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Bistable traveling wave solutions in a competitive recursion system with Ricker nonlinearity

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Abstract. Using an abstract scheme of monotone semiflows, the existence of bistable traveling wave solutions of a competitive recursion system is established. From the viewpoint of population dynamics, the bistable traveling wave solutions describe the strong inter-specific actions between two competitive species.

Keywords: monotone semiflows, strong competition, spreading speed, counter-propagation.

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1 Introduction

Bistable traveling wave solutions of evolutionary systems are useful for modeling biology invasion with Allee effect and phase transition with multi steady states [22]. In the past decades, the existence of bistable traveling wave solutions of *scalar* equations has been widely studied, we refer to [1–5, 8, 9, 12, 17, 22, 25] and the references cited therein. Very recently, Fang and Zhao [7] established an abstract scheme to prove the existence of bistable traveling wave solutions of evolutionary systems generating monotone semiflows. By the theory in [7], Zhang and Zhao [26, 27] obtained the existence of bistable traveling wave solutions in some coupled systems.

In this paper, we shall investigate the bistable traveling wave solutions of the following recursion system

$$\begin{cases} U_{n+1}(x) = \int_{\mathbb{R}} U_n(y) e^{r_1(1-U_n(y)-a_1V_n(y))} l_1(x-y)dy, \\ V_{n+1}(x) = \int_{\mathbb{R}} V_n(y) e^{r_2(1-V_n(y)-a_2U_n(y))} l_2(x-y)dy, \end{cases} \tag{1.1}$$

where $r_1 > 0, r_2 > 0$, $a_1 \geq 0$, $a_2 \geq 0$ are constants, $U_n(x)$ and $V_n(x)$ denote the densities of two competitors at time $n \in \mathbb{N} \bigcup \{0\}$ at location $x \in \mathbb{R}$ in population dynamics, l_1 and l_2 are probability functions describing the dispersal of individuals. When $a_1 < 1 < a_2$ in (1.1), Wang

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and Castillo-Chávez [23] considered its monostable traveling wave solutions and spreading speeds, and Li and Li [14] further studied the properties of its monostable traveling wave solutions. Recently, Pan and Lin [18] answered the existence and nonexistence of traveling wave solutions of (1.1) if $a_1, a_2 \in (0,1)$, see also Li and Li [15].

If $a_1, a_2 > 1$ in (1.1), then the corresponding difference system

$$\begin{cases} u_{n+1} = u_n e^{r_1(1-u_n-a_1v_n)}, \\ v_{n+1} = v_n e^{r_2(1-v_n-a_2u_n)}, \end{cases} \tag{1.2}$$

has four equilibria:

$$E_0 = (0,0),\ E_1 = (1,0),\ E_2 = (0,1),\ E_3 = \left(\frac{1-a_1}{1-a_1a_2}, \frac{1-a_2}{1-a_1a_2}\right) =: (k_1, k_2).$$

In particular, if $r_1, r_2 \in (0,1]$, then both E_1 and E_2 are stable while E_0, E_3 are unstable. In population dynamics, (1.2) is the Ricker competitive system [19], see [6, 11, 13, 20, 21] for its dynamics.

When E_1, E_2 are stable in (1.2), then a traveling wave solution connecting E_1 with E_2 is a bistable traveling wave solution of (1.1), and a traveling wave solution connecting E_0 (or E_3) with E_1 (or E_2) is a monostable traveling wave solution of (1.1), see [14, 23]. In this paper, we shall prove the existence of bistable traveling wave solutions of (1.1) by the theory in Fang and Zhao [7]. In particular, to verify the counter-propagation in what follows, the spreading speeds of several monostable subsystems of (1.1) are established by the results in Hsu and Zhao [10], Liang and Zhao [16] and Weinberger et al. [24].

2 Preliminaries

In this paper, we shall use the standard partial ordering and ordering interval in $\mathbb{R}$ or $\mathbb{R}^2$. Let $\mathcal{C} := C(\mathbb{R}, \mathbb{R}^2)$ be

$$C(\mathbb{R}, \mathbb{R}^2) = \{\, U \mid U\colon \mathbb{R} \to \mathbb{R}^2 \text{ is a uniformly continuous and bounded function} \,\}$$

equipped with the standard compact open topology, namely, $U_n \to U$ in $\mathcal{C}$ if and only if the sequence of $U_n(x) \in \mathcal{C}$ converges to $U(x) \in \mathcal{C}$ uniformly in any compact subset of $x \in \mathbb{R}$. If $U = (u_1(x), u_2(x))$, $V = (v_1(x), v_2(x)) \in \mathcal{C}$, then

$$\begin{aligned} U \geq (\leq) V \quad &\Leftrightarrow \quad u_i(x) \geq (\leq) v_i(x),\ i = 1,2, x \in \mathbb{R}; \\ U \gg (\ll) V \quad &\Leftrightarrow \quad U \geq (\leq) V \text{ and } u_i(x) > (<) v_i(x),\ i = 1,2, x \in \mathbb{R}. \end{aligned}$$

Moreover, if $A, B \in \mathbb{R}^2$ with $A \leq B$, then $\mathcal{C}_{[A,B]} = \{\, U : U \in \mathcal{C}, A \leq U(x) \leq B,\ x \in \mathbb{R} \,\}$.

To study the bistable traveling wave solutions of (1.1), we shall impose the following assumptions in this paper:

(H1) $r_1, r_2 \in (0,1]$ and $a_1, a_2 \in (1, \infty)$;

(H2) l_i is Lebesgue measurable and integrable such that $\int_{\mathbb{R}} l_i(y)dy = 1$ and $\int_{\mathbb{R}} l_i(y)e^{\lambda y}dy < \infty$ for all $\lambda \in \mathbb{R}$, $i = 1,2$;

(H3) $l_i(y) = l_i(-y) \geq 0, y \in \mathbb{R}$, $i = 1,2$.

To apply the theory of monotone semiflows, we make a change of variables $U_n(x) = 1 - U_n^*(x)$, $V_n(x) = V_n^*(x)$ and drop the star for the sake of simplicity, then (1.1) becomes

$$\begin{cases} U_{n+1}(x) = 1 - \int_{\mathbb{R}} (1 - U_n(y)) e^{r_1(U_n(y) - a_1 V_n(y))} l_1(x-y) dy, \\ V_{n+1}(x) = \int_{\mathbb{R}} V_n(y) e^{r_2(1 - a_2 - V_n(y) + a_2 U_n(y))} l_2(x-y) dy, \end{cases} \tag{2.1}$$

and the corresponding difference system of (2.1) is

$$\begin{cases} u_{n+1} = 1 - (1 - u_n) e^{r_1(u_n - a_1 v_n)}, \\ v_{n+1} = v_n e^{r_2(1 - a_2 - v_n + a_2 u_n)}. \end{cases} \tag{2.2}$$

Evidently, (2.2) has four equilibria

$$F_0 = (0,0),\ F_1 = (1,0),\ F_2 = (1 - k_1, k_2),\ F_3 = (1,1),$$

and F_0, F_3 are stable while F_1, F_2 are unstable. Then it suffices to study the bistable traveling wave solutions of (2.1) connecting F_0 with F_3. We now give the definition of traveling wave solutions as follows.

Definition 2.1. A traveling wave solution of (2.1) is a special solution of the form $U_n(x) = \phi(t)$, $V_n(x) = \psi(t)$, $t = x + cn$ with the wave speed $c \in \mathbb{R}$ and the wave profile $(\phi, \psi) \in \mathcal{C}$. Then (ϕ, ψ) and c must satisfy

$$\begin{cases} \phi(t+c) = 1 - \int_{\mathbb{R}} (1 - \phi(y)) e^{r_1(\phi(y) - a_1 \psi(y))} l_1(t-y) dy, \\ \psi(t+c) = \int_{\mathbb{R}} \psi(y) e^{r_2(1 - a_2 - \psi(y) + a_2 \phi(y))} l_2(t-y) dy, t \in \mathbb{R}. \end{cases} \tag{2.3}$$

For a bistable traveling wave solution (ϕ, ψ), it also satisfies

$$\lim_{t \to -\infty} (\phi(t), \psi(t)) = (0,0) =: \theta, \quad \lim_{t \to \infty} (\phi(t), \psi(t)) = (1,1) =: \mathbf{1}. \tag{2.4}$$

In what follows, we shall investigate the existence of (2.3)–(2.4) by Fang and Zhao [7]. Let $\theta \ll M \in \mathbb{R}^2$ and Q be a map from $\mathcal{C}_{[\theta,M]}$ to $\mathcal{C}_{[\theta,M]}$ with $Q(\theta) = \theta, Q(M) = M$. Also let F be the set of all spatially homogeneous steady states of Q restricted on $[\theta, M]$. We now list the conditions of [7, Theorem 3.1] as follows.

(A1) (*Transition invariance*) $T_y \circ Q[\Phi] = Q \circ T_y[\Phi]$ for any $\Phi \in \mathcal{C}_{[\theta,M]}$ and $y \in \mathbb{R}$, where $T_y[\Phi](x) = \Phi(x-y)$;

(A2) (*Continuity*) $Q\colon \mathcal{C}_{[\theta,M]} \to \mathcal{C}_{[\theta,M]}$ is continuous with respect to the compact open topology;

(A3) (*Monotonicity*) Q is order preserving in the sense that $Q[\Phi] \geq Q[\Psi]$ if $\Phi \geq \Psi$ with $\Phi, \Psi \in \mathcal{C}_{[\theta,M]}$;

(A4) (*Compactness*) $Q\colon \mathcal{C}_{[\theta,M]} \to \mathcal{C}_{[\theta,M]}$ is compact with respect to the compact open topology;

(A5) (*Bistability*) Two fixed points θ and M are strongly stable from above and below, respectively, for the map $Q\colon \mathcal{C}_{[\theta,M]} \to \mathcal{C}_{[\theta,M]}$, that is, there exist a number $\delta > 0$ and unit vectors E_4, E_5 with $\theta \ll E_4, E_5 \ll \mathbf{1}$ such that

$$Q[\eta E_4] \ll \eta E_4, Q[M - \eta E_5] \gg M - \eta E_5,\ \eta \in (0, \delta],$$

and the set $F \backslash \{\theta, M\}$ is totally unordered;

(A6) (*Counter-propagation*) For each $I \in F\backslash\{\theta, M\}$, $c_-^*(I, M) + c_+^*(\theta, I) > 0$, where $c_-^*(I, M)$ and $c_+^*(\theta, I)$ represent the leftward and rightward spreading speeds of the monostable subsystem $\{Q^n\}_{n\geq 0}$ restricted on $\mathcal{C}_{[I,M]}$ and $\mathcal{C}_{[\theta,I]}$, respectively.

In Fang and Zhao [7], under the assumptions (A1)–(A6), the existence of bistable traveling wave solutions of $\{Q^n\}_{n\geq 0}$ connecting θ with M has been proved, which is monotone increasing. That is, there exist a monotone decreasing function $\Psi \in \mathcal{C}$ and a constant $c \in \mathbb{R}$ such that

$$Q^n[\Psi](x) = \Psi(x + cn),\ x \in \mathbb{R},\ n \geq 0$$

and

$$\lim_{\xi \to -\infty} \Psi(\xi) = \theta,\ \lim_{\xi \to \infty} \Psi(\xi) = M.$$

3 Existence of bistable traveling wave solutions

We first present the main conclusion of this paper as follows.

Theorem 3.1. *Assume that (H1)–(H3) hold. Then there exist $c \in \mathbb{R}$ and $(\phi, \psi) \in \mathcal{C}_{[\theta,\mathbf{1}]}$ satisfying (2.3)–(2.4), which is monotone increasing and is a bistable traveling wave solution of (2.1).*

For $\Phi = (\phi, \psi) \in \mathcal{C}_{[\theta,\mathbf{1}]}$, we define $Q = (Q_1, Q_2)$ by

$$\begin{cases} Q_1(\phi, \psi)(t) = 1 - \displaystyle\int_{\mathbb{R}} (1 - \phi(y)) e^{r_1(\phi(y) - a_1 Y_n(y))} l_1(t - y) dy, \\ Q_2(\phi, \psi)(t) = \displaystyle\int_{\mathbb{R}} \psi(y) e^{r_2(1 - a_2 - \psi(y) + a_2 \phi(y))} l_2(t - y) dy. \end{cases} \tag{3.1}$$

To prove Theorem 3.1, we now take $M = (1, 1)$, $F = \{F_0, F_1, F_2, F_3\}$ and check (A1)–(A6) by several lemmas, throughout which (H1)–(H3) hold.

Lemma 3.2. *If Q is defined by (3.1), then it satisfies (A1).*

Proof. For any $y \in \mathbb{R}$ and $(\phi, \psi) \in \mathcal{C}_{[\theta,\mathbf{1}]}$, we have

$$\begin{aligned} T_y[Q_2(\phi, \psi)(t)] &= T_y \left[\int_{\mathbb{R}} \psi(t - s) e^{r_2(1 - a_2 - \psi(t-s) + a_2 \phi(t-s))} l_2(s) ds \right] \\ &= \int_{\mathbb{R}} \psi(t - y - s) e^{r_2(1 - a_2 - \psi(t-y-s) + a_2 \phi(t-y-s))} l_2(s) ds \\ &= Q_2(T_y[\phi], T_y[\psi])(t). \end{aligned}$$

Similarly, we obtain $T_y[Q_1(\phi, \psi)(t)] = Q_1(T_y[\phi], T_y[\psi])(t)$. The proof is complete. $\square$

Lemma 3.3. *If Q is defined by (3.1), then $Q\colon \mathcal{C}_{[\theta,\mathbf{1}]} \to \mathcal{C}_{[\theta,\mathbf{1}]}$ and satisfies (A2)–(A4).*

Proof. For any t, δ and $(\phi, \psi) \in \mathcal{C}_{[\theta,\mathbf{1}]}$, we have

$$\begin{aligned} &|Q_2(\phi, \psi)(t + \delta) - Q_2(\phi, \psi)(t)| \\ &= \left| \int_{\mathbb{R}} \psi(s) e^{r_2(1 - a_2 - \psi(s) + a_2 \phi(s))} \left[l_2(t + \delta - s) - l_2(t - s) \right] ds \right| \\ &\leq \int_{\mathbb{R}} \psi(s) e^{r_2(1 - a_2 - \psi(s) + a_2 \phi(s))} \left| l_2(t + \delta - s) - l_2(t - s) \right| ds \\ &\leq \int_{\mathbb{R}} \left| l_2(t + \delta - s) - l_2(t - s) \right| ds, \end{aligned} \tag{3.2}$$

which implies the equicontinuity of $Q_2(\phi,\psi)(t)$ by (H2). A similar result holds for $Q_1(\phi,\psi)(t)$.

Since $r_2 \in (0,1]$, we know that $ue^{r_2(1-a_2-u+a_2v)}$ is monotone increasing in $u, v \in [0,1]$ such that

$$0 \le ue^{r_2(1-a_2-u+a_2v)} \le 1,\ u \in [0,1],\ v \in [0,1],$$

which further implies that

$$0 = \int_{\mathbb{R}} 0 \cdot l_2(t-s)ds \le \int_{\mathbb{R}} \psi(s)e^{r_2(1-a_2-\psi(s)+a_2\phi(s))} l_2(t-s)ds \le \int_{\mathbb{R}} 1 \cdot l_2(t-s)ds = 1$$

for any $(\phi,\psi) \in \mathcal{C}_{[\theta,\mathbf{1}]}$. By a similar analysis of Q_1, we can prove that $Q\colon \mathcal{C}_{[\theta,\mathbf{1}]} \to \mathcal{C}_{[\theta,\mathbf{1}]}$.

Due to the continuity and the monotonicity of

$$ue^{r_2(1-a_2-u+a_2v)},\ 1-(1-u)e^{r_1(u-a_1v)},\ u,v \in [0,1],$$

and the verification of (3.2), then (A2)–(A4) are clear and we omit the details here. The proof is complete. $\square$

Lemma 3.4. *(A5) is true.*

Proof. Let

$$\delta = \min\left\{\frac{a_2-1}{4a_1a_2+4}, \frac{a_1-1}{4a_1a_2+4}\right\} > 0,\ E_4 = \left(\frac{2a_1}{\sqrt{1+4a_1^2}}, \frac{1}{\sqrt{1+4a_1^2}}\right).$$

It is clear that $\eta \in (0,\delta]$ leads to $\frac{r_1\eta a_1}{\sqrt{1+4a_1^2}} > 0$, then

$$\left(1-\frac{2\eta a_1}{\sqrt{1+4a_1^2}}\right)e^{\frac{r_1\eta a_1}{\sqrt{1+4a_1^2}}} > 1-\frac{2\eta a_1}{\sqrt{1+4a_1^2}},$$

and

$$1-\left(1-\frac{2\eta a_1}{\sqrt{1+4a_1^2}}\right)e^{\frac{r_1\eta a_1}{\sqrt{1+4a_1^2}}} < \frac{2\eta a_1}{\sqrt{1+4a_1^2}} = \eta\frac{2a_1}{\sqrt{1+4a_1^2}}.$$

On the other hand, the definition of δ implies that $\frac{2\eta a_1a_2}{\sqrt{1+4a_1^2}} < a_2 - 1$, then

$$1-a_2-\frac{\eta}{\sqrt{1+4a_1^2}}+\frac{2\eta a_1a_2}{\sqrt{1+4a_1^2}} < 0,$$

and

$$\frac{\eta}{\sqrt{1+4a_1^2}}e^{r_2\left(1-a_2-\frac{\eta}{\sqrt{1+4a_1^2}}+\frac{2\eta a_1a_2}{\sqrt{1+4a_1^2}}\right)} < \frac{\eta}{\sqrt{1+4a_1^2}}.$$

By what we have done, we obtain $Q[\eta E_4] \ll \eta E_4, \eta \in (0,\delta]$.

Furthermore, $Q[M-\eta E_5] \gg M-\eta E_5,\ \eta \in (0,\delta]$ can be similarly verified by letting

$$E_5 = \left(\frac{1}{\sqrt{1+4a_2^2}}, \frac{2a_2}{\sqrt{1+4a_2^2}}\right).$$

Moreover, F_1 and F_2 are unordered. The proof is complete. $\square$

Lemma 3.5. $c_-^*(F_1, F_3) + c_+^*(F_0, F_1) > 0$.

Proof. To compute $c_-^*(F_1, F_3)$, we consider the spreading speed of the following integrodifference equation
$$p_{n+1}(x) = \int_{\mathbb{R}} p_n(y) e^{r_2(1-p_n(y))} l_2(x-y) dy.$$
By (H1)–(H3) and Hsu and Zhao [10, Theorem 2.1], then
$$c_-^*(F_1, F_3) = \inf_{\mu>0} \frac{\ln(e^{r_2} \int_{\mathbb{R}} e^{\mu y} l_2(y) dy)}{\mu},$$
which implies that $c_-^*(F_1, F_3) > 0$ by (H2) and Liang and Zhao [16, Lemma 3.8].

To establish $c_+^*(F_0, F_1)$, define an integrodifference equation as follows
$$q_{n+1}(x) = 1 - \int_{\mathbb{R}} (1 - q_n(y)) e^{r_1 q_n(y)} l_1(x-y) dy. \tag{3.3}$$
Let $w_n(x) = 1 - q_n(x)$, then (3.3) becomes $w_{n+1}(x) = \int_{\mathbb{R}} w_n(y) e^{r_1(1-w_n(y))} l_1(x-y) dy$ and
$$c_+^*(F_0, F_1) = \inf_{\mu>0} \frac{\ln(e^{r_1} \int_{\mathbb{R}} e^{\mu y} l_1(y) dy)}{\mu} > 0.$$
The proof is complete. □

Lemma 3.6. $c_-^*(F_2, F_3) + c_+^*(F_0, F_2) > 0$.

Proof. We first consider $c_-^*(F_2, F_3)$. Letting $p_n(x) = U_n(x) - (1 - k_1)$, $q_n(x) = V_n(x) - k_2$, then (2.1) leads to
$$\begin{cases} p_{n+1}(x) = k_1 + \int_{\mathbb{R}} (p_n(y) - k_1) e^{r_1(p_n(y) - a_1 q_n(y))} l_1(x-y) dy, \\ q_{n+1}(x) = -k_2 + \int_{\mathbb{R}} (q_n(y) + k_2) e^{r_2(a_2 p_n(y) - q_n(y))} l_2(x-y) dy. \end{cases} \tag{3.4}$$
Consider the corresponding initial value problem of (3.4) with $0 \le p_0(x) \le k_1,\ 0 \le q_0(x) \le 1 - k_2,\ x \in \mathbb{R}$, in which $p_0(x), q_0(x)$ are uniformly continuous and admit nonempty compact supports. If $0 \le u \le k_1,\ 0 \le v \le 1 - k_2$, then
$$\begin{aligned} &0 \le k_1 + (u - k_1) e^{r_1(u - a_1 v)} \le k_1, \\ &0 \le -k_2 + (v + k_2) e^{r_2(a_2 u - v)} \le 1 - k_2 \end{aligned}$$
and both of them are monotone increasing in $u \in [0, k_1]$, $v \in [0, 1 - k_2]$. Using the comparison principle, we obtain $(p_n(x), q_n(x)) \in \mathcal{C}$, $n \in \mathbb{N}$ with $0 \le p_n(x) \le k_1$, $0 \le q_n(x) \le 1 - k_2$, $x \in \mathbb{R}$, $n \in \mathbb{N}$. Let $K^* = [k_1, 1 - k_2]$, then $\mathcal{C}_{[\theta, K^*]}$ is an invariant region of (3.4) and it is reasonable to restrict Q on $\mathcal{C}_{[F_2, F_3]}$.

For $\mu \ge 0$, define
$$B_\mu = \begin{bmatrix} (1 - r_1 k_1) \int_{\mathbb{R}} e^{\mu y} l_1(y) dy & a_1 r_1 k_1 \int_{\mathbb{R}} e^{\mu y} l_2(y) dy \\ a_2 r_2 k_2 \int_{\mathbb{R}} e^{\mu y} l_1(y) dy & (1 - r_2 k_2) \int_{\mathbb{R}} e^{\mu y} l_2(y) dy \end{bmatrix}.$$
We now consider the principle eigenvalue of B_μ, denoted by $\lambda\left(B_\mu\right)$. If
$$\int_{\mathbb{R}} e^{\mu y} l_1(y) dy \le \int_{\mathbb{R}} e^{\mu y} l_2(y) dy$$

holds, then

$$\begin{aligned}
&\begin{vmatrix} \int_{\mathbb{R}} e^{\mu y} l_1(y)dy - (1-r_1k_1)\int_{\mathbb{R}} e^{\mu y} l_1(y)dy & -a_1 r_1 k_1 \int_{\mathbb{R}} e^{\mu y} l_2(y)dy \\ -a_2 r_2 k_2 \int_{\mathbb{R}} e^{\mu y} l_1(y)dy & \int_{\mathbb{R}} e^{\mu y} l_1(y)dy - (1-r_2k_2)\int_{\mathbb{R}} e^{\mu y} l_2(y)dy \end{vmatrix} \\
&= r_1k_1 \int_{\mathbb{R}} e^{\mu y} l_1(y)dy \begin{vmatrix} 1 & -a_1 \int_{\mathbb{R}} e^{\mu y} l_2(y)dy \\ -a_2 r_2 k_2 & \int_{\mathbb{R}} e^{\mu y} l_1(y)dy - (1-r_2k_2)\int_{\mathbb{R}} e^{\mu y} l_2(y)dy \end{vmatrix} \\
&= r_1k_1 \int_{\mathbb{R}} e^{\mu y} l_1(y)dy \left[\int_{\mathbb{R}} e^{\mu y} l_1(y)dy - \int_{\mathbb{R}} e^{\mu y} l_2(y)dy + (1-a_1a_2) r_2 k_2 \int_{\mathbb{R}} e^{\mu y} l_2(y)dy \right] \\
&\le (1-a_1a_2) r_2k_2r_1k_1 \int_{\mathbb{R}} e^{\mu y} l_1(y)dy \int_{\mathbb{R}} e^{\mu y} l_2(y)dy \\
&< 0,
\end{aligned}$$

which implies that

$$\lambda\left(B_\mu\right) > \int_{\mathbb{R}} e^{\mu y} l_1(y)dy > 1,\ \lambda\left(B_0\right) > 1.$$

By what we have done, we obtain that

$$\inf_{\mu>0} \frac{\ln(\lambda(B_\mu))}{\mu} > 0,$$

and so $c_-^*(F_2, F_3) > 0$ by Weinberger et al. [24, Lemma 3.1].

If $\int_{\mathbb{R}} e^{\mu y} l_1(y)dy > \int_{\mathbb{R}} e^{\mu y} l_2(y)dy$, we also have $c_-^*(F_2, F_3) > 0$ by a similar discussion.

As F_0, F_2 are steady states of (2.1) and (2.1) is cooperative, thus $\mathcal{C}_{[F_0,F_2]}$ is an invariant region of (2.1). Let $U_n(x) = 1 - k_1 - t_n(x)$, $V_n(x) = k_2 - s_n(x)$, then

$$\begin{cases} t_{n+1}(x) = \int_{\mathbb{R}} (k_1 + t_n(y)) e^{r_1(-t_n(y)+a_1s_n(y))} l_1(x-y)dy, \\ s_{n+1}(x) = k_2 - \int_{\mathbb{R}} (k_2 - s_n(y)) e^{r_2(-a_2t_n(y)+s_n(y))} l_2(x-y)dy. \end{cases} \tag{3.5}$$

Evidently, (3.5) defines a cooperative system and $\mathcal{C}_{[F_0,F_2]}$ is an invariant region of (3.5). For $\mu \ge 0$, define

$$D_\mu = \begin{bmatrix} (1-r_1k_1)\int_{\mathbb{R}} e^{\mu y} l_1(y)dy & r_1a_1k_1 \int_{\mathbb{R}} e^{\mu y} l_2(y)dy \\ r_1a_1k_1 \int_{\mathbb{R}} e^{\mu y} l_1(y)dy & (1-r_2k_2)\int_{\mathbb{R}} e^{\mu y} l_2(y)dy \end{bmatrix}$$

Similar to the analysis of B_μ, we have

$$\inf_{\mu>0} \frac{\ln(\lambda(D_\mu))}{\mu} > 0,$$

and $c_+^*(F_0, F_2) > 0$. The proof is complete. □

Applying Fang and Zhao [7, Theorem 3.1], we finish the proof of Theorem 3.1.

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References

[1] D. G. Aronson, H. F. Weinberger, Nonlinear diffusion in population genetics, combustion, and nerve pulse propagation, In: *Partial Differential Equations and Related Topics* (Ed. by J. A. Goldstein), Lecture Notes in Mathematics, Vol. 446, pp. 5–49, Springer, Berlin, 1975. MR0427837

[2] P. W. Bates, A. Chmaj, A discrete convolution model for phase transitions, *Arch. Ration. Mech. Anal.* **150**(1999), 281–305. MR1741258; url

[3] H. Berestycki, F. Hamel, Generalized transition waves and their properties, *Comm. Pure Appl. Math.* **65**(2012), 592–648. MR2898886; url

[4] X. Chen, Existence, uniqueness and asymptotic stability of traveling waves in nonlocal evolution equation, *Adv. Differential Equations* **2**(1997), 125–160. MR1424765

[5] X. Chen, J. S. Guo, C. C. Wu, Traveling waves in discrete periodic media for bistable dynamics, *Arch. Ration. Mech. Anal.* **189**(2008), 189–236. MR2413095; url

[6] J. M. Cushing, S. Levarge, N. Chitnis, S. M. Henson, Some discrete competition models and the competitive exclusion principle, *J. Diff. Eqns. Appl.* **10**(2004), 1139–1151. MR2100718; url

[7] J. Fang, X.-Q. Zhao, Bistable traveling waves for monotone semiflows with applications, *J. Eur. Math. Soc.,* in press.

[8] P. C. Fife, J. B. McLeod, The approach of solutions of nonlinear diffusion equations to travelling front solutions, *Arch. Ration. Mech. Anal.* **65**(1977), 335–361. MR0442480; url

[9] W. Huang, Uniqueness of the bistable traveling wave for mutualist species, *J. Dynam. Diff. Eqns.*, **13**(2001), 147–183. MR1822216; url

[10] S. B. Hsu, X.-Q. Zhao, Spreading speeds and traveling waves for nonmonotone integrodifference equations, *SIAM J. Math. Anal.* **40**(2008), 776–789. MR2438786; url

[11] Y. Kang, H. Smith, Global dynamics of a discrete two-species Lottery–Ricker competition model, *J. Biol. Dyn.* **6**(2012), 358–376. MR2897879; url

[12] J. P. Keener, Propagation and its failure in coupled systems of discrete excitable cells, *SIAM J. Appl. Math.*, **47**(1987), 556–572. MR0889639; url

[13] J. Li, B. Song, X. Wang, An extended discrete Ricker population model with Allee effects, *J. Difference Equ. Appl.* **13**(2007), 309–321. MR2311062; url

[14] K. Li, X. Li, Asymptotic behavior and uniqueness of traveling wave solutions in Ricker competition system, *J. Math. Anal. Appl.* **389**(2012), 486–497. MR2876514; url

[15] K. Li, X. Li, Travelling wave solutions in integro-difference competition system, *IMA J. Appl. Math.* **78**(2013), 633–650. MR3064263; url

[16] X. Liang, X.-Q. Zhao, Asymptotic speeds of spread and traveling waves for monotone semiflows with applications, *Comm. Pure Appl. Math.* **60**(2007), 1–40. MR2270161; url

[17] K. MISCHAIKOW, V. HUTSON, Traveling waves for mutualist species, *SIAM J. Math. Anal.* **24**(1993), 987–1008. MR1226860; url

[18] S. PAN, G. LIN, Coinvasion-coexistence traveling wave solutions of an integro-difference competition system, *J. Difference Equ. Appl.*, in press. url

[19] E. RICKER, *Handbook of Computation for Biological Statistics of Fish Populations*, Bulletin 119 of the Fisheries Resource Board Canada, Ottawa, 1958.

[20] R. J. SACKER, A note on periodic Ricker maps, *J. Difference Equ. Appl.* **13**(2007), 89–92. MR2284308; url

[21] G. Q. SUN, G. ZHANG, Z. JIN, Dynamic behavior of a discrete modified Ricker & Beverton–Holt model, *Comput. Math. Appl.* **57**(2009), 1400–1412. MR2512317; url

[22] A. I. VOLPERT, V. A. VOLPERT, V. A. VOLPERT, *Traveling Wave Solutions of Parabolic Systems*, Translations of Mathematical Monographs 140, AMS, Providence, Rhode Island, 1994. MR1297766

[23] H. WANG, C. CASTILLO-CHAVEZ, Spreading speeds and traveling waves for non-cooperative integro-difference systems, *Discrete Contin. Dyn. Syst. Ser. B* **17**(2012), 2243–2266. MR2924460; url

[24] H. F. WEINBERGER, M. A. LEWIS, B. LI, Analysis of linear determinacy for spread in cooperative models, *J. Math. Biol.* **45**(2002), 183–218. MR1930974; url

[25] H. YAGISITA, Existence of traveling wave solutions for a nonlocal bistable equation: an abstract approach, *Publ. RIAM, Kyoto Univ.* **45**(2009), 955–979. MR2597125; url

[26] Y. ZHANG, X.-Q. ZHAO, Bistable travelling waves in competitive recursion systems, *J. Differential Equations* **252**(2012), 2630–2647. MR2860634; url

[27] Y. ZHANG, X.-Q. ZHAO, Bistable travelling waves for a reaction and diffusion model with seasonal succession, *Nonlinearity* **26**(2013), 691–709. MR3018938; url