: A Course in Universal Algebra.
McKenzie–McNulty–Taylor: Algebras, Lattices, Varieties.

MDPT214. Group theory
Multiplicative groups of fields. Permutation groups (primitive and multiply transitive groups, wreath product, Frobenius groups). Free groups (subgroups, rank, defining relations, Reidemeister-Schreirer theory). Solvable groups. p-groups. Nilpotent groups. The transfer. The Burnside problem. Matrix groups. Finite simple groups. Subgroup lattices.
Literature:
Aschbacher: Finite Group Theory.
M. Jr. Hall:The Theory of Groups.
Huppert: Endliche Gruppen.
Lyndon-Schupp: Combinatorial Group Theory.

MDPT3100. Regular semigroups
Congruences of regular semigroups: kernel and trace of a congruence, the lattice of congruences, special congruences. The fine structure of completely regular semigroups, Lallement's theorem, bands. Inverse semigroups:

E-unitary inverse semigroups, covering theorem and P-theorem. Orthodox semigroups: the Hall representation, E-unitary regular semigroups. Locally inverse semigroups: covering theorems by Pastijn and McAlister. Regular semigroups and bi-ordered sets. Generalizations of regular semigroups.

References:

Grillet, P. A.: Semigroups: An Introduction to the Structure Theory, Marcel Dekker, 1995
Howie, J. M.: An introduction to Semigroup Theory, Academic Press, 1976
Howie, J. M.: Fundamentals of Semigroup Theory, Clarendon Press, 1995
Lawson, M. V.: Inverse semigroups: The Theory of Partial Symmetries, World Sci., 1998
Nambooripad, K. S.S.: Structure of Regular Semigroups I, Mem. Am. Mat. Soc. 22, 1979
and selected research papers.

MDPT3101. Classes of semigroups from a universal algebraic point of view

The lattice of varieties of semigroups, several sublattices, finite basis property, word problem. Free completely regular semigroups, the lattice of varieties of completely regular semigroups, the lattice of band varieties. Free inverse semigroups, the lattice of varieties of inverse semigroups. There is no free regular or orthodox semigroup. Existence varieties of regular semigroups, bifree objects, existence varieties of orthodox semigroup. Notions of identity in several sublattices of the lattice of existence varieties of regular semigroups. Pseudovarieties of finite semigroups, profinite objects.

References:

Almeida, J.: Finite Semigroups and Universal Algebra, World Scientific, 1994
Howie, J. M.: Fundamentals of Semigroup Theory, Clarendon Press, 1995
Petrich, M.: Inverse Semigroups, John Wiley & Sons, 1984
Petrich-Reilly: Completely Regular Semigroups, John Wiley & Sons, 1999
and selected research papers.

MDPT3102. Congruence varieties

The significance of congruence distributivity, Baker's theorem. Jónsson terms, Day terms (Gumm terms). Mal'cev condition for lattice identities stronger than modularity. Characterization of Mal'cev classes and strong Mal'cev classes. Nation's identities, (3,3)-identities. Polin's counterexample and a characterization of $\models_c$. Some consequences of modularity in congruence varieties. Abelian lattices and congruence varieties determined by Abelian varieties (that is, by varieties derived from varieties of modules). Self-duality of Abelian congruence varieties. The rudiments of commutator

theory. Applications of commutator theory for congruence varieties: „congruence splitting”, diamond identities, Day and Kiss' sufficient conditions of abelianness. An example of a non-abelien congruence variety. Congruence varieties of local varieties.
Literature:
Freese and McKenzie: Commutator Theory
furthermore, selected paper by Jónsson, Day, Freese and others

MDPT3103. Coordinatizaton theory of lattices
Geometric lattices. Characterizing geomodular lattices and projective geometries. Lattice theoretic counterparts of Desargues' theorem. Coordinatization of (direct-product components of) Arguesian geometric lattices. Coordinatization of von Neumann frames and the complemented modular lattices they generate. Huhn diamond. The theory of n-distributive lattices. Subdirectly irreducible modular lattices generated by Huhn diamonds. Coordinatizing Arguesian lattice generated by diamond or frames. Von Neumann's dimension function. Proof theory of linear lattices.
Literature:
Crawley–Dilworth: Algebraic Theory of Lattices
Grätzer: General Lattice Theory
Neumann, J.: Continuous Geometries
furthermore, selected papers by Day, Freese, Haiman, Herrmann, Huhn, and others

MDPT3104. Finite ordered sets
Series-parallel posets. Dilworth's chain decomposition theorem. Dimension of posets. Schnyder's theorem. The relationship between finite distributive lattices and finite posets. Sperner type theorems. Dismantlable posets and the fixed point property. Roddy's theorem. Obstructions for posets. Monotone operations, Tardos's theorem. Irreducible posets. Order varieties. Arithmetics of posets. Hashimoto's theorem.
Literature:
Bogart–Freese–Kung (editors): The Dilworth Theorems. Selected papers of Robert P. Dilworth
Schröder: Ordered sets
Trotter: Combinatorics and Partially Ordered Sets. Dimension Theory
Selected papers from the mathematical journal Order

MDPT3105. Clones
Abstract clones and clones of operations. Galois connection. Connection of relational clones and clones of operations, the Baker–Pixley theorem. Comp-

leteness theorems for clones: Lagrange-interpolation in finite fields, the theorem of Werner and Wille, the theorem of Sheffer and Webb, Slupecki's theorem, Salomaa's theorem. Clone lattices over finite sets; the Janov–Mucnik theorem. Maximal clones; the Post theorem. Rosenberg's theorem and its applications: McKenzie's theorem, completeness of pattern functions. Sheffer functions; Rousseau's theorem. Minimal clones. Swierczkowski's lemma. Rosenberg's type-theorem. Primitive positive clones; Kuznecov's theorem.
Literature:
Pöschel–Kaluzsnyin: Funktionen- und Relationenalgebren (in German)
Szendrei Ágnes: Clones in Universal Algebra
Selected papers of various authors

MDPT3106. Finite algebra
Primal algebras and their generalizations. The Stone-Hu duality for primal algebras. The term condition, commutators, Abelian algebras. McKenzie's theorem on strictly simple algebras of congruence permutable varieties. Locally finite varieties. The spectrum of a variety. The relationship between relational clones and free algebras. Finitely based algebras. Theorems of Post and Lyndon, the Lyndon groupoid, the Murskii groupoid, inherently non-finitely based algebras. The Pálfy-Pudlák theorem, Pálfy's theorem. Minimal algebras, elements of Tame Congruence Theory.
Literature:
Burris–Sankappanavar: A Course in Universal Algebra
McKenzie–McNulty–Taylor: Algebras, Lattices, Varieties
Szendrei Ágnes: Clones in Universal Algebra
Hobby–McKenzie: The Structure of Finite Algebras
Selected papers of authors Baker–McNulty–Werner, Berman, McKenzie, Pálfy, Pudlák and others

MDPT3109 Field theory and Galois-theory
Field extensions. Simple algebraic and transcendental extensions. Field extensions of finite degree, the multiplicativity formula for degrees. Splitting fields and normal extensions. Finite fields. Perfect fields. The Galois group. The main theorem of Galois theory. Solving equations by radicals. The Ruffini-Abel theorem. Algebraic theory of geometric constructions.
Literature:
D. S. Dummit, R. M. Foote: Abstract Algebra
Garling: A Course in Galois Theory
Hungerford: Algebra

MDPT3110. Rings and modules

Morita theory. Morita equivalence; characterizations and applications for the structure theory and for the Brauer group. Morita duality; characterization, the dual, PF- and QF-rings, AB and linear compactness. Structure theory. Semi-perfect modules and rings. Perfect rings. The theorems of Bass and Björk. PI-rings. Basic notions. The theorems of Kaplansky and Amitsur-Levitzky.
Literature:
D. S. Dummit, R. M. Foote: Abstract Algebra
Jacobson: Basic Algebra I, Freeman, 1974
Selected papers from various authors

MDPT3112. Linear algebra
Eigenvalues, eigenvectors and the characteristic polynomial of a linear transformation. Euclidean spaces. Orthogonal and self-adjoint transformations. The principal axis theorem of quadratic forms. Unitary spaces, normal transformations. Modules. The fundamental theorem of finitely generated modules over principal ideal domains. The Jordan normal form of matrices over a field, the Cayley-Hamilton theorem.
Literature:
Birkhoff–MacLane: Algebra
D. S. Dummit, R. M. Foote: Abstract Algebra
Jacobson: Basic Algebra I

MDPT3117. Free lattices
Whitman's solution for the word problem, canonical term, continuity, fixed-point free translation, embedding $FL(\omega)$. Bounded homomorphisms, Day's interval-doubling, finite McKenzie-bounded lattices. D-relation, semidistributivity, splitting lattices. Weak atomicity.
Literature:
Freese-Jezek-Nation: Free lattices

MDPT3118. The algorithmic complexity of the graph homomorphism problem
Algorithmic complexity classes (P, NP, NP-complete). The problem class CSP and the dichotomy conjecture. Different presentations of the homomorphism problem (for relational structures, algebras, systems of equations and varieties). The Feder-Vardi reductions. Schaefer's dichotomy theorem. Special cases (of a semilattice, a near-unanimity or a Maltsev operation). Weak near-unanimity operations. The proof of a conjecture of Bang-Jensen and Hell. Characterization of bounded width problems. The relationship between CSP and MMSNP.

Literature:
Hell-Nesetril: Graphs and homomorphisms
Selected papers of Feder, Vardi, Bulatov, Jeavons, Dalmau, Kozik and Barto.

Analysis program:

(For referenced literature, see the end of this section.)

MDPT12. Measure and integral
Measure space, measurable functions. Definition of the integral, convergence theorems. Extension of a measure from a semialgebra to the generated σ-algebra. Constructions of measures on $\mathbf{R}^n$, the Lebesgue measure. Connections between the Riemann integral and the Lebesgue integral. Product of measure spaces, Fubini's theorem. Regularity of Borel measures. Theorems of Luzin and Jegorov. Hölder and Minkowski inequalities. The function spaces $L^p(\mu)$, the notions of Banach spaces and Hilbert spaces. Orthogonal complement of a subspace, the dual of a Hilbert space. Complex measures, total variation. Lebesgue decomposition of measures, Radon–Nikodym theorem. Polar decomposition, Hahn decomposition. Constructions of complex Borel measures on the real line, functions of bounded variation.
Compulsory literature: Kérchy László: Valós- és funkcionálanalízis, Polygon, Szeged, 2008.
Recommended literature: 58, 69.

MDPT221. Selected chapters from complex analysis
Mittag-Leffler's theorem on the decomposition of meromorphic functions into the sum of partial fractions, the decomposition of cotg πz. Weierstrass' theorem on the factorization of entire functions, the factorization of $\sin \pi z$. The gamma function. Approximation by rational functions, Runge's theorem. The Hardy spaces H^p of analytic functions on the open unit disk. Limit on the circle, Fatou's theorem. The F. and M. Riesz theorem, Szegő's theorem. Blashke products, inner and outer functions, the canonical factorization. The Banach algebra of functions analytic on the open unit disk and continuous on the closed unit disk. The shift-invariant subspaces of the Hilbert space H^2.
Compulsory literature: 69, 31
Recommended literature: 19, 24, 38, 58

MDPT223. Introduction to approximation theory
Approximation by positive operators, the theorems of Korovkin. The Weierstrass and the Weierstrass-Stone theorem. Moduli of continuity and smoothness, Jackson's theorem, direct theorems. Estimates of derivatives, Bernstein's inequality and inverse theorems. Characterization of best approximating polynomials, extremal signatures. L^p-approximation. Saturation of the Bernstein polynomials, the parabola method.
Compulsory literature: 43, 44

Recommended literature: 1, 11, 45, 49, 72

MDPT224. Fourier series I
Completeness of the trigonometric system. Bessel inequality, Parseval's formula. The convergence of Fourier series: Riemann-Lebesgue lemma, Dini's theorem, the localization principle, Dirichlet-Jordan theorem, Lebesgue constants. Summability of Fourier series: Fejér's theorem and its consequences, Lebesgue's theorem, Lebesgue points of integrable functions. The divergence of Fourier series: the examples of Fejér and Kolmogorov. Special trigonometric series, coefficients of which are monotonically tending to zero.
Compulsory literature: 79
Recommended literature: 4, 20, 69

MDPT225. Functional analysis
Orthonormal systems in Hilbert spaces, the dimension of a Hilbert space. Convergence of Fourier series, summation in the senses of Cesaro and Abel. The Hahn-Banach theorem and its applications, Banach limit, Banach integral and measure. Banach-Steinhaus theorems, the open mapping theorem and the closed graph theorem; their applications to Fourier series. The duals of L^p spaces, reflexivity. The dual of the space of continuous functions, Riesz representation theorem. The Weierstrass-Stone approximation theorem.
Compulsory literature: Kérchy László: Valós- és funkcionálanalízis, Polygon, Szeged, 2008.
*Recommended literature:*57, 58, 69

MDPT3200. Hilbert spaces, Banach spaces and their operators I
Orthonormal basis in Hilbert space, the subspace lattice. Shift-, multiplication- and integral-operators. Adjoint of an operator, normal operators, orthogonal projections. The ideal of compact operators. The spectrum of an element of a Banach algebra, spectral radius, the Riesz–Dunford functional calculus. Spatial spectra, spectrum of a compact operator. Abelian Banach algebras, the Gelfand transformation, the Gelfand–Naimark theorem. Operator-topologies, monotone sequences of selfadjoint operators. Spectral measure, the spectral theorem. Functional calculus and functional model for normal operators. Neumann's double commutant theorem, abelian von Neumann algebras. Invariant subspaces of compact operators, Lomonosov's theorem.
Compulsory literature: Kérchy László: Hilbert terek operátorai, Polygon, Szeged, 2003
Recommended literature: 13, 18, 26, 34, 59

MDPT3201. Hilbert spaces, Banach spaces and their operators II

Unbounded symmetric and selfadjoint operators, the Cayley transformation. The spectral theorem for unbounded normal operators. Stone's theorem for one-parameter unitary groups. Fredholm operators, Fredholm index, essential spectrum. C^*-algebras, the Gelfand–Maimark–Segal construction. Trace class operators, Hilbert–Schmidt operators, Bergman operators, subnormal operators. Positive and completely positive mappings. Reflexive and hyperreflexive operator algebras and operator spaces.
Compulsory literature: 13, 14
Recommended literature:
J.B. Conway: The theory of subnormal operators, Amer. Math. Soc., Providence, Rhode Island, 1991.
K.-J. Engel - R. Nagel: One-parameter semigroups for linear evolution equations, Springer, New York, 2000.

MDPT3202. Hilbert space contractions I
Wold decomposition of isometries. Sz.-Nagy's dilation theorem. Dilation theorems for commuting contractions. Unitary ρ-dilations. The structure of the minimal unitary dilation, residual parts. Dilation of the commutants, Lifting theorems. Classification according to the asymptotic behaviour of the iterates. Quasisimilarity, hyperinvariant subspaces, the class C_{11}. Spectral properties of the minimal unitary dilation. The H^∞-functional calculus. The class C_0, minimal function, spectrum, hyperinvariant subspaces, model of quasisimilarity, reflexivity.
Compulsory literature: 9, 71
Recommended literature: 23

MDPT3203. Hilbert space contractions II
Operator-valued analytic functions. Inner and outer functions, the canonical factorization. Scalar multiple. The characteristic function of a contraction, functional model. Relation of the spectrum to the characteristic function. Invariant subspaces induced by regular factorizations of the characteristic function. Hyperinvariant subspaces of C_{11}-contractions. Weak contractions.
Compulsory literature: 71
Recommended literature: 9, 23, 31

MDPT3204. Strong summability and approximation I
The Hardy–Littlewood theorem, theorems of Marcinkiewicz and Zygmund. Problem of Alexits and the result of Alexits and his coauthors, the order of strong approximation. Structural properties of functions arising from the order of strong approximation. Connection between the strong and the best approximation. Function classes and the approximation by Fourier series. Very strong and mixed approximation.

Compulsory literature: 41, 43
Recommended literature: 61, 79

MDPT3205. Strong summability and approximation II
The strong summability of orthogonal series. Strong approximation by orthogonal series, extra strong approximation, strong approximation with large exponents. Strong and very strong summability and approximation of orthogonal series by special summability methods (e.g. Abel, Cesàro, Euler, Hausdorff). Limit cases in the strong approximation. Connection between the usual and strong approximations in the case of orthogonal series.
Compulsory literature: 41, 43
Recommended literature: 61, 79

MDPT3210. Inequalities, numerical approximation
Classical inequalities for series and integrals, Hardy-Littlewood type inequalities, the Copson-inequality, the Graham Bennett inequality. Inverse Hölder inequalities for series and integrals, Bernoulli type inequalities. Inequalities with blocks and generalized exponents. Methods of numerical approximation.
Compulsory literature: 6, 27
Recommended literature: 8, 25, 75, 79

MDPT3211. Fourier series II
Absolute convergence of Fourier series: theorems of Bernstein, Zygmund, Wiener and Lévay. The existence of conjugate functions and Abel-Poisson means. The Fourier character of a conjugate series and the convergence of a Fourier series and conjugate series in L^1-norm. The Riesz-Thorin interpolation theorem. The Hausdofff-Young theorem and theorems of F. Riesz. Interpolation theorem of Marcinkiewicz, and Paley's theorem on the Fourier coefficients. Summability of multiple Fourier series.
Compulsory literature: 79
Recommended literature: 4, 20

MDPT3214. Complex harmonic analysis I
Representation of a holomorphic function on the open unit disk by Poisson integral. The holomorphic supplementation of a harmonic function, Herglotz integral. The H^p and h^p spaces on the open unit disk. The characterization of the h^1 space by Poisson-Stieltjes integral. The existence of the border function of a function in h^1. A holomorphic interpretation of the logarithm of a holomorphic function. Jensen and Poisson-Jensen formulas, the distribution of the zeros of a holomorphic function. The existence and characterization of a Blaschke product. Factorization theorems of F. Riesz and Nevalinna.

Factorization of an inner function. The existence of the border function of a function in N. The convergence to the border function in L^p-norm. Characterization of H^1 by Poisson integral, the F. and M. Riesz theorem and its equivalent reformulations. The existence of an outer function. The canonical factorization of a function in H^p.
Compulsory literature: 19
Recommended literature: 24, 31, 38, 74, 79

MDPT3215. Complex harmonic analysis II
Characterization of the spaces H^p and h^p by Poisson integral, $1 < p \leq \infty$. Characterization of the Nevanlinna class N by Poisson-Stieltjes integral. The completeness of the space H^p, $0 < p \leq \infty$, and its characterization by the approximation property, $0 < p < \infty$. Characterization of the Smirnov class N^+, Hardy inequality for functions in H^1, and Privalov's theorems for the holomorphic functions on the open unit disk whose border function is absolutely continuous. The Banach algebra (so-called disk algebra) of functions that are continuous on the closed unit disk and are holomorphic on the open unit disk, $p > 0$. The harmonic conjugate of a function in h^p, $p > 0$. The H^p and h^p spaces on the upper half plane.
Compulsory literature: 19
Recommended literature: 24, 31, 38, 74, 79

MDPT3216. Real harmonic analysis I
The monotone decreasing rearrangement f^* of the measurable function f and the elementary maximal function f^{**}. The Hardy-Littlewood maximal operator $f \to Mf$ is bounded from $L^1(R^n)$ into weak-$L^1(R^n)$ and it is bounded in $L^p(R^n)$, if $p > 1$. The proof of the theorem concerning the equivalence of f^{**} and $(Mf)^*$. The $L^1 + L^\infty$ and $L^1 \cap L^\infty$ spaces. The interpolation theorems of M. Riesz-Thorin and Marcinkiewicz. The classes $L \ln^+ L$ and $\exp L$ of Zygmund. The Calderón-Zygmund decomposition of L^1 functions.
Compulsory literature: 7
Recommended literature: 64, 74

MDPT3217. Real harmonic analysis II
Fourier series, conjugate series and conjugate function of a periodic function. The truncated conjugate function. Convergence of Fourier series in L^p norm and the existence of the conjugate function. The Hilbert transform of the characteristic function. Maximal Hilbert transform, Calderón operator, theorems of Kolmogorov and M. Riesz. Modified Hilbert transform of L^∞ functions. The BMO space and the John-Nirenberg inequality. The Hardy-Littlewood maximal function and Hilbert transform of the functions

belonging to BMO. The equivalence of the real and complex H^1 spaces, the atomic decomposition. The K-functional of Peetre. The duality theorem of Fefferman and the decomposition of the BMO functions in the form $\varphi_1 + \tilde{H}\varphi_2$ where $\varphi_1, \varphi_2 \in L^\infty$.
Compulsory literature: 7
Recommended literature: 64, 74

MDPT3218. Numerical Analysis
The eigenvalue problem: Orthogonal triangularization and similarity transformation of matrices to an upper Hessenberg form. The LR algorithm and its modification, and the QR algorithm. The inverse power iteration. The Moore-Penrose generalized inverse matrix: its calculation by rank-factorization, partition and orthogonal triangularization. Examination of the system of linear equations by means of the generalized inverse of the coefficient matrix: the existence and unicity of the normal solution. Solutions of nonlinear equations and systems of equations: the Sturm method for the approximation of all roots of a polynomial. The Lehmer-Schur method for the approximation of all complex roots of a polynomial. The multiple Newton-Raphson method, and the Bairstow method. The Caccioppoli-Banach fix point theorem for contraction operators. Minimization of functions without conditions: Slope methods and golden section. Solution of systems of linear equations by gradient method and conjugate gradient method. Approximations of functions: interpolation by algebraic polynomials, trigonometric polynomials and cubic splines. Approximation of periodic functions by the least squares method. Fast Fourier transformation. Quadrature formulas. Romberg integration method.
Compulsory literature: 47
Recommended literature: 65, 77

MDPT3219. Orthogonal polynomials I
Measures and orthogonal systems; orthogonal polynomials; recursion coefficients; differential equations; zeros; Gaussian quadrature; generating functions; classical orthogonal polynomials; orthogonal polynomials on the unit circle and their relation to real orthogonal polynomials; Szegő's theory.
Compulsory literature: 22, 68
Recommended literature: 62

MDPT3220. Orthogonal polynomials II
Fundamentals of potential theory; general orthogonal polynomials; n-th root asymptotics; regular measures and their characterization; Freud polynomials; orthogonal polynomials with unbounded recursion coefficients..

Compulsory literature: 62
Recommended literature: 22, 68

MDPT3221. Topics in approximation theory I
and
MDPT3222. Topics in approximation theory II
Approximation by operators; approximation by polynomials; Müntz' theory; best approximation; unicity; one-sided approximation; weighted approximation; weighted approximation with varying weights; splines; problems in several variables; radial functions; wavelets; dyadic analysis; signal analysis; convolutions; nonlinear approximation; interpolation; quadratures; continued fractions; moment problems.
*Compulsory literature:*16, 44
Recommended literature: 1, 2, 11, 17, 45, 49, 52, 66, 72

MDPT3223. Rational and complex approximation
Polynomials on the complex plane; the theorems of Bernstein and Mergelian; rational functions on the complex plane, Runge's theorem; Padé approximation, the theorems of Gonchar and Nuttall; real rational approximation and its connection to spline approximation; the theorems of Pekarskii; interpolation; wavelets, Schauder bases with bounded degree.
Compulsory literature: 52
Recommended literature: 43, 44, 58

MDPT3224. Approximation by operators
Positive operators; K-functionals, φ-moduli; direct approximation; inverse theorems; saturation; combinations of operators; operators of several variables; the strong converse theorem for Bernstein polynomials.
Compulsory literature: 16
Recommended literature: 17, 43, 44

MDPT3225. Approximation by polynomials
Trigonometric polynomials; Nikolskii's theory; Dzjadik's inverse theorems; characterization of best algebraic polynomial approximation via φ-moduli; discrete operators; potential theory and polynomials; approximation with varying weights; orthogonal polynomials and weighted approximation by polynomials; Müntz' theory and its generalizations.
Compulsory literature: 17, 44, 72
Recommended literature: 1, 43, 49

MDPT3226. Fractals and wavelets

Iterated systems and their limits; fractional dimension; fractals; representation; orthogonal systems and the Haar system; wavelets, Daubechie's construction; multiresolution analysis; image compression; nonlinear approximation, Schauder bases.
Compulsory literature: 5, 78
Recommended literature: 12

MDPT3227. Special functions
Orthogonal polynomials and continued fractions; hypergeometric functions; differential equations; generating functions; zeros; addition formulas; asymptotics of orthogonal polynomials; q-series and special functions; discrete orthogonal polynomials; root systems; combinatorics.
Compulsory literature: 68
Recommended literature: 22

MDPT3228. Potential theory and its applications
Logarithmic potentials; superharmonic functions; Riesz representation; principles; equilibrium measures and potentials; potentials with external fields; Riesz potentials; applications.
Compulsory literature: 55, 60
Recommended literature: 29, 76

MDPT3231. Orthogonal series I
Orthogonal series in L^2, the Riesz-Fischer theorem, the Bessel inequality, complete orthonormal systems, Parseval's formula. Special orthogonal systems: the trigonometric, the Haar, the Rademacher and the Walsh system; basic convergence theorems. Convergence of orthogonal series: the Rademacher-Mensov inequality and theorem, Tandori's theorem, the Mensov-Kaczmarz functions. Unconditional convergence of orthogonal series: the theorems of Orlicz and Tandori. Cesaro summability of orthogonal series and its connection to the convergence of partial series: the theorems of Kolmogorov, Kaczmarz and Zygmund, the Mensov-Kaczmarz theorem and Tandori's theorem. Strong summability.
Compulsory literature: 3
Recommended literature: 33, 36

MDPT3232. Fourier integrals
Fourier transform of L^1 functions. Cesàro, Abel and Gauss-Weierstrass summability of the Fourier integral, unicity and the inversion formula. Fourier transform of L^2 functions, theorem of Plancherel. Fourier transform of L^p functions for $1 < p < 2$, the Hausdorff-Young inequality, convolution theorem. Fourier transform of distributions.

Compulsory literature: 64, 73
Recommended literature: 69, 79

Literature for the Analysis program

[1.] N. I. Akhiezer: Lectures on the theory of approximation

[2.] N. I. Akhiezer: The classical moment problem

[3.] G. Alexits: Convergence problems of orthogonal series

[4.] N. K. Bary: A treatise on trigonometric series

[5.] M. Barnsley: Fractals everywhere

[6.] E. F. Beckenbach–R. Bellman: Inequalities

[7.] C. Bennett–R. Sharpley: Interpolation of operators

[8.] G. Bennett: Factorizing the Classical Inequalities

[9.] H. Bercovici: Operator theory and arithmetic in H^∞

[10.] F. F. Bonsall–J. Duncan: Complete normed algebras

[11.] P. Borwein–T. Erdelyi: Polynomials and Polynomial Inequalities

[12.] C. K. Chui: Wavelets

[13.] J. B. Conway: A course in functional analysis

[14.] J.B. Conway: A course in operator theory

[15.] Császár Ákos: Valós analízis I-II.

[16.] R. DeVore: Approximation by positive operators

[17.] Z. Ditzian–V. Totik: Moduli of smoothness

[18.] N. Dunford–J. Schwartz: Linear operators

[19.] P. L. Duren: Theory of H^p spaces

[20.] R. E. Edwards: Fourier series

[21.] L. Euler: Introduction to Analysis of the Infinite

[22.] G. Freud: Orthogonal polynomials

[23.] C. Foias–A.F. Frazho: The commutant lifting approach to interpolation problems

[24.] J. Garnett: Bounded analytic functions

[25.] K.-G. Grosse-Erdmann: The Blocking Technique, Weighted Mean Operators and Hardy's Inequalities

[26.] P. R. Halmos: A Hilbert space problem book

[27.] G. H. Hardy–J.E. Littlewood-G. Pólya: Inequalities

[28.] G.H. Hardy: Divergent series

[29.] L. L. Helms: Introduction to potential theory

[30.] E. Hille–R.S. Phillips: Functional analysis and semigroups
[31.] K. Hoffmann: Banach spaces of analytic functions
[32.] L. Jaconsen–O. Nostrad: Continued fractions
[33.] S. Kaczmarz-H. Steinhaus: Theorie der Orthogonalreihen
[34.] R. V. Kadison–J.R. Ringrose: Fundamentals of the theory of operator algebras, Volume I: elementary theory
[35.] Kalmár László: Bevezetés a matematikai analízisbe I-II
[36.] B. S. Kashin–S.A. Saakjan: Orthogonal series
[37.] M. Kline: Mathematical Thought from Ancient to Modern Times
[38.] P. Koosis: Introduction to H^p spaces
[39.] J. D. Lambert: Computational methods in ordinary differential equations
[40.] J. D. Lambert: Numerical methods for ordinary differential systems
[41.] L. Leindler: Strong approximation by Fourier series
[42.] Leindler László: Analízis
[43.] G. G. Lorentz: Approximation of functions
[44.] G. G. Lorentz–R. DeVore: Approximation theory
[45.] G. G. Lorentz–M. Von Golitschek-A. Makovez: Approximation Theory II.
[46.] A.I. Markusevich: Series
[47.] Móricz Ferenc: Numerikus módszerek az algebrában és az analízisben
[48.] Móricz Ferenc: Differenciálegyenletek numerikus módszerei
[49.] I.P. Natanson: Constructive function theory
[50.] Sz. Sz. Pontrjagin: Matematikai analízis középiskolák számára (orosz nyelven)
[51.] G. K. Pedersen: Analysis now
[52.] P. Petrushev–V. Popov: Rational approximation
[53.] Pintér Lajos: Analízis I-II.
[54.] Pólya György–Szeghő Gábor: Feladatok és tételek az analízis köréből I-II.
[55.] T. Ransford: Potentials Theory on the Complex Plane
[56.] W. Rendin: A matematikai analízis alapjai
[57.] F. Riesz–B. Szőkefalvi-Nagy: Functional analysis
[58.] W. Rudin: Real and complex analysis
[59.] W. Rudin: Functional analysis

[60.] E. B. Saff-V. Totik: Logarithmic Potentials with External Fields

[61.] S. R. Siha: Summability methods and their applications

[62.] H. Stahl–V. Totik: General orthogonal polynomials

[63.] E. M. Stein: Singular integrals and differentiability properties of functions

[64.] E. Stein–G. Weiss: Introduction to Fourier analysis on Euclidean spaces

[65.] J. Stoer–R. Bulirsch: Introduction to numerical analysis

[66.] J. Szabados–P. Vértesi: Interpolation theory

[67.] Szász Pál, A differenciál- és integrálszámítás elemei 1, 2

[68.] G. Szegő: Orthogonal polynomials

[69.] Szőkefalvi-Nagy Béla: Valós függvények és függvénysorok

[70.] Szőkefalvi-Nagy Béla: Komplex függvénytan

[71.] B. Szőkefalvi-Nagy–C. Foias: Harmonic analysis of operators on Hilbert spaces

[72.] M. Timan: The approximation of real functions

[73.] E. C. Titchmarsh: Introduction to the theory of Fourier integrals

[74.] A. Torchinsky: Real variable methods in harmonic analysis

[75.] H. Triebel: Interpolation theory, function spaces, differential operators

[76.] M. Tsuji: Potential theory in modern function theory

[77.] J. H. Wilkinson: The algebraic eigenvalue problem

[78.] J. Wojsztaczyk: A Mathematical Introduction to Wavelets

[79.] A. Zygmund: Trigonometric series

Dynamical systems program:

MDPT231. Ordinary differential equations I
and
MDPT232. Ordinary differential equations II

Differential equations on manifolds. Existence and uniqueness theorems. Differential equations in spaces of infinite dimension. Linear systems. Infinitesimal generator. Integral manifolds. Linearization, the Hartman–Grobman theorem. Perturbation theory. Non-autonomous systems. Periodic and almost periodic equations. The method of averaging. Boundary value problems. Sturm-Liouville theory. Second order equations, oscillation. Limit sets and limit cycles. Poincaré-Bendixson theorem. Stability. The second method of Lyapunov. Invariance principles. First order partial differential equations. The Hamilton–Jacobi theory.

Compulsory literature:

H. Amann, Ordinary Differential Equations, DeGruyter, 1990.

V. I. Arnold, Ordinary Differential Equations, Springer-Verlag, 1992.

J. K. Hale, Ordinary Differential Equations, Wiley-Interscience, 1969.

Recommended literature:

D. V. Anosov, V. I. Arnold, Dynamical Systems I, Ordinary Differential Equations and Smooth Dynamical Systems, Springer-Verlag, 1991.

C. Chicone, Ordinary Differential Equations with Applications, Springer, 1999.

Ph. Hartman, Ordinary Differential Equations, Birkhäuser, 1982.

M. Hirsch, S. Smale, Differential Equations, Dynamic Systems and Linear Algebra, Academic Press, 1974.

M. A. Naimark, Linear Differential operators, Nauka, 1969 (in Russian).

V. V. Nemytskii, V. V. Stepanov, Qualitative Theory of Differential Equations, Dover Publications, 1954.

J. Palis, W. DeMelo, Geometric Theory of Dynamical Systems, Springer-Verlag, 1982.

V. A. Pliss, Integral Manifolds of Periodic Systems of Differential Equations, Nauka, 1977 (in Russian).

MDPT233. Partial differential equations I

Distributions (generalized functions). Sobolev spaces. Fourier transform of distributions. Fundamental solutions of PDE's. Partial differential operators. Classical and generalized solutions. Hypo-elliptic differential operators. Well-posed problems in half space for linear systems of PDE's. Existence, uniqueness and stability of solutions of boundary value problems for elliptic, hyperbolic, and parabolic PDE's in Sobolev spaces.

Compulsory literature:

V. Sz. Vlad'imirov, Introduction to the theory of partial differential equations.
L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 20, AMS, Providence, Rhode Island, 1998.
Recommended literature:
O. A. Ladyzhenskaya, The Boundary Value Problems of Mathematical Physics, Springer-Verlag, 1985.
I. Pazy, Semigroups of Linear Operators and Applications to Partial Differential Equations, Springer-Verlag, 1986.

MDPT234. Dynamical systems I
and
MDPT235. Dynamical systems II
Existence and smoothness of invariant manifolds. Behaviour of solutions near fixed points and periodic orbits. Linearization. Orbital stability. Poincaré maps. Averaging. Limit sets. Asymptotically smooth maps and semi-groups. α-contracting semi-groups. Stability of invariant sets. Dissipative systems. Global attractors. Fix point theorems. Morse-Smale maps. Dimension of a global attractor. Periodic flows. Gradient systems. Examples: retarded FDE's, neutral FDE's, parabolic and hyperbolic PDE's.
Compulsory literature:
M. Hirsch and S. Smale, Differential Equations, Dynamic Systems and Linear Algebra, Academic Press, 1974.
J. Palis, W. DeMelo, Geometric Theory of Dynamical Systems: an Introduction, Springer-Verlag, 1982.
S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer-Verlag, 1990.
Recommended literature:
J. Guckenheimer and P.J. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983.
J. Hale, L. Magalhaes, W. Oliva, An Introduction to Infinite Dimensional Dynamical Systems — Geometric Theory, Springer-Verlag, 1984.
J. Hale, Asymptotic Behavior of Dissipative Systems, AMS, 1986.
D. Henry, Geometric Theory of Semilinear Parabolic Equations, Springer-Verlag, 1981.
M. Hirsch, C. Pugh, M. Shub, Invariant Manifolds, Springer-Verlag, 1977.
V. V. Nemytskii, V. V. Stepanov, Qualitative Theory of Differential Equations, Dover Publications, 1954.
H. L. Smith, Monotone Dynamical Systems, AMS, 1995.
R. Temam, Infinite-Dimensional Dynamical Systems in Mechanics and Physics, Springer, 1997.

MDPT3300. Functional differential equations I

and

MDPT3301. Functional differential equations II

The abstract theory of phase spaces, trajectories, solutions. Existence and uniqueness theorems. Continuous dependence on the initial data. New phenomena in comparison with the ordinary differential equations. Continuability and compactness of solutions. Linear functional differential equations. Oscillation for first and second order equations. Stability. Integro-differential equations. Neutral equations. Geometric theory of autonomous equations. The existence of periodic solutions. Applications from biology, mechanics, and other sciences.

Compulsory literature:

O. Dickmann, S. A. Van Gils, S. M. Verduyn Lunel, H.-O. Walter, Delay Equations, Springer, 1995.

J. Hale, Theory of Functional Differential Equations, Springer-Verlag, 1977.

Recommended literature:

T. A. Burton, Stability and Periodic Solutions of Ordinary and Functional Differential Equations, Academic Press, 1985.

G. Gripenberg, S.-O. Londen, O. Staffans, Volterra Integral and Functional Equations, Cambridge University Press, 1990.

I. Győri, G. Ladas, Oscillation Theory of Delay Differential Equations, Clarendon Press, 1991.

Y. Hino, S. Murakami, T. Naito, Functional Differential Equations with Infinite Delay, Springer-Verlag, 1991.

V. B. Kolmanovskii, V.R. Nosov, Stability of Functional Differential Equations, Academic Press, 1986.

T. Krisztin, H.-O. Walter, J. Wu, Shape, Smoothness and Invariant Stratification of an Attracting Set for Delayed Monotone Positive Feedback, AMS, 1999.

S. H. Saperstone, Semidynamical Systems in Infinite Dimensional Spaces, Springer-Verlag, 1981.

MDPT3302. Partial differential equations II

Integral equations. The Fredholm alternative in Hilbert space. Potential theory. Elliptic, hyperbolic, parabolic (with variable coefficients) partial differential equations: existence, uniqueness, stability; asymptotic solutions of equations with small and large parameters. Pseudodifferential operators, Fourier integral operators. Propagation of singularities. Foundations of nonlinear partial differential equations.

Compulsory literature:

L. C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 20, AMS, Providence, Rhode Island, 1998.

A. Haraux, Nonlinear Evolution Equations-Global Behavior of Solutions, Springer-Verlag, 1981.
M. Renardy, R.C. Rogers, An Introduction to Partial Differential Equations, Springer-Verlag, 2004.
Recommended literature:
M. H. Holmes, Introduction to Perturbation Methods, Springer, 1995.
S. G. Krein, Linear Differential Equations in Banach Spaces, Nauka, 1967 (in Russian).
L. Hörmander, The Analysis of Linear Partial Differential Operators, I-IV, Springer-Verlag, 1983-85.
S. A. Lomov, Introduction to the Theory of Singular Perturbations, Nauka, 1981 (in Russian).
B. R. Vainberg, Asymptotic Methods of the Equations of Mathematical Physics, Moscow State Univ., 1982 (in Russian).

MDPT3303. Stability theory I
and
MDPT3304. Stability theory II

Lyapunov stability and asymptotic stability. Stability of linear systems. Lyapunov exponents, spectrum. Regular systems. Stability in the first approximation; critical cases. Bifurcations. Dichotomies. The direct method of Lyapunov. Incariance principle for autonomous systems. The Barbashin–Krasovskii theorems and applications. Non-autonomous systems; localization theorems for limit sets. Stability of periodic solutions for autonomous and non-autonomous systems; Poincaré maps. Stability of equilibria and stationary motions in mechanics. Partial stability. Structural stability. Local structural stability. Invariant manifolds, transversality. Generic properties. Hiperbolic closed trajectories, the Kupka–Smale theorem. Morse–Smale vector fields. The first prolongation and prolongation limit sets. Recurrence properties (Poisson stability, non-wandering points, Lagrange stability). Dispersive properties, parallelizability.
Compulsory literature:
N. P. Bhatia, G. P. Szego, Stability Theory of Dynamical Systems, Springer, 1970.
W. Hahn, Stability of Motion, Springer, 1967.
N. Rouche, P. Habets, M. Laloy, Stability Theory by Liapunov's Direct Method, Springer-Verlag, 1977.
T. Yoshizawa, Stability Theory by Lyapunov's Second. Method, Math. Soc. Japan, 1966.
Recommended literature:
B. F. Bylov, R. E. Vinograd, D. M. Grobman, V. V. Nemytskii, Theory of Lyapunov Exponents, Nauka, 1966 (in Russian).

W. A. Coppel, Stability and Asymptotic Behavior of Differential Equations, D.C. Heath and Company, 1965.
Ju. L. Daletskii, M. G. Krein, Stability of Solutions of Differential Equations in Banach Spaces, Nauka, 1970 (in Russian).
B. P. Demidovich, Lectures on Mathematical Theory of Stability, Nauka, 1967 (in Russian).
V. B. Kolmanovskii, V. R. Nosov, Stability of Functional Differential Equations, Academic Press, 1986.
N. N. Krasovskii, Stability of Motion, Stanford University Press, 1963.
V. Lakshmikantham, S. Leela, Differential and Integral Inequalities, I-II, Academic Press, 1969.
J. P. LaSalle, The Stability of Dynamical Systems, SIAM, 1976.
D. Merkin, Introduction to the Theory of Stability, Springer, 1997.

MDPT3305. Bifurcation theory, chaos I
and
MDPT3306. Bifurcation theory, chaos II

Local bifurcations: center manifolds, normal forms, codimension-one bifurcations of fixed points, codimension-one bifurcations of maps and periodic orbits. Poincaré maps. Averaging. The Melnikov method: perturbations of two dimensional homoclinic orbits, perturbations of subharmonic orbits and Hamiltonian systems. The Smale horseshoe. Symbolic dynamics. The Conley–Moser conditions. Global bifurcations: homoclinic bifurcations, global bifurcations from local codimension-two bifurcations. Lyapunov exponents. Chaos. Global attractors.

Compulsory literature:
J. Guckenheimer, P. J. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983.
Yu. A. Kuznetsov, Elements of Applied Bifurcation Theory, Springer-Verlag, 1998.

Recommended literature:
V. I. Arnold, A differenciálegyenletek elméletének geometriai fejezetei, Muszaki Könyvkiadó, 1988.
S.-N. Chow, J. K. Hale, Methods of Bifurcation Theory, Springer-Verlag, 1982.
J. K. Hale, H. Kocak, Dynamics and Bifurcation, Springer, 1991.
G. Ioss, D. D. Joseph, Elementary Stability and Bifurcation Theory, Springer, 1980.
S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer-Verlag, 1990.
S. Wiggins, Global Bifurcations and Chaos, Springer-Verlag, 1988.

MDPT3307. Introduction to control theory

Mathematical formulation of the basic problem of control theory. Connection with the calculus of variations. Linear optimal control theory. Existence theorems with convexity conditions. The maximum principle for linear equations. The existence of the optimal control in the non-convex case. Maximum principle in the nonlinear case. Application for second-order systems. Optimal control via Krasovskii's method. Applications. Optimal control for symmetric missiles. Adaptive systems.
Compulsory literature:
E. B. Lee, L. Markus, Foundations of Optimal Control Theory, Wiley, 1966.
L. Sz. Pontrjagin, Optimális folyamatok elmélete, Közgazdasági és Jogi Könyvkiadó, 1968.
Recommended literature:
L. D. Berkovitz, Optimal Control Theory, Springer-Verlag, 1974.
V. N. Fomin, A. L. Fradkov, B. A. Yakubovich, Adaptive Control of Dynamical Objects, Nauka, 1981.
J. Warga, Optimal Control of Differential and Functional Equations, Academic Press, 1972.

MDPT3308. Applications of differential equations
Applications in mechanics. Stability of satellites and gyroscopes. Oscillation in resistant medium. Gyroscopes, monorail. Pendulum with variable length. Parametric resonance. Dynamics of electric circuits. Stability of a betatron. Stability and motion of bodies containing fluids. Modelling of tsunami waves, travelling waves. Problems in chemical reactions. Heat reactors, nuclear reactors. Reaction–diffusion equations. Chemical oscillators. Epidemic spread; models for AIDS. River pollution. Traffic models. Automatic control, feedback. Stability of regulators. Automatic pilot. Applications in economics. Leontief's model for macroeconomics. Hicks–Samuelson theory on stability of equilibrium. Business-cycle models. .
Compulsory literature:
V. I. Arnold, Mathematical methods of classical mechanics. Graduate Texts in Mathematics, Vol. 60. 2nd Edition. Springer-Verlag, 1989.
E. Beltrami, Mathematics for Dynamic Modeling, Academic Press, 1987.
M. Braun, Differential Equations and Their Applications, Springer- Verlag, 1975.
Differential Equation Models, Edited by M. Braun, C.S. Coleman D.A. Drew, Springer- Verlag, 1978.
T. P. Dreyer, Modelling with Ordinary Differential Equations, CRC Press, 1993.
A. Friedman, Mathematics in Industrial Problems, Vol. 10, Springer, 1998.
Modules in Applied Mathematics, Edited by W.F. Lucas, Springer-Verlag, 1976.

W. B. Zhang, Differential Equations, Bifurcations, and Chaos in Economics (Series on Advances in Mathematics for Applied Sciences), World Scientific, 2005.

MDPT3309. Numerical methods of differential equations
Initial value problems for ordinary differential equations: the method of successive approximation, existence theorems, method of Taylor series. One-step methods: truncation error, order, consistence and convergence. Error estimates. Runge–Kutta methods. Linear difference equations: general solution of the homogeneous difference equation. Stability of solutions. Particular solution of the inhomogeneous difference equation. Linear multistep methods: truncation error, order, consistence, stability and convergence. Adams formulas, Störmer formulas, formulas derived from quadrature formulas. Predictor–corrector methods. Matrix theoretical background: irreducible and weakly diagonally dominant matrices, positive and monotone matrices. Iterative methods for solving large linear systems: Jacobi overrelaxation and successive overrelaxation method. Boundary value problems for ordinary differential equations: reduction to initial value problems, shooting method. Finite-difference method, error analysis. Partial differential equations: elliptic, hyperbolic and parabolic equations in mathematical physics. Finite-difference method, the Ritz–Galerkin variational method. Minimization problems and their numerical solution with the finite element method; weak formulation of partial differential equations, Galerkin's variational method and its application for partial differential equations modelling problems from physics; solution methods for large systems.
Compulsary literature:
Móricz Ferenc: Differenciálegyenletek numerikus módszerei. Polygon.
Recommended literature:
S.C. Brenner, L.R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 2008.
J. van Kan, A. Segal, F. Vermolen, Numerical Methods in Scientific Computing, VSSD, 2006.
J.D. Lambert: Computational methods in ordinary differential equations
J.D. Lambert: Numerical methods for ordinary differential systems
K.W. Morton, D.F. Mayer, Numerical Solution of Partial Differential Equations, Cambridge University Press, 2005.
J. Stoer and R. Bulirsch: Introduction to numerical analysis

MDPT3310. Difference equations
Difference calculus. Existence and uniqueness theorems. Linear systems of equations (generator function, Bernoulli method, theorems of Poincaré and Perron). Stability. The Lyapunov method. Comparison theorems. Oscilla-

tions. Riccati type problems. Difference equations in population dynamics and economics.
Compulsory literature:
S. N. Elaydi, An Introduction to Difference Equations, Springer, 1996.
S. Goldberg, Introduction to Difference Equations, Dover Publications, 1958.
Recommended literature:
R. Agarwal, Differential Equations and Inequalities, Marcel Dekker, 1992.
W. G. Kelley, A. C. Peterson, Difference Equations, Academic Press, 1991.

MDPT3311. Differential and integral inequalities
Mean values, well-known inequalities (Cauchy, Hölder, Jensen, etc.) and their applications. The Gronwall–Bellman inequality and generalizations (Bihari inequality, multivariable case, discrete case, inequalities with Stieltjes integrals). Applications to some ordinary, functional, partial differential equations, and some integral equations. Comparison theorems for ordinary, functional and partial differential equations.
Compulsory literature:
V. Lakshmikantham, S. Leela, Differential and Integral Inequalities I- II, Academic Press, 1969.
Recommended literature:
R. Agarwal, Differential Equations and Inequalities, Marcel Dekker, 1992.
E. F. Beckenbach, R. Bellman, Inequalities, Springer-Verlag, 1961.
G. H. Hardy, J.E. Littlewood, G. Pólya, Inequalities, Cambridge University Press, 1934.
W. Walter, Differential and Integral Inequalities, Springer-Verlag, 1970.

MDPT3312. Classical mechanics
The Hamilton variational principle. Lagrange's equation of motion of the second kind. Lagrange mechanics on manifolds. Oscillations. Rigid body. The Hamilton' equations of motions. The Poincaré–Cartan integral invariant. The Hamilton–Jacobi theory. Problems of the celestial mechanics.
Compulsory literature:
V. I. Arnold, Mathematical methods of classical mechanics, Springer-Verlag, 1974.
Recommended literature:
R. Abraham, J. Marsden, Foundations of Mechanics, Benjamin/Cummings, 1978.
V. I. Arnold, Mathematical Aspects of Classical and Celestial Mechanics, Springer, 1997.
F. Gantmacher, Lectures in Analytical Mechanics, Mir, 1975.
H. Goldstein, Classical Mechanics, Addison-Wesley Press, Inc., 1975.
L. D. Landau, E. M. Lifshitz, Mechanics, Nauka, 1973 (in Russian).

D. R. Merkin, Introduction to the Theory of Stability, Springer, 1987.

MDPT3314. Dynamical models in biology
Population dynamics: discrete and differential equation models. The effect of delayed feedback. Coexistence of species (predator-pray, competing, co-operating models). Structured populations, metapopulations. Mathematical epidemiology: compartmental systems, structured models, species spreading diseases, microparasite systems. Case studies: influenza, AIDS, rabies. Population genetics: Hardy–Weinberg laws, mutation, selection and recombination. Evolutionary dynamics, Fisher equation. Propagation in space, the Fisher–Kolmogorov model, diffusion, traveling waves. Reaction diffusion equations, pattern formation. Neural networks.
Compulsory literature:
J. D. Murray, Mathematical Biology I-II 3rd ed. Springer IAM vol 17-18, 2002/03.
M. Farkas, Dynamical Models in Biology, Academic Press 2001.
O. Diekmann, J. A. P. Heesterbeek, Mathematical Epidemiology of Infectious Diseases, Wiley, 2000.
Recommended literature:
Y. Kuang, Delay differential equations with applications in population dynamics, Academic Press MSE 191, 1993.
F. Brauer, P. van den Driessche, J. Wu (eds), Mathematical epidemiology (Lecture Notes in Mathematics / Mathematical Biosciences Subseries), Springer, 2008.
F. Brauer, C. Castillo-Chavez, Mathematical Models in Population Biology and Epidemiology, Springer, 2001.
H. Thieme, Mathematics in Population Biology, Princeton University Press, 2003.
J. Wu, Introduction to Neural Dynamics and Signal Transmission Delay, De Gruyter Series in Nonlinear Analysis and Applications, 6, 2001.

Geometry, combinatorics, and theoretical computer science program

MDPT13. Topology

Topological spaces. Compact and locally compact spaces, partition of unity. Topological manifolds. Homotopy and simplicial complexes. The fundamental group. Classification of triangulable 2-manifolds. Topological groups, transformation groups, Identification spaces induced by a subgroup. Homogeneous spaces, differentiable and analytic manifolds. Lie-groups.

References:

L. Auslander–R. E. MacKenzie, Introduction to Differentiable Manifolds, Dover, 1977

M. W. Hirsch, Differential Topology, Springer, 1976.

N. Steenrod, The topology of fiber bundles, Princeton, 1951.

MDPT14. Discrete mathematics

Enumeration problems: formal power series, recursions. Sets and multisets. Enumerating subsets, binomial coefficients. permutations and some of their statistics. Partitions of sets, Bell numbers, Stirling numbers of second kind. Groups acting on a finite set, Pólya–Redfield method. Inclusion-exclusion principle and its applications. Partially ordered sets and their Möbius function. Linear recursion, examples, Fibonacci numbers. Graph theory: Graphs, loops, parallel edges, degrees. Connectivity, trees, enumerating spanning trees. 2-connected graphs, k-connected graphs, flows, Menger's theorems. Matchings in bipartite graphs, Kőnig's theorem, Hungarian method. Matchings in general graphs, Tutte's and Berge's theorem, Edmonds' algorithm. Coloring of graphs, chromatic number, Brooks' theorem, perfect graphs. Drawing graphs: Kuratowski's theorem, crossing parameter and crossing lemma. Extremal graph theory: Turán's and Ramsey's theorem and their applications. Complexity of combinatorial problems: the NP class and NP-complete problems.

Recommended readings:

R.P. Stanley, Enumerative combinatorics Vol. 1., Corrected reprint of the 1986 original, Cambridge Studies in Advanced Mathematics vol. 49., Cambridge University Press, Cambridge, 1997.

László Lovász, Combinatorial problems and exercises. Second edition. North-Holland Publishing Co., Amsterdam, 1993.

R. Diestel, Graph theory, Fourth edition, Graduate Texts in Mathematics vol. 173, Springer, Heidelberg, 2010.

MDPT241. Combinatorial methods in geometry

Designs: definition of block designs, their parameters, and divisibility constrains. Steiner triples and their various constructions. Hadamard matrices.

Resolvable designs, Baranyai's theorem. Finite projective and affine geometries: latin squares, parameters of finite geometries. Desargues and Pappos planes, coordinatization of geometries Finite reflection groups, Coxeter groups and complexes. Buildings.

References:

M.Jr. Hall, Combinatorial theory, Waltham, Mass. 1967.

P.J. Cameron, Combinatorics: topics, techniques, algorithms, Cambridge University Press, Cambridge, 1994

S. Jukna, Extremal combinatorics, with applications in computer science, Second edition, Texts in Theoretical Computer Science, An EATCS Series, Springer, Heidelberg, 2011.

K. Brown, Buildings, Springer-Verlag, London, 1989.

MDPT242. Riemannian geometry

Riemannian metric, Levi–Civita connection. Geodesics, convex neighborhood, normal coordinates. Variations of geodesics, Jacobian vector fields, conjugate points. Hopf–Rinow and Hadamard theorems. Morse index theorem. Sectional curvature, curvature tensor, scalar curvature. Constant curvature spaces.

References:

M. P. do Carmo, Riemannian Geometry, Birkhäuser, 1992.

J. Milnor, Morse Theory, Princeton University Press, 1963.

W. Klingenberg, D. Gromoll, W. Meyer: Riemannsche Geometrie im Grossen, Springer, 1968.

J. Cheeger, D. Ebin: Comparison Theorems in Riemannian Geometry, North-Holland, 1975.

MDPT243. Convex bodies an classical integral geometry

Basic properties of convex sets, Charathéodory, Radon, Helly theorems. Separation, Euler relation, duality. Approximations of convex sets, Blaschke's selection theorem. Mixed volumes, Brunn–Minkowski theorem, Minkowski and Fenchel–Alexandrov inequalities. Density for points, lines, kinematic density, integral formulas in the plane. Steiner's formula, quermassintegrals, Blaschke and Poincaré formulas. Curvature integrals and their applications.

References:

L.A. Santalo: Integral Geometry and Geometric Probability, Encyclopedia of Math., Addison–Wesley, London, 1976.

T. Bonnesen, W.Fenchel: Theorie der konvexen Körper, Springer, Berlin, 1934.

W. Blaschke: Vorlesungen über Integralgeometrie, Berlin, 1955.

H. Busemann: Convex surfaces, Interscience, London, 1958.

MDPT244. Computational geometry

Special data structures in geometry. Geometric searching. Encoding polytopes and hyperplane arrangements, permutation tables. Dissection of point sets. Zones in arrangements. The complexity of families of cells. Convex hull algorithms in two and higher dimensions. The average case analysis of the algorithms. The geometry of linear programming. Planar point location search. Largest convex subsets, minimum measure simplices. Vector–sum maximization problems. Algorithms for determining similarities Voronoi–diagrams. Triangulating point sets, finding closest neighbors, minimum spanning trees, shapes of point sets. Separation and intersection in the plane. Design of geometric algorithms.
References:
M. de Berg, M. van Kreveld, M. Overmas, O. Schwarzkopf: Computational Geometry, 2nd. revised edition, Springer 2000.
H. Edelsbrunner, Algorithms in Combinatorial Geometry, Springer, New York, 1987.

MDPT245. Geometric algebra
Affine and projective planes. Desargues' Theorem and the coordinate field, Pappus' Theorem and the commutativity, the characteristic of the coordinate field, Fano configuration. Collineations and semilinear transformations. Symplectic and orthogonal geometry, the structure of the symplectic and orthogonal groups. Clifford algebras.
References:
E. Artin, Geometric Algebra, Princeton University, 1957.
R. Baer, Linear Algebra and Projective Geometry, Academic Press, 1952.
D. R. Hughes, F. C. Piper: Projective Planes, Springer, 1970.
J. Dieudonné, La Géométrie des Groupes Classiques, Springer, 1955.

MDPT246. Algebraic topology
Homotopy and simplicial complexes. Subdivision and the simplicial approximation theorem. The fundamental group and methods of calculation. Classification of triangulable 2-manifolds. Singular homology groups and methods of calculation: simplicial homology, exact sequences. Homology groups with arbitrary coefficients, and the Lefschetz Fixed–Point Theorem. Cohomology groups and calculation theorems. The Alexander-Poincaré duality theorem. Homotopy groups and CW-complexes. The theorem of J. H. C. Whitehead and the cellular approximation theorem. Homology and Cohomology of CW-Complexes. The Hurewicz theorem. Cohomology products
References:
S. Eilenberg, N. Steenrod, Foundations of Algebraic Topology, Princeton, 1952.
E. Spanier, Algebraic Topology, McGraw–Hill, New York, 1966.

C. R. F.Maunder, Algebraic Topology, Van Nostrand Reinold, London, 1970.
W. S. Massey, Singular Homology Theory, Springer, 1980.

MDPT3400. Integral geometry of Gelfand type
Radon transform on real affine spaces (invertibility, support theorems, Plancherel formula, Paley–Wiener theorem, connections to other transforms). Radon transform of distributions, Radon transform on complex domain. Radon transform and differentiation, Radon-type transforms on spaces of constant curvature and on Lorentz spaces.
References:
I. M. Gel'fand, M.I.Graev and N.Ya.Vilenkin: Generalized functions I
V. S. Helgason: Radon transform
S. Helgason: Groups and geometric analysis
V. G. Romanov: Integral geometry and inverse problems for Hyperbolic equations
F. John: Plane waves and spherical means

MDPT3401. Geometric analysis
Fourier analysis on spaces of constant curvature. Invariant measures on manifolds. Invariant differential operators on manifolds. Spherical transforms (spherical function series, Paley–Wiener theorem, inverse formulas).
References:
S. Helgason: Groups and geometric analysis
V. S. Varadarajan: Lie groups, Lie algebras and their representation,
S. Helgason: Differential geometry and symmetric spaces,
E. Hewitt and K. A. Ross: Abstract harmonic analysis

MDPT3402. Graph theory
Connectivity: digraphs and their connectivity, nowhere-0-flows. Matchings: Gallai–Edmonds structural theorem, Edmonds polytope, random algorithms in matching theory, permanent of matrices, Van der Waerden conjecture and its proof. Graph colorings: Hajós' theorem, the Kneser graph and its chromatic number, unit distance graph and coloring the space. Independent sets in graphs: τ-critical graphs, vertex packing polytope, perfect graphs, Shannon capacity of graphs. Eigenvalues of graphs: random walks on graphs, expanding parameter of graphs and its connection to eigenvalues.
Recommended readings:
L. Lovász and M.D. Plummer, Matching theory, Akadémiai Kiadó, Budapest, 1986.
Béla Bollobás, Modern graph theory, Graduate Texts in Mathematics vol. 184., Springer-Verlag, New York, 1998.
R. Diestel, Graph theory, Fourth edition, Graduate Texts in Mathematics vol. 173, Springer, Heidelberg, 2010.

MDPT3403. Convex geometry
Combinatorial properties of convex sets, Carathéodory, Radon, Helly theorems and their generalizations and applications. Separation of convex sets, duality. Approximations of convex sets, Blaschke's selection theorem. Operations on convex sets, mixed volumes. The isoperimetric theorem. Bodies of constant width. Valuations on convex bodies. Zonoids.
References:
H. G. Eggleston, Convexity, Cambridge Univ. Press 47 (1958).
L. Danzer, B. Grünbaum, V. Klee, Helly's theorem and its relatives, Proc. Symp. Pure Math., 7 (Convexity) (1963), 101–180.
B. Grünbaum, Convex Polytopes, John Wiley & Sons, London, 1967.
P. M. Gruber, J. M. Wills, Convexity and its applications, Birkhäuser, 1983.

MDPT3405. Integrable Systems
Hamilton systems. Darboux theorem. Symplectic manifolds. Legendre transform. Free particle in pseudo-Riemannian space. Moment map. Symmetry reduction method. Liouville theorem. Adler–Kostant–Symes theorem. Integrable mechanical systems, examples.
References:
A. M. Perelomov: Integrable Systems of Classical Mechanics and Lie Algebras, Birkhäuser, 1990.
R. Abraham, J. Marsden: Foundations of Mechanics, Benjamin, 1978.
V.I. Arnold, Mathematical Methods of Classical Mechanics, Springer, 1989.
J. M. Souriau: Structure des Systemes Dynamiques, Dunod, 1970.

MDPT3407. Combinatorics of polytopes
Theorems of Carathéodory, Radon, Helly and its applications. The inductive construction of polytopes, Gale-transforms and Gale-diagrams. Euler's relation, the Dehn–Sommerville equations. Upper bounds for the number of k-faces. Combinatorial types of 3-polytopes, Steinitz's theorem. Properties of boundary complexes, the van Kampen-Flores theorem. Characterization of the f-vector. Addition and decomposition of polytopes. Hamiltonian paths and circuits on polytopes. Regular polytopes.
References:
H. Hadwiger, H. Debrunner, V. Klee: Combinatorial Geometry in the Plane, Holt, Reinhardt and Winston, New York, 1964.
L. Danzer, B. Grünbaum, V. Klee: Helly's theorem and its relatives, Proc. Symp. Pure Math., 7 (Convexity) (1963), 101 - 180.
B. Grünbaum: Convex Polytopes, John Wiley & Sons, London, 1967.

MDPT3408. Set systems
The ν and τ parameters of set systems: Continuous relaxations, Greedy algorithm for finding small covering set, Kőnig property of set systems, normal

hypergraphs, Erdős–Pósa [property of hypergraphs. Coloring and discrepancy. Extremal problems: Intersecting families, Erdős–Ko–Rado theorem and its generalizations, set system with restricted intersection sizes, Ray-Chaudhury–Wilson theorem and its applications. The tensor product method, Bollobás' theorem. Katona–Kruskal theorem, combinatorial isoperimetric problems.

Recommended readings:

Claude Berge, Hypergraphs, Combinatorics of finite sets, North-Holland Mathematical Library vol. 45., North-Holland Publishing Co., Amsterdam, 1989.

Ian Anderson, Combinatorics of Finite Sets, Clarendon Press, Oxford, 1989.

L. Babai and P. Frankl, Linear Algebra methods in Combinatorics with Applications to Geometry and Computer Science, Preliminary Version, Department of Computer Science, The University of Chicago, 1992.

MDPT3409. Theory of connections and holonomy groups

Connections on principal fibre bundles. Parallelism. Holonomy groups and theorem. Reduction theorem. Infinitesimal holonomy group. Linear connections. Holonomy groups of Riemannian manifolds. De Rham's decomposition theorem. Invariant connections on reductive homogeneous and symmetric spaces. Invariant Riemann metrics and complex structures.

References:

S. Kobayashi, K. Nomizu, Foundations of Differential Geometry, I, II, Interscience Publ., 1963, 1969.

A. Lichnerowicz, Théorie Globale des Connexions et des Groupes d'Holonomie, Cremonese, 1955.

K. Nomizu, Lie Groups and Differential Geometry, Publ. Math. Soc. Japan, 1956.

A. Lichnerowicz, Géométrie des Groupes de Transformations, Dunod, Paris, 1958.

MDPT3410. Symmetric spaces

Theorems of variations and components, pinched manifolds, locally symmetric spaces, symmetric and two-point homogeneous spaces, groups of isometries, canonical connections, Jacobi equation, total geodesic submanifolds, homogeneous Riemann manifolds, Riemannian symmetric spaces of rank one, manifolds of geodesics

References:

S. Helgason, Lie groups and symmetric spaces

I. Chavel, Riemannian symmetric spaces

J. A. Wolf, Spaces of constant curvature

S. Kobayashi and K. Nomizu, Foundation of differential geometry II.

A. L. Besse, Manifolds all of whose geodesics are closed

MDPT3411. Enumeration problems
Formal power series. Major index of permutations. Enumerating subspaces of finite geometries, q-analogues of classical combinatorial objects. Partitions of integers, Jacobi formulas, Ramanujan–Rodgers identities. Möbius function of partially ordered sets and methods for its computation. Asymptotical formulas. Linear extension of partially ordered sets, dimension of partially ordered sets. Enumerating linear extensions of a partially ordered sets, complexity of enumeration problems. Log-concave sequences. Jeu-de-taquin, tableaux, symmetric functions.
Recommended readings:
Richard P. Stanley, Enumerative combinatorics Vol. 1., Corrected reprint of the 1986 original, Cambridge Studies in Advanced Mathematics vol. 49., Cambridge University Press, Cambridge, 1997.
Richard P. Stanley, Enumerative combinatorics. Vol. 2., Cambridge Studies in Advanced Mathematics vol. 62., Cambridge University Press, Cambridge, 1999.
W.T. Trotter, Combinatorics and partially ordered sets, Dimension theory, Johns Hopkins Series in the Mathematical Sciences, Johns Hopkins University Press, Baltimore, MD, 1992.

MDPT3412. Special graph classes
Outerplanar graphs, series-parallel graphs. Planar graphs and their various characterizations. Major graph theoretical problems (coloring, multicommodity flow problems) in these graphs classes. Minor closed graph classes, minor monotone graph parameters (Path-width, tree-width). Well-quasi-ordered sets, the basics of the Seymour–Robertson theory. Graph classes closed for induced subgraphs: Line graphs, interval graphs, split graphs. Perfect graphs and special subclasses of them, characterizations. Symmetric graphs: strongly regular graphs, friendship theorem, transitive graphs, Cayley graphs. Expander graphs and their existence.
Recommended readings:
R. Diestel, Graph theory, Fourth edition, Graduate Texts in Mathematics vol. 173, Springer, Heidelberg, 2010.
P.J. Cameron and J.H. van Lint, Graph theory, Coding theory and block designs, Cambridge University Press, 1980.

MDPT3413. Combinatorial optimization
Linear programming; simplex algorithms, ellipsoid algorithm and interior-point methods. Integer programming. Semidefinite programming and its applications in combinatorics. Convex programming. Greedy algorithms,

dynamic programming, methods based on augmentations. Polyhedral method for combinatorial problems. LP and semidefinite relaxations. Branch and bound methods. Applications.

Recommended readings:

Bernhard Korte and Jens Vygen,Combinatorial optimization, Theory and algorithms, Algorithms and Combinatorics, vol. 21., Springer-Verlag, Berlin, 2000.

William J. Cook, William H. Cunningham, William R. Pulleyblank and Alexander Schrijver, Combinatorial optimization, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, Inc., New York, 1998.

Martin Grötschel, László Lovász and Alexander Schrijver, Geometric algorithms and combinatorial optimization, Second edition, Algorithms and Combinatorics vol. 2., Springer-Verlag, Berlin, 1993.

MDPT3414. Special classes of set systems

Convex geometries: basic notions, various axiomatic descriptions. Geometric parameters of convex geometries, and their relations. The happy end problem in convex geometries. Geometric set systems. Vapnik–Chervonenkis dimension of set systems. Discrepancy problems for geometric set systems. Simplicial complexes and their f- and h-vectors. Boolean functions as set systems. monotone Boolean functions as simplicial complexes. Arithmetical set systems. Additive combinatorics. Roth's theorem on $r_3(n)$. Discrepancy in arithmetic sets systems.

Recommended readings:

Handbook of combinatorics, Vol. 1 and 2, Edited by R. L. Graham, M. Grötschel and L. Lovász, Elsevier Science B.V., Amsterdam; MIT Press, Cambridge, MA, 1995.

Bernhard Korte, László Lovász, Rainer Schrader, Greedoids, Algorithms and Combinatorics vol. 4., Springer-Verlag, Berlin, 1991.

János Pach and Pankaj K. Agarwal, Combinatorial geometry, Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, Inc., New York, 1995.

MDPT3415. Designs and codes

Steiner triples and their various constructions. Symmetrical block designs, finite projective planes, Ryser–Chowla theorem. Resolvable block designs. t-designs. Finite geometries. Basic notions of coding theory: size, effectiveness, weight counting polynomial. Gilbert–Varaslimov bound. Linear codes, Mac Williams' theorem. Hamming codes. Self dual codes. Projective codes. Goley codes and their relations to designs.

Recommended readings:

Welsh, Dominic Codes and cryptography, Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1988.
Thomas Beth, Dieter Jungnickel and Hanfried Lenz, Design theory, Vol. I., Second edition, Encyclopedia of Mathematics and its Applications, vol. 69., Cambridge University Press, Cambridge, 1999.
Thomas Beth, Dieter Jungnickel and Hanfried Lenz, Design theory, Vol. II., Second edition, Encyclopedia of Mathematics and its Applications, vol. 78., Cambridge University Press, Cambridge, 1999.
P.J. Cameron and J.H. van Lint, Designs, graphs, codes and their links, London Mathematical Society Student Texts, vol. 22., Cambridge University Press, Cambridge, 1991.

MDPT3416. Theory of matroids
Matroid theoretical notions and different axiomatic descriptions of matroids. Operations on matroids: contractions, restrictions, dual, direct sum, sum, homomorphism. Intersection of matroids. Minimax theorems and their relations. Coordinatization of matroids. Characterization of binary matroids. Ternary matroids. Graphic matroids. Unimodular matrices and matroids, representable over arbitrary fields. Matroids and combinatorial optimization. Algorithms on matroids. Submodular functions.
Recommended readings:
D. J. A. Welsh, Matroid Theory, Academic Press, London, 1976.
James G. Oxley, Matroid theory, Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1992.

MDPT3417. The probabilistic method in combinatorics
Probabilistic method. Basic applications: bounding Ramsey numbers, 2-coloring of hypergraphs, discrepancy. The first moment method, the second moment method and the exponential moment method. Bounds from probability theory: Chernoff's bound. Lovász' local lemma, its oombinatorial and algorithmic proof. Applications. Martingales, and their applications in combinatorics. The semirandom method, and its applications. Different models of random graphs. Erdős–Rényi model. Threshold function for different monotone properties. Evolution of random graphs. Randomized algorithms, randomized complexity classes: BPP, RP, PP. Examples: primality testing, checking polynomial identities, testing the existence of perfect matching on a parallel machine, s-t connectivity, computing the volume.
Recommended readings:
N. Alon, J. Spencer, The probabilistic method, Third edition, With an appendix on the life and work of Paul Erdős. Wiley-Interscience Series in Discrete Mathematics and Optimization, John Wiley & Sons, Inc., Hoboken, NJ, 2008.

Béla Bollobás, Modern graph theory, Graduate Texts in Mathematics vol. 184., Springer-Verlag, New York, 1998.
R. Motwani, P. Raghavan, Randomized algorithms, Cambridge University Press, Cambridge, 1995.

MDPT3418. Combinatorial methods in complexity I.
Boole decision trees: evasive functions. Rivest–Vuillemin theorem. Topological method: Kahn–Saks–Sturtevan theorem, Yao's theorem. Random and non-deterministic decision trees. Sensitivity of Boolean functions. Communication complexity: the rank bound, Möbius functions. Random communication complexity. Formulas: Size of a formula, depth of a circuit, and their relation. Computing symmetric function with small formula. Nečiprok's theorem. Ramsey theoretical method; Hodes–Specker–Pudlák theorem. Method of random restrictions: Subotovskaja's method, Andreev's theorem. Monotone formulas. Method of random restrictions: Karchmer-Wigderson theorem. Linear algebraic method: Razborov's theorem. Applications of communication complexity for formulas: Raz–Wigderson theorem.
Recommended readings:
Handbook of Theoretical Computer Science, Volume A: Algorithms and complexity, (Ed. J. van Leeuwen), R. Boppana, M. Sipser, Chapter 14, MIT Press, 1990.
Paul E. Dunne, The complexity of Boolean networks, Academic Press 1988.
I. Wegener, The complexity of Boolean functions, Wiley-Teubner, 1987.
Lovász László, Bonyolultságelmélet, ELTE jegyzet.
S. Arora, B. Barak, Computational complexity, A modern approach, Cambridge University Press, Cambridge, 2009.

MDPT3419. Combinatorial methods in complexity II.
Circuits: Size of a circuit, Turing machine complexity, and their relation. Non-uniform Turing machines. Linear lower bounds on circuit complexity. Constant depth circuits. Hastad lemma and the random restriction method. Razborov's approximation method. Razborov's and Smolenski's theorem. Monotone circuits, Razborov's methods and its applications for various functions, Andreev's theorem. Branching programs, their complexity, and its relation to space complexity of Turing machines: Masek's theorem. Constant width branching programs, Branching programs with bounded access to the input.
Recommended readings:
Handbook of Theoretical Computer Science, Volume A: Algorithms and complexity, (Ed. J. van Leeuwen), R. Boppana, M. Sipser, Chapter 14, MIT Press, 1990.
Paul E. Dunne, The complexity of Boolean networks, Academic Press 1988.

I. Wegener, The complexity of Boolean functions, Wiley-Teubner, 1987.
S. Arora, B. Barak, Computational complexity, A modern approach, Cambridge University Press, Cambridge, 2009.

MDPT3421. Elementary combinatorics
Combinatorial/bijective proofs for identities, inequalities and divisibilities. Famous sequences, combinatorial and number theoretical properties of them. Polynomials and formal power series. Graph theoretical notions: connectivity, trees, coloring, independent sets, cliques. Graph theory in problem solving contests. Set systems and their appearance in elementary mathematics.
Recommended readings:
A.M.Jaglom, I.M.Jaglom, Challenging mathematical problems with elementary solutions, Combinatorial analysis and probability, Dover Publ. Inc., New York, 1987
Engel, Problem-solving strategies, Springer-Verlag, New York–Berlin, 1998
A.T. Benjamin, J.J. Quinn, Proofs that really count, The art of combinatorial proof, The Dolciani Mathematical Expositions vol. 27, Mathematical Association of America, Washington, DC, 2003.
R.A. Beeler, How to count, An introduction to combinatorics and its applications, Springer, Cham, 2015.

MDPT3422. Elementary complexity theory
Comparison based decision trees, measuring problems, sorting and searching algorithms. Boolean decision trees, graph properties, lower bounds based on adversary strategies. Evasive Boole functions and proof techniques for showing evasiveness, Invariant method. Arithmetic complexity. Complexity of addition and multiplication. Matrix arithmetic and its investigation from the point of complexity.
Recommended readings:
P. Gács Péter, L. Lovász László, Complexity of algorithms, http://www.cs.elte.hu/ lovasz/complexity.pdf

MDPT3423. Coxeter groups
Symmetry groups of regular polytopes. Root systems. Standard presentation of reflection groups. Classification of finite reflection groups. Coxeter graphs. Wythoff construction, Wythoff polytopes. Affine Weyl groups, extended Dynkin diagrams. Coxeter systems. Parabolic subgroups. Coxeter complex. Geometric representation of Coxeter groups. Bruhat order. The role of Coxeter groups in the classification of simple Lie algebras. Coxeter matroids. Abstract regular polytopes and C-groups.
References:

A. Björner and F. Brenti: Combinatorics of Coxeter groups, Graduate Texts in Mathematics, vol. 231, Springer, New York, 2005.
Bourbaki: Lie Groups and Lie Algebras, Springer, Chapters 4-6, Springer, 2002.
J. E. Humphreys: Reflection groups and Coxeter groups, Cambridge University Press, 1990.

MDPT3424. Discrete geometry
Basic concepts of lattices, special lattices, symmetries of lattices, Minkowski's theorems, Blichfeldt's theorem, packing and covering problems for convex bodies, properties of density, packings of spheres in d-dimensions, Blichfeldt's method, Roger's simplex bound, Minkowski-Hlawka theorem, Rogers-Shephard theorem, successive minima, finite packing and covering problems, parametric density.
References:
J. Pach, P. Agarwal: Combinatorial Geometry, John Wiley & Sons, Inc., 1995.
P. M. Gruber: Convex and discrete geometry, Springer, 2007.
L. Fejes Tóth: Regular Figures, Pergamon Press, 1964.
C. A. Rogers: Packing and Covering, Cambridge University Press, 1964.

MDPT3425. Stochastic geometry
Random closed sets, random measures and point processes, Poisson point processes, Palm distributions, convex hull of random points, random projections of polytopes, extremal problems for probability and expectation, approximation of convex bodies by random polytopes, economic cap covering and its applications, central limit theorems for random polytopes.
References:
R. Schneider, W. Weil, Stochastic and Integral Geometry, Probability and Its Applications, Springer-Verlag, Berlin, Heidelberg, 2008.
Baddeley, A.; Bárány, I.; Schneider, R.; Weil, W.: Stochastic geometry. Lectures given at the C.I.M.E. Summer School held in Martina Franca, September 13–18, 2004. With additional contributions by D. Hug, V. Capasso and E. Villa. Edited by W. Weil. Lecture Notes in Mathematics, 1892. Springer-Verlag, Berlin, 2007.
Santaló, Luis A.: Integral geometry and geometric probability. Second edition. With a foreword by Mark Kac. Cambridge Mathematical Library. Cambridge University Press, Cambridge, 2004.

MDPT3426. Groups and geometries
The classical fields and their automorphisms. Affine and projective spaces. Projective linear groups. The Poincaré-Birkhoff-Witt Theorem. Orthogonal,

unitary and symplectic inner products and the related linear groups. Quadratic and Hermitian varieties. Polar spaces and generalized quadrangles. Isomorphisms between classical groups. Quaternion and octonions. Designs. Multiply transitive finite permutation groups. The geometry of sporadic groups. Topological groups, Lie groups, algebraic groups. Permutation groups on the computer.

References:

J. Dieudonné, La Géométrie des Groupes Classiques, Springer, 1955.
D. E. Taylor, The geometry of classical groups, Heldermann, Berlin, 1992.
The GAP Group, GAP — Groups, Algorithms, and Programming, Version 4.4.12; 2008 (http://www.gap-system.org)

MDPT3427. Combinatorics of finite point sets

Gallai lines, Gallai–Sylvester theorem, counting Gallai lines. Crossing number, crossing lemma, Szemerédi–Trotter theorem and its applications in combinatorial geometry. k-sets and their numbers, basic bounds and constructions. Ungar's theorem. The number of unit distances, norms, the case points in convex position. Counting distinct distances. Erdős-Szekeres theorem, empty convex k-tuples. Planar problems in higher dimension.

Recommended readings:

J. Pach, P.E. Agarwal, Combinatorial geometry. Wiley-Interscience Series in Discrete Mathematics and Optimization, A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1995.
S. Felsner, Geometric graphs and arrangements, Some chapters from combinatorial geometry, Advanced Lectures in Mathematics. Friedr. Vieweg & Sohn, Wiesbaden, 2004.
J. Matoušek, Lectures on discrete geometry, Graduate Texts in Mathematics, volume 212, Springer, New York, 2002.

Stochastics program

MDPT15. Probability theory
Kolmogorov's axioms, 0–1 laws, Borel–Cantelli lemmas. Random variables, distribution function and density function, moments. The most important probability distributions. Definitions and properties of the characteristic and the moment generating function. Strong and weak laws of large numbers, central limit theorem, law of iterated logarithm. Kolmogorov's three-series theorem.
References:
Shiryaev: Probability, Springer-Verlag, New York, 1996.
Klenke: Probability theory. A comprehensive course. Springer-Verlag, London, 2008.
S. Csörgő: Fifty-three lectures on probability, Ann Arbor, 1991.

MDPT251. Probability theory I
Kolmogorov's axioms. Random vectors, multivariate distributions and distribution functions. Stochastic processes, Kolmogorov existence theorem. Independence and product spaces. Discrete, continuous and singular distributions, Lebesgue decomposition. Distribution function of sums and functions of random variables. Expected value, moments, variance, covariance, and correlation. The most important probability distributions. Types of convergence. Laws of large numbers, 0–1 laws, Kolmogorov's three-series theorem. Conditional probability and expectation, conditional deistribution.
References:
Shiryaev: Probability. Springer-Verlag, New York, 1996.
Billingsley: Probability and measure. John Wiley & Sons, New York, 1995.
S. Csörgő: Fifty-three lectures on probability, Ann Arbor, 1991.

MDPT252. Probability theory II
Weak convergence and convergence in distribution. Helly's theorem, tightness. characteristic functions. Central limit theorems. Multivariate normal distributions and multivariate central limit theorems. Local limit theorems and asymptotic expansions. The random walk. The properties of discrete time martingales. Gaussian processes, existence and properties of the Wiener process. Laws of the iterated logarithm, fluctuation. The arcsine law.
References:
Shiryaev: Probability. Springer-Verlag, New York, 1996.
Billingsley: Probability and measure. John Wiley & Sons, New York, 1995.
S. Csörgő: Fifty-three lectures on probability, Ann Arbor, 1991.

MDPT253. Mathematical statistics I

Statistical sample, realisation, sample space, statistics. Empirical distributions, Glivenco–Cantelli theorem. Efficient, unbiased, consistent and sufficient point estimator. Fisher information and the Cramér–Rao inequality. Rao–Blackwell–Kolmogorov theorem. Exponential ditribution families. Methods of estimation: method of moments, minimal distance estimation, and maximum likelihood estimation. Asymptotic properties of the maximum likelihood estimator: consistency, asymptotic normality, efficiency. Bayes estimators and their properties. Exact and asymptotic confidence intervals. Statistical hypothesis testing: test statistics, significance, power. The Neyman–Person lemma, uniformly most powerful unbiased tests. Likelihood-ratio tests.
References:
Borovkov: Mathematical statistics. Gordon and Breach, Amsterdam, 1998.
Cox, Hinkley: Theoretical Statistics. Chapman and Hall, London, 1974.
Keener: Theoretical statistics. Springer, New York, 2010.
Lehmann, Casella: Theory of point estimation. Springer, New York, 1998.

MDPT254. Mathematical statistics II
Estimation theory: the most important statistics and their properties, confidence intervals. Statistical hypothesis testing: test statistics, significance, power. Tests for the parameters of the normal distribution and the related confidence intervals. Testing goodness-of-fit: parametric and nonparametric methods. Nonparametric statistical tests. Analysis of variance, regression and linear regression. Multivariate methods: principal components analysis and discriminant analysis. Contingency table. Bootstrap methods. Statistical software packages.
References:
Lehmann, Romano: Testing statistical hypotheses, Springer, New York, 2005.
Efron, Tibshirani: An introduction to the bootstrap. Chapman and Hall, New York, 1993.

MDPT257. Stochastic processes I
Discrete-time Markov chains, transition probabilities, equivalent definitions. Multi-step transition probabilities, Chapman–Kolmogorov equations. Communication classes, periodic states. Strong Markov property, recurrence times, types of states. Stationary distribution and ergodicity. Continuous-time Markov chains: definition, transition probabilities, Chapman–Kolmogorov equations. Infinitesimal generator and Kolmogorov equations. Jump chain and holding times. Birth-death processes and queueing systems.
References:

Norris: Markov chains. Cambridge University Press, Cambridge, 1998.
Karlin, Taylor: A first course in stochastic processes. Academic Press, New York, 1975.
Feller: An introduction to probability theory and its applications. Vol. I. John Wiley & Sons, New York, 1968.

MDPT258. Stochastic processes II
Filtration, stopping times, continuous-time martingales. The Brownian motion and its properties. The Itô stochastic integral and Itô's lemma. Kolmogorov equations and the Feynman–Kac formula. Weak and strong solutions of stochastic differential equations. Explicit solutions for certain stochastic differential equations. Feller's test for explosion.
References:
Karatzas, Shreve: Brownian motion and stochastic calculus. Springer-Verlag, New York, 1988.
Mikosch: Elementary stochastic calculus, World Scientific. River Edge, 1998.

MDPT3500. Classical limit theorems
Representation of infinitely divisible distributions. The conditions of convergence to an infinitely divisible distribution. Convergence to the Poisson distribution. Limit distribution of sums of identically distributed random variables: stable distributions, domains of attraction, and domains of partial attraction. Large deviations theory.
References:
Gnedenko, Kolmogorov: Limit distributions for sums of independent random variables. Addison-Wesley, Reading, 1968.
Petrov: Sums of independent random variables. Springer-Verlag, New York, 1975.

MDPT3501. Convergence of probability measures
Borel sets and probability measures on metric spaces, random elements. Weak convergence: the portmanteau theorem and the mapping theorem. Relative compactness and tightness, Prokhorov's theorem. Weak convergence in the space $C[0,1]$, Donsker's theorem on the partial sums process. Skorokhod topology on the space $D[0,1]$, weak convergence, Donsker's theorem on the empirical process.
References:
Billingsley: Convergence of probability measures. John Wiley & Sons, New York, 1999.

MDPT3502. Gaussian approximations in stochastics
The distribution of the most important functions of the Wiener process and the Brownian bridge. Skorokhod's representation and embedding theorem.

Lévy's modulus of continuity theorem. Strassen's and Brillinger's approximation for the partial sums process and the empirical process. Komlós–Major–Tusnády approximations.
References:
M. Csörgő, Révész: Strong approximations in probability and statistics. Akadémiai Kiadó, Budapest, 1981.
M. Csörgő, Horváth: Weighted approximations in probability and statistics. John Wiley & Sons, Chichester, 1993.

MDPT3503. Empirical and quantile processes
Statistical sample, empirical distribution function, the Glivenko–Cantelli theorem. Kolmogorov–Szmirnov-, Cramér–von Mises-, and Anderson–Darling-statistics. Exact and asymptotic statistical distributions. Brillinger and Komlós–Major–Tusnády approximation. The uniform and the general quantile process, approximations. Bahadur–Kiefer theorem..
References:
M. Csörgő, S. Csörgő, Horváth: An asymptotic theory for empirical reliability and concentration processes. Springer-Verlag, Berlin, 1986.
M. Csörgő, Révész: Strong approximations in probability and statistics. Akadémiai Kiadó, Budapest, 1981.
Shorack, Wellner: Empirical processes with applications to statistics. John Wiley & Sons, New York, 1986.

MDPT3506. Extreme value theory
Maximum of independent and identically distributed random variables: Gnedenko's theorem about the limit distribution. Extreme value distributions, domains of attraction. Extremal processes and records. Point processes.
References:
Resnik: Extreme values, regular variation, and point processes. Springer-Verlag, New York, 1987.
Resnik: Heavy-tail phenomena. Springer, New York, 2007.
Beran, Feng, Ghosh, Kulik: Long-memory processes. Springer, Heidelberg, 2013.

MDPT3508. Elements of stochastic processes
Selected topics in the theory of Markov processes, branching processes and birth-death processes. Renewal processes and the Poisson process. Discrete-time martingales.
References:
Brzezniak, Zastawniak: Basic stochastic processes. Springer-Verlag, London, 1999.
Dobrow: Introduction to stochastic processes with R. John Wiley & Sons, Hoboken, 2016.

Karlin, Taylor: A first course in stochastic processes. Academic Press, New York, 1975.

MDPT3510. Branching processes
Generating functions, the Galton–Watson process and the extinction criterion. Multitype branching processes, mean matrix, asymptotic behavior in the critical, the subcritical and the supercritical case. Continuous-time and age-dependent processes. Applications in biology.
References:
Athreya, Ney: Branching processes. Springer-Verlag, New York, 1972.
Kimmel, Axelrod: Branching processes in biology. Springer-Verlag, New York, 2002.

MDPT3511. Martingales
Discrete- and continuous-time martingales, sub- and supermartingales. Stopping times. The optional stopping theorem and the martingale convergence theorem. Square integrable martingales and the martingale central limit theorem. Martingale representation theorems. Optimal strategies.
References:
Revuz, Yor: Continuous martingales and Brownian motion. Springer-Verlag, Berlin, 1999.
Shiryaev: Probability. Springer-Verlag, New York, 1996.

MDPT3513. Stochastic analysis
Martingales and point processes. Stochastic integration with respect to martingale and point process, Itô's lemma. Martingale representation theorems. Stochastic differential equations: weak and strong solutions, existence and uniqueness, boundary conditions. Diffusions.
References:
Bichteler: Stochastic integration with jumps. Cambridge University Press, Cambridge, 2002.
Ikeda, Watanabe: Stochastic differential equations and diffusion processes. North-Holland, Amsterdam, 1981.

MDPT3516. Mathematical methods in statistical physics
Selected topics in the theory of statistical physics: thermodynamics and heat equations, phase transition. Diffusion processes and their applications. The Ising modell. Simulation methods: Monte Carlo method, bootstrap.
References:
Reichl: A modern course in statistical physics. John Wiley & Sons, New York, 1998.
Huang: Introduction to statistical physics. Taylor & Francis, New York, 2001.

Landau, Binder: A Guide to Monte Carlo simulations in statistical physics. Cambridge University Press, Cambridge, 2000.
Newman, Barkema: Monte Carlo methods in statistical physics. The Clarendon Press Oxford University Press, New York, 2001.

MDPT3517. Ergodic theory
Stationary processes, measure-preserving transformations and invariant sets. Birkhoff's and von Neumann's ergodic theorem. Ergodic and mixing transformations. The theory of discrete spectrum transformations. Dynamical systems and their applications.
References:
S. Csörgő: Fifty-three lectures on probability, Ann Arbor, 1991.
Billingsley: Ergodic theory and information. John Wiley & Sons, New York, 1965.
Halmos: Lectures on ergodic theory. The Mathematical Society of Japan, Japan, 1956.
Cornfeld, Fomin, Sinai: Ergodic theory. Springer-Verlag, New York, 1982.

MDPT3518. Multivariate statistical analysis
Statistics for the multivariate normal distribution. Parameter estimation and hypotheses testing in the multivariate normal model, testing normality. Linear models, analysis of variance and covariance. Partial and multivariate regression, canonical correlation, tests of independence. Discriminant, factor and principal component analysis.
References:
Bilodeau, Brenner: Theory of multivariate statistics. Springer-Verlag, New York, 1999.
Eaton: Multivariate statistics. A vector space approach. John Wiley & Sons, New York, 1983.
Johnson, Wichern: Applied multivariate statistical analysis. Prentice Hall, Englewood Cliffs, 1992.
Rencher: Methods of multivariate analysis. John Wiley & Sons, New York, 2002.

MDPT3519. Linear statistical models
The Gauss–Markov theorem. Linear regression, analysis of variance and covariance, logit, probit and log-linear models. Generalized linear models, components, residuals. Continuous and binary data. Likelihood and quasi-likelihood functions and estimators, optimality.
References:
Neter, Kutner, Nachtsheim, Wasserman: Applied linear regression models, McGraw-Hill, Chicago, 1990.

McCullagh, Nelder: Generalized linear models. Chapman & Hall, London, 1989.
Seber, Lee: Linear regression analysis. John Wiley & Sons, Hoboken, 2003.

MDPT3520. Time series analysis
Stationary processes in weak and strong sense. Spectral theory of stationary Gaussian processes. ARMA and ARIMA models: parameter estimation and model selection. Decomposition: trend, regression and seasonal components. Models with heteroscedastic errors: ARCH and GARCH processes. Multivariate time series. The Kalman filter with applications. Stochastic control.
References:
Brockwell, Davis: Time series: theory and methods, New York, 1996.
Box, Jenkins, Reinsel: Forecasting and control. Prentice Hall, Englewood Cliffs, 1994.
Hamilton: Time series analysis. Princeton University Press, Princeton, 1994.

MDPT3521. Statistics of stochastic processes
Statistical problems in the theory of stochastic processes. Likelihood theory: likelihood function, information matrix, local asymptotic normality. Linear and nonlinear filters, optimal filters. Statistical methods for diffusion processes.
References:
Küchler, Sorensen: Exponential families of stochastic processes. Springer-Verlag, New York, 1997.
Liptser, Shiryaev: Statistics of random processes, Vol. I.-II. Springer-Verlag, Berlin, 2001.
Rao: Stochastic processes—inference theory. Springer, Cham, 2014.
Fuchs: Inference for diffusion processes. Springer, Heidelberg, 2013.

MDPT3522. Nonparametric statistics
Estimators of density and regression functions: histograms and kernel density estimators. Consistency, bias, asymptotic distribution and efficiency. Bandwidth selection. Ordered sample, rank statistics and their asymptotic distribution. Testing goodness-of-fit: simple and composite hypothesis. Tests of independence.
References:
Ahsanullah, Nevzorov, Shakil: An introduction to order statistics. Atlantis Press, Paris, 2013.
Devroye: A course in density estimation. Birkh'́auser, Boston, 1987.
Hájek, Sen, Sidák: Theory of rank tests. Academic Press, San Diego, 1999.
Shorack, Wellner: Empirical processes with applications to statistics. John Wiley & Sons, New York, 1986.

MDPT3524. Resampling and simulation methods in statistics
Random number generation, sampling merhods. The Monte Carlo method. Resampling methods: jackknife and bootstrap. Some applications of the bootstrap: testing hypotheses, confidence intervals, censored samples, linear regression, discriminant analysis. Implementation in statistical software packages.
References:
Efron, Tibshirani: An introduction to the bootstrap. Chapman and Hall, New York, 1993.
Devroye: Non-uniform random variate generation, Springer, New York, 1986.
Thompson: Simulation, a modeler's approach. John Wiley & Sons, New York, 2000.

MDPT3525. Asymptotic methods in mathematical statistics
Parameter estimation methods: the method of moments and the maximum likelihood estimation. M-, Z- and U- statistics and their properties. Likelihood-ration tests. Nonparametric methods: rank and sign statistics. Further selected topics.
References:
Le Cam, Yang: Asymptotics in statistics. Springer-Verlag, New York, 2000.
van der Vaart: Asymptotic statistics. Cambridge University Press, Cambridge, 1998.
Höpfner: Asymptotic statistics. De Gruyter, Berlin, 2014.
Serfling: Approximation theorems of mathematical statistics, John Wiley & Sons, New York, 1980.

MDPT3529. Graph limits
Graph parameters and connection matrix. Graph homomorphisms. The cutnorm of dense weighted graph, the properties of convergence in this norm. Connection to statistical physics. Szemerédy's lemma. Convergence of graphs of bounded ranks.
References:
Lovász: Large networks and graph limits. American Mathematical Society, Providence, 2012.

MDPT35xx. Information theory
Information measures. Noiseless channels and Huffman coding. Discretetime memoryless channels: channel capacity and Shannon entropy. Error detection, error correcting codes and the Hamming bound. Continuos-time channels.
References:
Ash: Information theory, John Wiley & Sons, New York, 1965.
Gray: Entropy and information theory, Springer-Verlag, New York, 1990.

Yaglom, Yaglom: Probability and information, Dordrecht, 1983.

MDPT35xx. Markov processes

Transition probabilities and communication classes of discrete-time Markov processes. Atoms and small sets. Harris and topological recurrence, invariant distributions. Ergodicity and geometric ergodicity, Foster–Lyapunov conditions.

References:

Meyn, Tweedie: Markov chains and stochastic stability, Cambridge University Press, Cambridge, 2009.

MDPT35xx. Diffusion processes

Continuous-time Markov processes: transition probabilities and inifinitesimal generator. Diffusion processes, Kolmogorov equations. Partial differential equations, the heat equation. Examples: Brownian motion, Ornstein–Uhlenbeck process, and CIR process. Lévy processes and the Lévy–Hincsin representation. Stable processes and their properties. The Lévy–Itô representation. Subordinators. Potential theory.

References:

Karatzas, Shreve: Brownian motion and stochastic calculus. Springer-Verlag, New York, 1988.
Ikeda, Watanabe: Stochastic differential equations and diffusion processes. North-Holland, Amsterdam, 1981.
Karlin, Taylor: A second course in stochastic processes. Academic Press, New York, 1981.
Sato: Lévy processes and infinitely divisible distributions. Cambridge University Press, Cambridge, 1999.
Ibe: Markov processes for stochastic modeling. Elsevier, Amsterdam, 2009.

MDPT35xx. Stochastic methods in mathematical finance

The basics of mathematical finance: market, derivative, strategy, arbitrage. Discrete-time markets: no-arbitrage condition and equivalent martingale measures, complete markets. The binomial model and the CRR formula. Continuous-time markets: the Black–Scholes and the Bachelier model. Term-structure models: short rate models and the Heath–Jarrow–Morton model.

References:

Musiela, Rutkowski: Stochastic modelling and applied probability, Springer-Verlag, Berlin, 2005.
Filipovic: Term-structure models, Springer-Verlag, Berlin, 2009.

MDPT35xx. Renewal theory

Counting and renewal processes, the Poisson process. The elementary renewal theorem. Convolutions of sequences and distribution functions. Discrete and general renewal equations: existence and uniqueness of the solutions, asymptotic properties. Risk processes and Cramér–Lundberg approximations. Methods of the theory of martingales.

References:

Karlin, Taylor: A first course in stochastic processes. Academic Press, New York, 1975.

Karlin: On the renewal equation. Pacific J. Math. 5 (1955), 229–257.

Asmussen: Applied probability and queues, Springer-Verlag, New York, 2003.

MDPT35xx. Queueing theory

Discrete- and continuous-time Markov chains, birth-death processes. The basics of queueing theory: server, customer, equilibrium, explosion. Kendall's notation for queueing models. $M/M/a/b$ systems: PASTA property, Little's equations and the Polaczek–Khinchin formula. Renewal processes and $M/G/1$ systems. G/M and G/G systems.

References:

Ross: Introduction to probability models. Elsevier, Amsterdam, 2007.

Asmussen: Applied probability and queues, Springer-Verlag, New York, 2003.

Kleinrock: Queueing systems, John Wiley & Sons, New York, 1996.

Durret: Essentials of stochastic processes, Springer-Verlag, New York, 2001.

MDPT35xx. Bayesian statistics

Basics of Bayesian statistics: a priori and a posteriory distribution, loss function and risk function, Bayes estimator. Bayesian confidence intervals. Bayesian tests. Applications in regression models, time series analysis, spatial and multilevel data analysis, survival models, longitudinal data analysis, epidemiological models.

References:

Congdon: Applied Bayesian modelling. John Wiley & Sons, Chichester, 2003.

Geweke: Contemporary Bayesian econometrics and statistics. John Wiley & Sons, Hoboken, 2005.

MDPT35xx. Survival analysis

Survival and hazard function: definition and properties. Censored samples. Nonparametric estimation of the survival distribution. Asymptotic properties of the Kaplan–Meier estimator. The proportional hazards and multiplicative intensity models. Confidence bands for the survival distribution. Weighted logrank statistics for testing survival distributions.

References:

Fleming, Harrington: Counting processes and survival analysis, Wiley, New York, 1991.
Barlow, Proschan: Mathematical theory of reliability. Society for Industrial and Applied Mathematics, Philadelphia, 1996.

MDPT35xx. Data mining
Basics of statistical data mining. Data pre-processing: missing values, discretization, normalization, dimensionality reduction. Visual data mining. Measures of similarity and distance. Classification methods: discriminant analysis and cluster analysis. Regression methods. Variable selection methods. Decision trees.
References:
Han, Kamber: Data mining, concepts and techniques. Morgan Kaufmann, 2011.
Adriaans, Zantinge: Data mining. Addison–Wesley, 1996.

Matematikadidaktikai kurzusok:

(Didactics of Mathematics: this is **NOT OFFERED in English!**)

Subjects of the complex examination and their details

Doctoral School of Mathematics and Computer Science
University of Szeged

February 2, 2017; reconsidered April 9, 2019

There are 11 subjects, as listed below in the section named Contents. The Council of the Doctoral School determines two subjects for the candidate such that the following hold. The first subject consists of two subfields while the second subject consists only of one subfield. The eleven subjects are numbered by a natural numbers 1, . . . , 11 while their subfields have double indices. When the Council determines the subfields, significant (that is, at least 50 percent) overlaps must be avoided. For example, 1.1 (Structures in classical algebra) and 2.1 (Finite groups and field) significantly overlap and cannot be given to the same PhD student. Similarly, 3.1 (Measure and integral theory) and 4.1 (Theory of real functions) significantly overlap.

The first subject (the one with two subfields) cannot be 11. Didactics of mathematics. (Anyhow, the Didactics of mathematics is not offered in English; the magenta-colored part of this document is irrelevant!) Only a PhD student doing research in the Didactics of mathematics can get Subject 11 (Didactics of mathematics) as a second subject, but such a student can get a second subject different from Subject 11 as well.

For example, the Council may decide that a PhD student should pass the theoretical part of the complex exam from the following two subjects:

- 2. Group theory and semigroup theory (2.2 Group theory, 2.3. Semigroup theory), and
- 6. Differencial equations (6.3. Dinamical systems).

If this PhD student works in the didactics of mathematics, then (continuing our example), he can get (but need not get) 11. Didactics of mathematics (which consists of a single subfield) as his second subject instead of 6. Differencial equations.

Contents

1 Universal algebra and lattice theory

1.1 Structures in classical algebra

- General algebraic concepts and their connections (substructure, generating set, isomorphism, homomorphism, quotient structure, direct product). Isomorphism theorems.
- The general algebraic algebraic concepts and their connections for groups and rings, in particular.
- Simple groups and rings.
- Classical direct decomposition theorems for groups and rings.
- Principal ideal domains.
- Distributive and modular lattices.
- Representation theorems for groups, rings and Boolean algebras.
- Developing number concepts.

1.2 Universal algebra

- Universal algebraic concepts and their connections.
- Direct product, further notions of product, Birkhoff's subdirect decomposition theorem.
- Closure operators, closure systems.
- Congruence lattices.
- Free algebras.
- Varieties.
- Properties characterized by identities for varieties. Theorems of Malcev and Pixley.
- Minimal varieties.
- Varieties generated by primal algebras.

1.3 Clones

- Galois-connections.
- Abstract clones, function clones and relational clones; their connections.

- Completeness theorems for clones.
- Clone lattices on finite sets.
- Maximal clones.
- Minimal clones.
- Primitive positive clones.

1.4 Finite algebra

- Rosenberg's theorem and its applications for functionally completeness.
- Stone-Hu duality for primal algebras.
- Generalizations of primal algebras.
- Locally finite varieties.
- Spectrum of a variety.
- Relationship between relational clones and free algebras.
- Finitely based algebras. Theorems of Post és Lyndon.
- Tame congruence theory.

1.5 Lattice theory

- Basic concepts of lattice theory, duality, complete lattices.
- Algebraic lattices, subalgebra lattices.
- Distributive lattices: Birkhoff and Stone's representation theorems, the structure of finite distributive lattices.
- Birkhoff's criterion and Dedekind's criterion, the free modular and the free distributive lattice on three generators.
- Congruence of lattices.
- Modular lattices: decompositions of intervals and elements.
- Geometric lattices and complemented modular lattices.
- Projective geometries as modular lattices.
- Varieties of lattices.

1.6 Theory of coordinatization of lattices

- Geometric lattices and projective geometries.
- Coordinatization of (direct factors) of Arguesian geometric lattices.
- Coordinatization with von Neumann frames.

- Huhn diamond and és n-distributive lattices.
- Subdirectly irreducible lattices finitely presented by Huhn diamonds.
- Coordinatizing Arguesian lattice generated by Huhn diamonds.
- Von Neumann's dimension function.
- Proof theory of linear lattices.

2 Group theory and semigroup theory

2.1 Finite groups and fields

- Sylow-theorems, finite p-groups.
- Finite nilpotent and solvable groups.
- Field extensions, splitting field and normal extension.
- Finite fields.
- Perfect fields.
- The main theorem of Galois theory.
- Solving equations by radicals.
- Algebraic theory of geometric constructions.

2.2 Group theory

- Multiplicative groups of fields and division rings.
- Permutation groups (primitive and multiply transitive groups, wreath product, Frobenius groups).
- Free groups.
- Solvable groups.
- p-groups. Nilpotent groups.
- The transfer.
- The Burnside problem.
- Matrix groups. Finite simple groups.
- Subgroup lattices.

2.3 Semigroup theory

- Basic concepts of semigroup theory, representations by transformation semigroups.
- Green's relations.
- Regular $\mathcal{D}$-classes, Green's relations in regular semigroups.
- 0-simple semigroups, principal factors.
- Completely 0-simple semigroups.
- Completely regular semigroups.
- Basic properties of inverse semigroups, the Wagner–Preston representation, the natural partial order.
- Fundamental inverse semigroups, the Munn representation.

2.4 Regular semigroups

- Congruences of regular semigroups, the lattices of congruences.
- The fine structure of completely regular semigroups (Lallement's theorem).
- E-unitary inverse semigroups: covering theorem, P-theorem.
- Fundamental orthodox semigroups, the Hall representation.
- E-unitary regular semigroups.
- Locally inverse semigroups.
- Fundamental regular semigroups (Grillet's és Nambooripad's approaches).
- Generalizations of regular semigroups.

2.5 Regular semigroups

- Congruences of regular semigroups, the lattices of congruences.
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- Locally inverse semigroups.
- Fundamental regular semigroups (Grillet's és Nambooripad's approaches).
- Generalizations of regular semigroups.

3 Functional analysis

3.1 Measure and integral

- Measurable space, measure, measurable function, integral with respect to a measure, sets of zero measure.
- Convergence theorems: Lebesgue's dominated and monotone convergence theorems, Fatou's lemma.
- Positive linear functionals as integrals with respect to Borel measures, regularity of Borel measures.
- The Lebesgue measure on $\mathbf{R}^n$, change of variables, integration by parts.
- Theorems of Luzin and Jegorov, Hölder and Minkowski inequalities, completeness of the function spaces L^p.
- Product of measure spaces, Fubini's theorem, convolution.
- Complex measures, total variation, Lebesgue decomposition, Radon–Nikodym derivative, Hahn decomposition.
- Differentiation of Borel measures on $\mathbf{R}^n$, distribution functions of Borel measures on $\mathbf{R}$, functions of bounded variation.
- The dual spaces of L^p and $C_0(X)$.

3.2 Topological vector spaces

- Locally convex spaces, Hahn–Banach separation and extension theorems, Banach limit.
- Weak and weak-* topologies, metrizability, local compactness, Alaoglu's theorem.
- Convex sets, Krein–Milman and Krein–Smulian theorems.
- Topological groups, Haar measure.
- Reflexivity of normed spaces, the dual spaces of L^p and $C(X)$.
- The open mapping theorem, closed graph theorem, Banach–Steinhaus theorems.
- Hilbert spaces, orthogonal complement, orthonormal systems, the dual of a Hilbert space.

3.3 Banach algebras

- Spectrum, its dependence on subalgebras, spectral radius.

- Commutative Banach algebras, Gelfand transform, the Banach algebra $C(X)$ of continuous functions, Wiener's theorem on absolute convergent Fourier series.
- The Riesz–Dunford holomorphic functional calculus.
- Commutative C^*-algebras, functional calculus for normal elements.
- Representations of C^*-algebras on Hilbert spaces, the Gelfand–Naimark–Segal construction.

3.4 Operator theory

- Compact operators, spectrum, invariant subspaces.
- Fredholm operators, Fredholm index, essential spectrum.
- Normal, selfadjoint and unitary operators on Hilbert spaces, polar decomposition, the lattice of projections, the strong and weak operator topologies.
- Spectral theorem for normal operators, and for abelian systems of normal operators.
- Functional calculus and functional model for normal operators, multiplicity theory.
- Von Neumann algebras, the double commutant theorem, Kaplansky's density theorem, abelian von Neumann algebras.
- Unbounded operators, symmetric and selfadjoint operators, the Cayley transformation.
- Spectral theorem for unbounded normal operators, Stone's theorem for one-parameter unitary groups.

4 Classical analysis

4.1 Elements of real analysis

- Metric spaces, normed spaces, topological spaces. Convergence and continuity
- Sequences and series of functions, Stone–Weierstrass theorem
- Lebesgue integral (Beppo Lévi theorem, Lebesgue's dominated convergence theorem, Fatou's lemma)
- Canonical decomposition of monotone functions, differentiability of monotone functions

- Riemann–Stieltjes integral, Lebesgue–Stieltjes integral
- The function space $C[0,1]$ and its dual
- The function space $L^2(\Omega)$, $\Omega \subseteq \mathbf{R}$ (Riesz–Fischer theorem, orthonormal systems, Parseval's formula)
- The function space $L^p(\Omega)$ and its dual

4.2 Complex analysis

- Analytic functions (Cauchy's integral theorem, Taylor and Laurent expansions, Morera's theorem)
- Cauchy's integral formula and its consequences (maximum principle, Poisson formula, three-circles theorem)
- Zeros of analytic functions (isolation, the fundamental theorem of algebra, Rouché's theorem)
- Harmonic functions (mean value theorem, harmonic conjugate, analytic supplement)
- Factorization of entire functions, gamma function
- Convergent sequences of analytic functions (Weierstrass' theorem, the Vitali–Montel theorem)
- Riemann's conformal mapping theorem (continuity on the boundary)
- Approximation by rational functions and polynomials (Runge's theorem)
- Mergelyan's theorem, the Mittag-Leffler theorem

4.3 Fourier series

- Pointwise convergence of Fourier series (Dini, Dirichlet–Jordan and Dini–Lipschitz criterions)
- Pointwise summability of Fourier series (theorems of Fejér and Lebesgue, Lebesgue points)
- Convergence of Fourier series in L^2 norm (best approximation property, completeness, Parseval's formula)
- Characterization of function classes by Fejér and Abel–Poisson means of Fourier series
- Summability of conjugate series, conjugate functions
- Boundedness of partial sums of Fourier series in L^p norm
- Divergence of Fourier series (examples of Fejér and Kolmogorov)

- Absolute convergence of Fourier series (theorems of Bernstein, Wiener and Lévi)

4.4 Fourier integrals

- Fourier transform of L^1 functions (summability, unicity, inversion formula)
- Fourier transform of L^2 functions (Plancherel's theorem, summability)
- Fourier transform of L^p functions (Hausdorff–Young inequality, convolution-theorem)
- Fourier transform of a distribution
- Hardy–Littlewood maximal operator and theorem
- Calderón–Zygmund decomposition of L^1 functions
- Interpolation of linear operators: M. Riesz–Thorin theorem
- Interpolation of linear operators: Marcinkiewicz's theorem
- The existence of the Hilbert transform of L^p functions
- Properties of the Hilbert transform: theorems of Kolmogorov and M. Riesz

4.5 Harmonic analysis

- The H^p and h^p spaces on the complex unit disk, their characterizations by Poisson integral
- The existence of the radial limit of h^p function, Fatou's theorem
- The distribution of zeros of a holomorphic function, Jensen formula
- Blaschke product, factorization theorems of F. Riesz and Nevanlinna
- The theorem of F. and M. Riesz and its equivalent reformulations
- Canonical factorization in H^p and in the class N
- Hardy-Littlewood maximal operator and theorem
- Interpolation of linear operators: M. Riesz–Thorin theorem and Marcinkiewicz's theorem
- The existence of the Hilbert transform of L^p functions
- Properties of the Hilbert transform: theorems of Kolmogorov and M. Riesz
- The BMO and VMO spaces, maximal function and the behavior of the Hilbert transform in BMO

- The equivalence of the real and complex H^1 spaces, atomic decomposition, Fefferman's duality theorem

4.6 Orthogonal series

- The space L^2, orthogonal systems, Riesz–Fischer theorem, Parseval's formula
- Orthogonal polynomials and quadrature formuli of Gauss type
- Haar systems, the uniform and a.e. convergence of an expansion
- The Rademacher and Walsh system, the a.e. convergence and summability of an expansion
- A.e. convergence of an orthogonal series: Rademacher-Mensov theorem, Tandori's theorem
- The role of Lebesgue functions in the investigation of the convergence of orthogonal series
- Unconditional convergence of orthogonal series: theorems of Orlicz and Tandori
- $(C, 1)$-summability of orthogonal series
- Strong and absolute summability of orthogonal series
- The convergence of singular integrals: theorems of Lebesgue, Faddeev and Tandori

5 Constructive analysis

5.1 Approximation by trigonometric and algebraic polynomials

- The existence and unicity of best approximation
- Moduli of smoothness and their modifications
- Direct theorems
- Inverse theorems
- Interpolation

5.2 Approximation by linear processes

- Positive operators and the theorems of Korovkin

- Convolutions
- Moduli of smoothness and their generalizations
- Direct and inverse theorems

5.3 Orthogonal polynomials

- Measures, recursion coefficients and orthogonal polynomials
- Classical orthogonal polynomials
- Zeros of orthogonal polynomials
- General orthogonal polynomials
- Orthogonal polynomials on the unit circle
- Orthogonal polynomials on infinite intervals
- Quadratures

5.4 Potential theory and its applications

- Logarithmic potentials
- Harmonic functions
- Subharmonic functions
- The Dirichlet problem, balayage, Green's functions
- Potentials with external fields
- The theory of Freud-type orthogonal polynomials
- Approximation with varying weights

5.5 Summability theory

- Matrix methods
- Summability of trigonometric series
- Summability of orthogonal series
- Strong summability
- Strong approximation of Fourier series

5.6 Nonlinear approximation

- Approximation by rational functions
- Splines
- Fractals
- Wavelets
- n-widths

6 Differential equations

6.1 Fundamental theory of ordinary differential equations

- Existence and uniqueness of solutions of initial value problems, dependence on the initial data
- Differential inequalities
- Linear systems
- Second order linear equations
- Stability
- The theory of first integrals; integral manifolds
- The Poincaré–Bendixson theorem
- Differential equations on manifolds
- Boundary value problems

6.2 Fundamental theory of partial differential equations

- Well posed and ill posed problems
- Maximum principles
- Representation theorems for the solutions of the wave, heat and Laplace equations
- Harmonic functions
- Classical and generalised solutions of mixed problems for hyperbolic and parabolic equations
- Fundamental solutions of the operators $\partial_1^2 - \partial_2^2, \partial_0 - \sum_{i=1}^n \partial_i^2, \sum_{i=1}^n \partial_i^2$
- The Fourier method

6.3 Dynamical systems

- Local properties of dynamical systems
- Limit properties of dynamical systems
- Structural stability. The Grobman–Hartman theorem
- Existence and smoothness of local invariant manifolds
- Dissipative dynamical systems
- Properties of the global attractor
- Averaging
- Local bifurcation theory
- Chaos
- Hamiltonian systems

6.4 Stability theory

- Stability of linear systems. Lyapunov's first method. Dichotomies
- The direct method of Lyapunov
- Stability by linearisation. Critical cases
- Stability of periodic motions
- The problem of stabilization
- Stability of mechanical equilibria
- Total stability
- Structural stability
- Recursive properties. Ergodic theory

6.5 Functional differential equations

- Existence, uniqueness, smoothness, continuation of solutions
- Properties of the solution operator
- Linear, linear autonomous and linear periodic systems
- Stability. Lyapunov methods
- Local invariant manifolds
- Behaviour of the solutions near equlibria and periodic orbits
- Existence of periodic solutions
- Neutral equations
- Geometric theory of autonomus equations

6.6 Partial differential equation in function spaces

- Existence, uniqueness of the (classical, strong, weak) solutions of the Dirichlet and Neumann problems
- Expansion with respect to eigenfunctions
- Regularity of the solutions at inner and boundary points
- Perturbation and variational methods for nonlinear elliptic equations
- The method of monotone operators for nonlinear elliptic equations
- Energy methods for parabolic and hyperbolic equations
- Semigroup methods: the Hille–Yosida theorem, analytic semigroups

7 Convex and discrete geometry

7.1 Convex bodies and classical integral geometry

- Mixed volumes, Brunn–Minkowski inquality, Minkowski and Fenchel–Alexandrov inequalities
- Density for points and lines, kimenatic density, integral formulas in the plane
- Steiner's formula, quermassintegrals, Blaschke and Poincaré's basic formulas
- Curvature integrals and their applications

7.2 Computational geometry

- Encoding polytopes and hyperplane arrangements, permutation tables, dissection of point sets, zones in arrangements, the complexity of families of cells
- Data structures and geometric searching
- Convex hull algorithms, the worst case and average case analysis of the algorithms
- The geometry of linear programming
- Planar point location search, largest convex subsets, minimum measure simplices
- Algorithms for determining similarities
- Voronoi-diagrams

- Triangulating point sets, finding closest neighbors, minimum spanning trees, shapes of point sets
- Separation and intersection in the plane

7.3 Geometric algebra

- Affine and projective planes
- Desargues' Theorem and the coordinate field, Pappus' Theorem and the commutativity, the characteristic of the coordinate field, Fano configuration
- Collineations and semilinear transformations
- Symplectic and orthogonal geometry, the structure of the symplectic and orthogonal groups
- Clifford algebras

7.4 Convex geometry

- Basic properties of convex sets
- Charateodory, Radon, and Helly theorems, their generalizations and applications
- Separation, Euler relation, duality
- Approximations of convex sets, Blaschke's selection theorem
- Operations on convex sets
- The isoperimetric theorem
- Bodies of constant width
- Valuations on convex bodies
- Zonoids

7.5 Combinatorics of polytopes

- The inductive construction of polytopes, Gale-transforms and Gale-diagrams
- Euler's relation, the Dehn–Sommerville equations
- Upper bounds for the number of k-faces
- Combinatorial types of 3-polytopes, Steinitz's theorem
- Properties of boundary complexes, the van Kampen-Flores theorem

- Characterization of the f-vector
- Addition and decomposition of polytopes
- Hamiltonian paths and circuits on polytopes
- Regular polytopes

7.6 Combinatorial methods in geometry

- Block designs: Parameters of block designs, divisibility conditions. Steiner systems. Solvable block desings. Baranyai's theorem.
- Matroids: Matroid operations, coordinatization of matroids, characterization of binary matroids, graphic matroids,
- Finite projective geometries: Latin squares, parameters of finite geometries, and their relations. Desargues and Pappos planes and their relation to coordinatization, finite affine planes, Hadamard matrices.
- Finite reflection groups: Coxeter groups and complexes, buildings.

7.7 Algebraic topolgy

- Homotopy and simplicial complexes
- Subdivision and the simplicial approximation theorem
- The fundamental group and methods of calculation
- Classification of triangulable 2-manifolds
- Singular homology groups and methods of calculation: simplicial homology, exact sequences
- Homology groups with arbitrary coefficients, and the Lefschetz Fixed–Point Theorem
- Cohomology groups and calculation theorems
- The Alexander-Poincaré duality theorem
- Homotopy groups and CW-complexes
- The theorem of J. H. C. Whitehead and the cellular approximation theorem
- Homology and Cohomology of CW-Complexes
- The Hurewicz theorem
- Cohomology products

8 Differential geometry

8.1 Topology

- Topological spaces
- Compact and locally compact spaces, partition of unity
- Topological manifolds
- Homotopy and simplicial complexes, the fundamental group
- Classification of triangulable 2-manifolds
- Topological groups, transformation groups, identification spaces induced by a subgroup
- Homogeneous spaces, differentiable and analytic manifolds
- Lie-groups

8.2 Lie groups and Lie algebras, symmetric spaces

- Analytic manifolds and Frobenius theorem
- Lie groups and closed subgroups
- Exponential map, Taylor series of multiplication, Campbell–Hausdorff-formulas
- Adjoint representation of Lie groups and Lie algebras
- Fundamental theorem of Lie
- Nilpotent and solvable Lie algebras
- Semisimple Lie algebras
- Cartan subgroups and Cartan subalgebras
- Structure theory
- Classic Lie algebras and Lie groups
- Theorems of variations and components
- Pinched manifolds, locally symmetric spaces, symmetric and two-point homogeneous spaces
- Groups of isometries
- Canonical connections, Jacobi equation
- Total geodesic subgroups
- Homogeneous Riemann manifolds, geodesic manifolds of Riemannian symmetric spaces of rank one

8.3 Riemannian manifolds, theory of connections and holonomy groups

- Riemannian metric
- Levi–Civita connection
- Geodesics, convex neighborhood, normal coordinates
- Variations of geodesics, Jacobian vector fields, conjugate points
- Hopf–Rinow and Hadamard theorems
- Morse index theorem
- Sectional curvature, curvature tensor, scalar curvature
- Constant curvature spaces
- Connections on principal fibre bundles
- Parallelism
- Holonomy groups and theorem
- Reduction theorem
- Infinitesimal holonomy group
- De Rham's decomposition theorem
- Invariant connections on reductive homogeneous and symmetric spaces
- Invariant Riemann metrics and complex structures

8.4 Integral geometry of Gelfand type and geometric analysis

- Radon transform on real affine spaces, invertibility, support theorems, Plancherel formula, Paley–Wiener theorem, connections to other transforms
- Radon transform of distributions
- Radon transform on complex domain
- Radon transform and differentiation
- Radon-type transforms on spaces of constant curvature and on Lorentz spaces
- Fourier analysis on spaces of constant curvature
- Invariant measures on manifolds
- Invariant differential operators on manifolds
- Spherical transforms, Spherical function series, Paley–Wiener theorem, inverse formulas

8.5 Web geometry

- Quasigroups, loops and nets
- Coordinatization, closure configurations and their theorems
- Proectivities and collineations
- Moufang and Bol loops, Moufang and Bol nets
- Smooth webs and nets
- The tangent algebras of loops
- Chern connection
- Characterization of closure configurations with curvature and torsion
- Moufang-Lie loops and Malcev algebras

8.6 Integrable Systems

- Hamilton systems
- Darboux theorem
- Symplectic manifolds
- Legendre transform
- Free particle in pseudo-Riemannian space
- Lie–Poisson bracket
- Coadjoint orbits of Lie groups
- Moment map
- Symmetry reduction methods
- Liouville theorem
- Action-angle coordinates
- Adler–Kostant–Symes theorem
- Integrable mechanical systems, examples

9 Combinatorics and graph theory

9.1 Graph theory

- Connectivity: digraphs and their connectivity, nowhere-0-flows.
- Matchings: Gallai–Edmonds structural theorem, Edmonds polytope, random algorithms in matching theory.

- Graph colorings: Hajós' theorem, the Kneser graph and its chromatic number, unit distance graph and coloring the space.
- Independent sets in graphs: τ-critical graphs, vertex packing polytope, perfect graphs, Shannon capacity of graphs.
- Eigenvalues of graphs: random walks on graphs, expanding parameter of graphs and its connection to eigenvalues.
- Symmetric graphs: strongly regular graphs, friendship theorem, transitive graphs, Cayley-graphs.
- Random graphs.

9.2 Set systems

- Intersecting set systems, generalizations of the ErdHos–Ko–Rado theorem.
- Katona–Kruskal theorem, discrete isoperimetric problems.
- FKG inequality and its applications.
- Set systems with conditions on the intersection sizes. Ray–Chaudhuri–Wilson theorem. Applications: counterexample to Borsuk's conjecture.
- Tensor product method: Bollobás' theorem, extensions of Lovász.
- Applications of set systems in theoretical computer science: communication complexity, formula complexity, Razborov's theorem.

9.3 Designs and codes

- Steiner triples and their various constructions.
- Symmetrical block designs, resolvable block designs.
- t-designs.
- Finite projective planes, Ryser–Chowla theorem.
- Basic notions of coding theory: size, effectiveness, weight counting polynomial. Gilbert–Varaslimov bound.
- Hadamard codes.
- Linear codes, Mac Williams' theorem.
- Hamming codes. Self dual codes. Projective codes.

9.4 Enumeration problems

- Enumeration of linear extensions of partially ordered sets. Mixed volume, log-concave sequences, dimension of partially ordered sets.
- Jeu-de-taquin, tableaux, symmetric functions, Hopf algebras.
- Major index of permutations, enumerating subspaces of finite vector spaces, q-analogues of classical combinatorial objects, and identities.
- Sperner property of partially ordered sets. f-vectors of partially ordered sets.

9.5 Complexity theory

- Circuits: Size of a circuit, Turing machine complexity, and their relation. General lower bounds on circuit complexity. Constant depth circuits. Hastad lemma and the random restriction method. Lower bounds based on the approximation method. Razborov's and Smolenski's theorem. Monotone circuits, The approximation method and its applications for various functions. Limits of the approximation methods. Andreev's theorem.
- Branching programs, their complexity, and its relation to space complexity of Turing machines: Masek's theorem. Constant width branching programs.
- Formulas: Size of a formula, depth of a circuit, and their relation. Computing symmetric function with small formula. Nečiprok's theorem. Ramsey theoretical method; Hodes–Specker–Pudlák theorem. Method of random restrictions: Subotovskaja's method, Andreev's theorem. Monotone formulas. Method of random restrictions: Karchmer-Wigderson theorem. Linear algebraic method: Razborov's theorem. Applications of communication complexity for formulas: Raz–Wigderson theorem.
- Communication complexity: Rank method. Möbius function. Randomized and distributional communication complexity.
- Boolean decision trees: Examples for evasiveness. Rivest-Vuillemin theorem. Topological methods, Kahn-Saks-Sturtevan theorem. Random and non-deterministic decision trees. Sensivity of Boolean functions.

9.6 Combinatorial methods in geometry

- Designs: designs, their parameters and divisibility constrains, Steiner triples and their various constructions. Hadamard matrices. Resolvable

designs, Baranyai's theorem.
- Matroids: Matroid operations, coordinatization of matroids, characterization of binary matroids, graphic matroids,
- Finite projective geometries: Latin squares, parameters of finite geometries, and their relations. Desargues and Pappus planes and their relation to coordinatization, finite affine planes, Hadamard matrices.
- Finite reflection groups: Coxeter groups and complexes, buildings.

10 Stochastics

10.1 Strong laws in probability theory

- The elements of Kolmogorov's probability theory
- Kolmogorov's definition for conditional expectation and distribution
- The most important probability distributions and their properties
- Borel–Cantelli lemmas, Kolmogorov's 0–1 law
- The three-series theorem
- Kolmogorov's inequality, strong laws of large numbers
- The law of the iterated logarithm
- Birkhoff's ergodic theorem

10.2 Limit theorems

- Weak convergence of measures, Prokhorov's theorem
- Characteristic functions, inversion formula and continuity theorem
- The Lindeberg central limit theorem and the local limit theorem
- The Berry–Esséen theorem
- Cramér's theorem on the probability of large deviations
- Convergence to the Poisson distribution

10.3 Elements of mathematical statistics

- Empirical distribution, Glivenko–Cantelli theorem
- The Cramér–Rao and the Blackwell–Kolmogorov–Rao inequality
- The maximum likelihood estimation and its properties
- Confidence intervals

- The theory of statistical hypothesis testing, Neyman–Pearson lemma, likelihood-ratio tests
- Sequential methods
- Testing goodness-of-fit, the Kolmogorov–Smirnov test
- Wishart distribution, parameter estimation and hypothesis testing in the multivariate normal model
- Generalized linear models

10.4 Applied statistics

- Basics of the estimation theory and the statistical hypothesis testing
- Distributions derived from the normal distribution, classical tests for the parameters of the normal distribution
- Analysis of variances
- Linear and nonlinear regression, logistic regression
- Linear discriminant analysis, cluster analysis
- Factor analysis and principal component analysis
- Nonparametric estimation methods, nonparametric tests
- Contingency tables
- Survival analysis: censored samples, the Kaplan–Meier estimator
- The basics of Bayesian methods

10.5 Stochastic processes with discrete state space

- General theory of stochastic processes, Kolmogorov existence theorem
- Discrete-time Markov chains: Chapman–Kolmogorov equations, strong Markov property, types of the states, stationary distribution, ergodicity
- Pólya's theorem on the random walk
- Continuous-time Markov chains: Chapman–Kolmogorov equations, stochastic semigroups, infinitesimal generator, Kolmogorov equations, jump chain and holding times, stationary distribution, ergodicity
- The Poisson process, birth-death processes, queueing models, $M/M/a/b$ systems
- Renewal processes and the elementary renewal theorem

10.6 Stochastic processes with continuous state space

- Discrete- and continuous-time martingales: Doob–Meyer decomposition, optional stopping theorem, martingale convergence theorem
- Existence and properties of the Wiener process
- Stochastic integration with respect to square integrable continuous martingales, Itô's lemma
- Stochastic differential equations, weak and strong solutions
- The exponential Brownian motion and the Ornstein–Uhlenbeck process
- Diffusion processes, Kolmogorov equations
- The Feynman–Kac formula

11 Didactics of mathematics: NOT offered in English)