Monochromatic structures in two-edge-colored ordered complete graphs

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joint work with András Gyárfás and Géza Tóth

Co-authors

Preliminary results

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Every 2-colored complete graph has a monochromatic spanning tree.

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Every 2-colored complete graph K_{3n-1} contains a monochromatic matching M_n and this is not true for K_{3n-2}.

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Theorem (KPT)

Every 2-colored complete geometric graph has a monochromatic plane spanning tree.

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Every 2-colored complete geometric graph K_{3n-1} contains a monochromatic plane matching M_n.

Here a plane subgraph is one, whose edges in the embedding do not have common internal points.

Our starting point

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Conjecture: Every 2-colored complete simple drawing has a monochromatic plane spanning tree.

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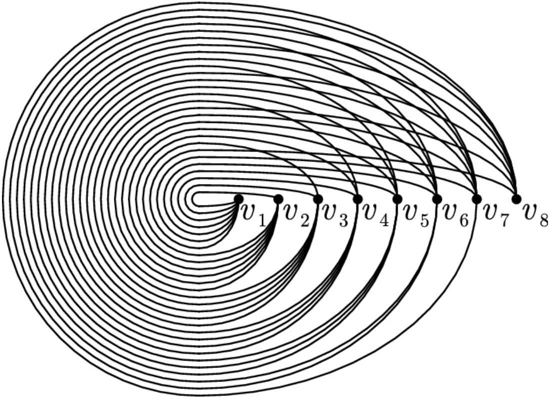

Ordered graphs

Ordered graphs

An ordered graph G is a simple graph with $V(G) = [m] = \{1, 2, \ldots, m\}$.
We also use $[i, j] = \{i, i + 1, \ldots, j\}$
The vertex set is considered with the natural ordering and the edges are denoted by (i, j), where we always assume $i < j$.
The length of (i, j) is $j - i$.

Independent edges

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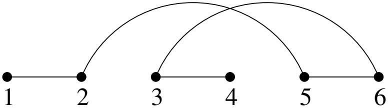
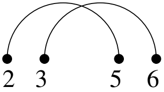
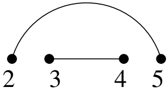
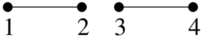
- Edges (a, b) and (c, d) are crossing if either $a < c < b < d$ or $c < a < d < b$.
- Edges (a, b) and (c, d) are nested if either $a < c < d < b$ or $c < a < b < d$.
- Edges (a, b) and (c, d) are separated if either $a < b < c < d$ or $c < d < a < b$.



an ordered graph







6 types of questions

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	length of the	longest cycle	
	lower bound	upper bound	
nested	3	3	
crossing	?	$n/2$	
separated	3	3	
non−nested	?	$2n/3$	$\rightarrow$ **twisted drawing**
non−crossing	$\sqrt{n/2}$	$\sqrt{n}$	$\rightarrow$ convex drawing
non−separated	$n/4$	$2n/3$	

Monochromatic spanning trees

Monochromatic spanning trees

Theorem (JB, AGy, GT)

In every 2-coloring of the ordered complete graph, there exists
(i) a monochromatic non-crossing spanning tree.
(ii) a monochromatic non-nested spanning tree.
(iii) a monochromatic non-separated spanning tree.

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(ii) connection to twisted drawings

Non-crossing spanning tree

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If all edges $(i, i+1)$, $i = 1, \dots, n-1$ have the same color, then we are done, they form the desired spanning tree.

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In every 2-coloring of the ordered complete graph on $[n]$, there exists (i) a monochromatic non-crossing spanning tree.

If all edges $(i, i+1)$, $i = 1, \ldots, n-1$ have the same color, then we are done, they form the desired spanning tree.
Otherwise there is an i, $1 < i < n$ such that $(i-1, i)$ and $(i, i+1)$ have different colors.

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Otherwise there is an i, $1 < i < n$ such that $(i-1, i)$ and $(i, i+1)$ have different colors.
Delete vertex i. Use induction.

Non-nested spanning tree

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In every 2-coloring of the ordered complete graph on $[n]$, there exists
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In every 2-coloring of the ordered complete graph on $[n]$, there exists (ii) a monochromatic non-nested spanning tree.

Let c be any 2-coloring of the edges. $(1, 2)$ is blue. If $(1, i)$ is blue for every i, $2 \leq i \leq n$, then we are done,

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In every 2-coloring of the ordered complete graph on $[n]$, there exists (ii) a monochromatic non-nested spanning tree.

Let c be any 2-coloring of the edges. $(1, 2)$ is blue. If $(1, i)$ is blue for every i, $2 \leq i \leq n$, then we are done,
Otherwise, let s be the smallest number such that $(1, s)$ is red. We now change the coloring c to $\tilde{c}$ as follows: we recolor each edge induced by $[s-1]$ blue, and keep c otherwise. Consider the coloring $\tilde{c}$ on $[2, n]$ and apply the induction hypothesis.

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Suppose first that we find a blue spanning tree B without nested edges. Delete the edges in B induced by $[2, s-1]$. The resulting graph can also be found in the original coloring c. Now add the blue edges $(1,2),(1,3),\ldots,(1,s-1)$.

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Suppose first that we find a blue spanning tree B without nested edges. Delete the edges in B induced by $[2, s - 1]$. The resulting graph can also be found in the original coloring c. Now add the blue edges $(1, 2), (1, 3), \ldots, (1, s-1)$.
Suppose now that we found a red spanning tree R on $[2, n]$. It cannot contain any edges induced by $[2, s-1]$ since they are blue. So, R can also be found in the original coloring c. Simply add edge $(1, s)$.

Non-separated spanning tree

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In every 2-coloring of the ordered complete graph on $[n]$, there exists
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Let c be any 2-coloring of the edges. $(1, n)$ is blue. If $(1, i)$ is blue for every i, $2 \leq i \leq n$, then we are done,
Otherwise, let s be the largest number such that $(1, s)$ is red.

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In every 2-coloring of the ordered complete graph on $[n]$, there exists (iii) a monochromatic non-separated spanning tree.

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Suppose now that we found a red spanning tree R on $[2, n]$. It cannot contain any edges induced by $[s+1, n]$ since they are blue. So, R can also be found in the original coloring c. Simply add edge $(1, s)$.

Monochromatic spanning subgraphs

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Theorem (JB, AGy, GT)

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Proposition (JB, AGy, GT)

(i) There is a 2-coloring of the ordered complete graph on $[n]$, which does not contain a non-crossing monochromatic subgraph with n edges.
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Let H be a non-crossing subgraph with red edges.

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Consider the convex drawing of K_n, where the blue edges are on the outer cycle. Now H becomes a planar subgraph of the convex drawing.

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Let us add all edges of the outer cycle ($n-1$ blue and possibly one red) to H to get H'.

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It is still an outerplanar graph. Therefore, H' has at most $2n-3$ edges, $n-1$ of them are blue.

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Thus H can have at most $n-2$ edges.

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The argument is the same for the blue edges.

Non-separated subgraph

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In any 2-coloring of the ordered complete graph on $[n]$, there is a non-separated monochromatic subgraph of $\lfloor n^2/8 \rfloor$ edges.

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majority color

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Theorem (JB, AGy, GT)

If an ordered complete graph on $[3n-1]$ contains either
(i) a red K_{2n-1} or
(ii) a blue $K_{n-1,2n}$ as a subgraph, then there is a monochromatic non-nested M_n.

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Consider the complete bipartite graph on $A = [1, n]$ and $B = [2n, 3n-1]$.

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Consider the complete bipartite graph on $A = [1, n]$ and $B = [2n, 3n{-}1]$.
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check the new edge and the non-separated property

Singular matchings

Singular matchings

Let $R_{nest}(t,n), R_{cr}(t,n), R_{sep}(t,n)$ be the smallest m such that every t-coloring of the edges of the ordered complete graph on $[m]$ there is a monochromatic nested, crossing, separated matching, respectively, of size n.

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Theorem (JB, AGy, GT)

For $t, n \geq 2$ we have $R_{nest}(t,n) = 2(t(n-1)+1)$.
For $t, n \geq 2$ we have $R_{cr}(t,n) = 2t(n-1)+1$.
For $t, n \geq 2$ we have $R_{sep}(t,n) = 2(t(n-1)+1)$.

Preliminary results on cycles

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Every n-vertex graph with $(k-1)(n-1)/2+1$ edges contains a cycle of length at least k.

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Every 2-coloring of the complete geometric graph with n vertices contains a cycle of length at least $\sqrt{k/2}$.

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Cycles and colorings

	length of the	longest cycle
	lower bound	upper bound
nested	3	3
crossing	?	$n/2$
separated	3	3
non−nested	?	$2n/3$
non−crossing	$\sqrt{n/2}$	$\sqrt{n}$
non−separated	$n/4$	$2n/3$

Open questions

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Problem 1

What is the minimum number m such that every 2-colored ordered complete graph on $[m]$ contains a monochromatic non-nested M_n?

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Problem 2

Show that every 2-colored ordered complete graph on $[11]$ contains a monochromatic non-nested M_4.