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# Positive Solutions for Systems of $n$th Order Three-point Nonlocal Boundary Value Problems

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**Abstract**

Intervals of the parameter $\lambda$ are determined for which there exist positive solutions for the system of nonlinear differential equations, $u^{(n)} + \lambda a(t)f(v) = 0,\ v^{(n)} + \lambda b(t)g(u) = 0$, for $0 < t < 1$, and satisfying three-point nonlocal boundary conditions, $u(0) = 0, u'(0) = 0, \ldots, u^{(n-2)}(0) = 0,\ u(1) = \alpha u(\eta), v(0) = 0, v'(0) = 0, \ldots, v^{(n-2)}(0) = 0,\ v(1) = \alpha v(\eta)$. A Guo-Krasnosel'skii fixed point theorem is applied.

**Key words and phrases:** Three-point nonlocal boundary value problem, system of differential equations, eigenvalue problem.
**AMS (MOS) Subject Classifications**: 34B18, 34A34

## 1 Introduction

We are concerned with determining intervals of the parameter $\lambda$ (eigenvalues) for which there exist positive solutions for the system of differential equations,

$$\begin{aligned} u^{(n)} + \lambda a(t)f(v) = 0, \quad 0 < t < 1, \\ v^{(n)} + \lambda b(t)g(u) = 0, \quad 0 < t < 1, \end{aligned} \tag{1}$$

satisfying the three-point nonlocal boundary conditions,

$$\begin{aligned} u(0) = 0, u'(0) = 0, \ldots, u^{(n-2)}(0) = 0, \quad u(1) = \alpha u(\eta), \\ v(0) = 0, v'(0) = 0, \ldots, v^{(n-2)}(0) = 0, \quad v(1) = \alpha v(\eta), \end{aligned} \tag{2}$$

where $0 < \eta < 1, 0 < \alpha\eta^{n-1} < 1$ and

(A) $f, g \in C([0,\infty),[0,\infty))$,

(B) $a, b \in C([0,1],[0,\infty))$, and each does not vanish identically on any subinterval,

(C) All of $f_0 := \lim_{x\to 0^+} \frac{f(x)}{x}$, $g_0 := \lim_{x\to 0^+} \frac{g(x)}{x}$, $f_\infty := \lim_{x\to\infty} \frac{f(x)}{x}$ and $g_\infty := \lim_{x\to\infty} \frac{g(x)}{x}$ exist as real numbers.

There is currently a great deal of interest in positive solutions for several types of boundary value problems. While some of the interest has focused on theoretical questions [5, 9, 13, 26], an equal amount of interest has been devoted to applications for which only positive solutions have meaning [1, 8, 17, 18]. While most of the above studies have dealt with scalar problems, some recent work has addressed questions of positive solutions for systems of boundary value problems [3, 12, 14, 15, 16, 19, 22, 25, 27, 30]. In addition, some studies have been directed toward positive solutions for nonlocal boundary value problems; see, for example, [4, 6, 10, 17, 18, 19, 21, 22, 20, 24, 26, 28, 29, 30].

Additional attention has been directed toward extensions to higher order problems, such as in [2, 4, 7, 8, 11, 23, 29]. Recently Benchohra *et al.* [3] and Henderson and Ntouyas [12] studied the existence of positive solutions of systems of nonlinear eigenvalue problems. Here we extend these results to eigenvalue problems for systems of higher order three-point nonlocal boundary value problems.

The main tool in this paper is an application of the Guo-Krasnosel'skii fixed point theorem for operators leaving a Banach space cone invariant [9]. A Green's function plays a fundamental role in defining an appropriate operator on a suitable cone.

# 2 Some preliminaries

In this section, we state some preliminary lemmas and the well-known Guo-Krasnosel'skii fixed point theorem.

**Lemma 2.1** *[4] Let $0 < \eta < 1, 0 < \alpha\eta^{n-1} < 1$; then for any $u \in C[0,1]$ the following boundary value problem*

$$u^{(n)}(t) = 0, \quad 0 < t < 1 \tag{3}$$

$$u(0) = 0, u'(0) = 0, \ldots, u^{(n-2)}(0) = 0, \quad u(1) = \alpha u(\eta), \tag{4}$$

*has a unique solution*

$$u(t) = \int_0^1 k(t,s) u^{(n)}(s) ds$$

*where* $k(t,s):[0,1]\times[0,1]\to\mathbb{R}^+$ *is defined by*

$$k(t,s)=\begin{cases}\frac{a(\eta,s)t^{n-1}}{(n-1)!}, & 0\le t\le s\le 1,\\ \\ \frac{a(\eta,s)t^{n-1}+(t-s)^{n-1}}{(n-1)!}, & 0\le s\le t\le 1,\end{cases}\tag{5}$$

*and*

$$a(\eta,s)=\begin{cases}-\frac{(1-s)^{n-1}}{1-\alpha\eta^{n-1}}, & \eta\le s,\\ \\ -\frac{(1-s)^{n-1}-(\eta-s)^{n-1}}{1-\alpha\eta^{n-1}}, & s\le\eta.\end{cases}$$

**Lemma 2.2** *[4] Let* $0<\alpha^{n-1}<1$. *Let* $u$ *satisfy* $u^{(n)}(t)\le 0, 0<t<1$, *with the nonlocal conditions* (2). *Then*

$$\inf_{t\in[\eta,1]} u(t)\ge\gamma\|u\|,$$

*where* $\gamma=\min\left\{\alpha\eta^{n-1},\frac{\alpha(1-\eta)}{1-\alpha\eta},\eta^{n-1}\right\}$.

Define $\theta(s)=\max_{t\in[0,1]}|k(t,s)|$. From Lemma 1.2 in [4], we know that

$$|k(t,s)|\ge\gamma\theta(s),\quad t\in[\eta,1],\ s\in[0,1].\tag{6}$$

By simple calculation we have (see [11])

$$\theta(s)=\max_{t\in[0,1]}|k(t,s)|\le\frac{(1-s)^{n-1}}{(1-\alpha\eta^{n-1})(n-1)!},\quad s\in(0,1).\tag{7}$$

We note that a pair $(u(t),v(t))$ is a solution of eigenvalue problem (1), (2) if, and only if,

$$u(t)=-\lambda\int_0^1 k(t,s)a(s)f\left(-\lambda\int_0^1 k(s,r)b(r)g(u(r))dr\right)ds,\quad 0\le t\le 1,\tag{8}$$

where

$$v(t)=-\lambda\int_0^1 k(t,s)b(s)g(u(s))ds,\quad 0\le t\le 1.$$

Values of $\lambda$ for which there are positive solutions (positive with respect to a cone) of (1), (2) will be determined via applications of the following fixed point theorem.

**Theorem 2.1** *Let $\mathcal{B}$ be a Banach space, and let $\mathcal{P} \subset \mathcal{B}$ be a cone in $\mathcal{B}$. Assume $\Omega_1$ and $\Omega_2$ are open subsets of $\mathcal{B}$ with $0 \in \Omega_1 \subset \overline{\Omega}_1 \subset \Omega_2$, and let*

$$T : \mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1) \to \mathcal{P}$$

*be a completely continuous operator such that, either*

(i) $||Tu|| \leq ||u||, u \in \mathcal{P} \cap \partial\Omega_1$, *and* $||Tu|| \geq ||u||, u \in \mathcal{P} \cap \partial\Omega_2$, *or*

(ii) $||Tu|| \geq ||u||, u \in \mathcal{P} \cap \partial\Omega_1$, *and* $||Tu|| \leq ||u||, u \in \mathcal{P} \cap \partial\Omega_2$.

*Then $T$ has a fixed point in $\mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1)$.*

# 3 Positive solutions in a cone

In this section, we apply Theorem 2.1 to obtain solutions in a cone (that is, positive solutions) of (1), (2). For our construction, let $\mathcal{B} = C[0,1]$ with supremum norm, $\|\cdot\|$, and define a cone $\mathcal{P} \subset \mathcal{B}$ by

$$\mathcal{P} = \left\{ x \in \mathcal{B} \mid x(t) \geq 0 \text{ on } [0,1], \text{ and } \min_{t \in [\eta,\ 1]} x(t) \geq \gamma \|x\| \right\}.$$

For our first result, define positive numbers $L_1$ and $L_2$ by

$$L_1 := \max\left\{ \left[\gamma^2 \int_\eta^1 \theta(r)a(r)f_\infty dr\right]^{-1}, \left[\gamma^2 \int_\eta^1 \theta(r)a(r)g_\infty dr\right]^{-1} \right\},$$

and

$$L_2 := \min\left\{ \left[\int_0^1 \theta(r)a(r)f_0 dr\right]^{-1}, \left[\int_0^1 \theta(r)b(r)g_0 dr\right]^{-1} \right\}.$$

**Theorem 3.1** *Assume conditions* (A), (B) *and* (C) *are satisfied. Then, for each $\lambda$ satisfying*

$$L_1 < \lambda < L_2, \tag{9}$$

*there exists a pair $(u, v)$ satisfying* (1), (2) *such that $u(x) > 0$ and $v(x) > 0$ on $(0,1)$.*

**Proof.** Let $\lambda$ be as in (9). And let $\epsilon > 0$ be chosen such that

$$\max\left\{ \left[\gamma^2 \int_\eta^1 \theta(r)a(r)(f_\infty - \epsilon)dr\right]^{-1}, \left[\gamma^2 \int_\eta^1 \theta(r)a(r)(g_\infty - \epsilon)dr\right]^{-1} \right\} \leq \lambda$$

and

$$\lambda \leq \min \left\{ \left[ \int_0^1 \theta(r)a(r)(f_0+\epsilon)dr \right]^{-1}, \ \left[ \int_0^1 \theta(r)b(r)(g_0+\epsilon)dr \right]^{-1} \right\}.$$

Define an integral operator $T : \mathcal{P} \to \mathcal{B}$ by

$$Tu(t) := -\lambda \int_0^1 k(t,s)a(s)f\left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr\right)ds, \quad u \in \mathcal{P}. \tag{10}$$

We seek suitable fixed points of $T$ in the cone $\mathcal{P}$.

By Lemma 2.2, $T\mathcal{P} \subset \mathcal{P}$. In addition, standard arguments show that $T$ is completely continuous.

Now, from the definitions of $f_0$ and $g_0$, there exists an $H_1 > 0$ such that

$$f(x) \leq (f_0+\epsilon)x \text{ and } g(x) \leq (g_0+\epsilon)x, \quad 0 < x \leq H_1.$$

Let $u \in \mathcal{P}$ with $\|u\| = H_1$. We first have from (7) and choice of $\epsilon$,

$$\begin{aligned}
-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\
&\leq \lambda \int_0^1 \theta(r)b(r)(g_0+\epsilon)u(r)dr \\
&\leq \lambda \int_0^1 \theta(r)b(r)dr(g_0+\epsilon)\|u\| \\
&\leq \|u\| \\
&= H_1.
\end{aligned}$$

As a consequence, we next have from (7), and choice of $\epsilon$,

$$\begin{aligned}
Tu(t) &= -\lambda \int_0^1 k(t,s)a(s)f\left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr\right)ds \\
&\leq \lambda \int_0^1 \theta(s)a(s)f\left(-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr\right)ds \\
&\leq \lambda \int_0^1 \theta(s)a(s)(f_0+\epsilon)\left[-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr\right]ds \\
&\leq \lambda \int_0^1 \theta(s)a(s)(f_0+\epsilon)H_1 ds \\
&\leq H_1 \\
&= \|u\|.
\end{aligned}$$

So, $\|Tu\| \leq \|u\|$. If we set

$$\Omega_1 = \{x \in \mathcal{B} \mid \|x\| < H_1\},$$

then

$$\|Tu\| \leq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_1. \tag{11}$$

Next, from the definitions of $f_\infty$ and $g_\infty$, there exists $\overline{H}_2 > 0$ such that

$$f(x) \geq (f_\infty - \epsilon)x \text{ and } g(x) \geq (g_\infty - \epsilon)x, \quad x \geq \overline{H}_2.$$

Let

$$H_2 = \max\left\{2H_1, \frac{\overline{H}_2}{\gamma}\right\}.$$

Let $u \in \mathcal{P}$ and $\|u\| = H_2$. Then,

$$\min_{t\in[\eta,1]} u(t) \geq \gamma\|u\| \geq \overline{H}_2.$$

Consequently, from (8) and choice of $\epsilon$,

$$\begin{aligned}
-\lambda\int_0^1 k(s,r)b(r)g(u(r))dr &\geq \lambda\gamma\int_\eta^1 \theta(r)b(r)g(u(r))dr \\
&\geq \lambda\gamma\int_\eta^1 \theta(r)b(r)g(u(r))dr \\
&\geq \lambda\gamma\int_\eta^1 \theta(r)b(r)(g_\infty - \epsilon)u(r)dr \\
&\geq \lambda\gamma\int_\eta^1 \theta(r)b(r)(g_\infty - \epsilon)dr\gamma\|u\| \\
&\geq \|u\| \\
&= H_2.
\end{aligned}$$

And so, we have from (8) and choice of $\epsilon$,

$$\begin{aligned}
Tu(\eta) &\geq \lambda\gamma\int_\eta^1 \theta(s)a(s)f\left(-\lambda\int_\eta^1 k(s,r)b(r)g(u(r))dr\right)ds \\
&\geq \lambda\gamma\int_\eta^1 \theta(s)a(s)(f_\infty - \epsilon)\left[-\lambda\int_\eta^1 k(s,r)b(r)g(u(r))dr\right]ds \\
&\geq \lambda\gamma\int_\eta^1 \theta(s)a(s)(f_\infty - \epsilon)H_2 ds
\end{aligned}$$

$$
\begin{aligned}
&\geq \lambda\gamma^2 \int_\eta^1 \theta(s)a(s)(f_\infty - \epsilon)H_2 ds \\
&\geq H_2 \\
&= \|u\|.
\end{aligned}
$$

Hence, $\|Tu\| \geq \|u\|$. So, if we set

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_2\},$$

then

$$\|Tu\| \geq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \tag{12}$$

Applying Theorem 2.1 to (11) and (12), we obtain that $T$ has a fixed point $u \in \mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1)$. As such, and with $v$ defined by

$$v(t) = -\lambda \int_0^1 k(t,s)b(s)g(u(s))ds,$$

the pair $(u, v)$ is a desired solution of (1), (2) for the given $\lambda$. The proof is complete.□

Prior to our next result, we introduce another hypothesis.

(D) $g(0) = 0$ and $f$ is an increasing function.

We now define positive numbers $L_3$ and $L_4$ by

$$L_3 := \max\left\{\left[\gamma^2 \int_\eta^1 \theta(r)a(r)f_0 dr\right]^{-1}, \left[\gamma^2 \int_\eta^1 \theta(r)a(r)g_0 dr\right]^{-1}\right\},$$

and

$$L_4 := \min\left\{\left[\int_0^1 \theta(r)a(r)f_\infty dr\right]^{-1}, \left[\int_0^1 \theta(r)b(r)g_\infty dr\right]^{-1}\right\}.$$

**Theorem 3.2** *Assume conditions* (A)–(D) *are satisfied. Then, for each* $\lambda$ *satisfying*

$$L_3 < \lambda < L_4, \tag{13}$$

*there exists a pair* $(u, v)$ *satisfying* (1), (2) *such that* $u(x) > 0$ *and* $v(x) > 0$ *on* $(0, 1)$.

**Proof.** Let $\lambda$ be as in (13). And let $\epsilon > 0$ be chosen such that

$$\max\left\{\left[\gamma^2 \int_\eta^1 \theta(r)a(r)(f_0 - \epsilon)dr\right]^{-1}, \left[\gamma^2 \int_\eta^1 \theta(r)a(r)(g_0 - \epsilon)dr\right]^{-1}\right\} \leq \lambda$$

and

$$\lambda \leq \min\left\{\left[\int_0^1 \theta(r)a(r)(f_\infty+\epsilon)dr\right]^{-1},\ \left[\int_0^1 \theta(r)b(r)(g_\infty+\epsilon)dr\right]^{-1}\right\}.$$

Let $T$ be the cone preserving, completely continuous operator that was defined by (10).

From the definitions of $f_0$ and $g_0$, there exists $H_1 > 0$ such that

$$f(x) \geq (f_0-\epsilon)x \text{ and } g(x) \geq (g_0-\epsilon)x, \quad 0 < x \leq H_1.$$

Now $g(0) = 0$ and so there exists $0 < H_2 < H_1$ such that

$$\lambda g(x) \leq \frac{H_1}{\int_0^1 \theta(r)b(r)dr}, \quad 0 \leq x \leq H_2.$$

Choose $u \in \mathcal{P}$ with $\|u\| = H_2$. Then

$$\begin{aligned} -\lambda \int_0^1 k(s,r)b(r)g(u(r))dr &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\ &\leq \lambda \int_0^1 \theta(r)b(r)g(u(r))dr \\ &\leq \frac{\int_0^1 \theta(r)b(r)H_1 dr}{\int_0^1 \theta(r)b(s)ds} \\ &\leq H_1. \end{aligned}$$

Then, by (8) and (D)

$$\begin{aligned} Tu(\eta) &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)f\left(\lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))dr\right)ds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0-\epsilon)\lambda\gamma \int_\eta^1 \theta(r)b(r)g(u(r))drds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0-\epsilon)\lambda\gamma^2 \int_\eta^1 \theta(r)b(r)(g_0-\epsilon)\|u\|drds \\ &\geq \lambda\gamma \int_\eta^1 \theta(s)a(s)(f_0-\epsilon)\|u\|ds \\ &\geq \lambda\gamma^2 \int_\eta^1 \theta(s)a(s)(f_0-\epsilon)\|u\|ds \\ &\geq \|u\|. \end{aligned}$$

So, $\|Tu\| \geq \|u\|$. If we put

$$\Omega_1 = \{x \in \mathcal{B} \mid \|x\| < H_2\},$$

then

$$\|Tu\| \geq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_1. \tag{14}$$

Next, by definitions of $f_\infty$ and $g_\infty$, there exists $\overline{H}_1$ such that

$$f(x) \leq (f_\infty + \epsilon)x \text{ and } g(x) \leq (g_\infty + \epsilon)x, \quad x \geq \overline{H}_1.$$

There are two cases, (a) $g$ is bounded, and (b) $g$ is unbounded.

For case (a), suppose $N > 0$ is such that $g(x) \leq N$ for all $0 < x < \infty$. Then, for $u \in \mathcal{P}$

$$-\lambda \int_0^1 k(s,r)b(r)g(u(r))dr \leq N\lambda \int_0^1 \theta(r)b(r)dr.$$

Let

$$M = \max\left\{f(x) \mid 0 \leq x \leq N\lambda \int_0^1 \theta(r)b(r)dr\right\},$$

and let

$$H_3 > \max\left\{2H_2, M\lambda \int_0^1 \theta(s)a(s)ds\right\}.$$

Then, for $u \in \mathcal{P}$ with $\|u\| = H_3$,

$$\begin{aligned} Tu(t) &\leq \lambda \int_0^1 \theta(s)a(s)Mds \\ &\leq H_3 \\ &= \|u\|, \end{aligned}$$

so that $\|Tu\| \leq \|u\|$. If

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_3\},$$

then

$$\|Tu\| \leq \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \tag{15}$$

For case (b), there exists $H_3 > \max\{2H_2, \overline{H}_1\}$ such that $g(x) \leq g(H_3)$, for $0 < x \leq H_3$. Similarly, there exists $H_4 > \max\left\{H_3, \lambda \int_0^1 \theta(r)b(r)g(H_3)dr)\right\}$ such that $f(x) \leq f(H_4)$, for $0 < x \leq H_4$. Choosing $u \in \mathcal{P}$ with $\|u\| = H_4$, we have by (D) that

$$Tu(t) \leq \lambda \int_0^1 \theta(s)a(s)f\left(\lambda \int_0^1 \theta(r)b(r)g(H_3)dr\right)ds$$

$$
\begin{aligned}
&\le \lambda \int_0^1 \theta(s)a(s)f(H_4)ds \\
&\le \lambda \int_0^1 \theta(s)a(s)ds(f_\infty + \epsilon)H_4 \\
&\le H_4 \\
&= \|u\|,
\end{aligned}
$$

and so $\|Tu\| \le \|u\|$. For this case, if we let

$$\Omega_2 = \{x \in \mathcal{B} \mid \|x\| < H_4\},$$

then

$$\|Tu\| \le \|u\|, \text{ for } u \in \mathcal{P} \cap \partial\Omega_2. \tag{16}$$

In either of the cases, application of part (ii) of Theorem 2.1 yields a fixed point $u$ of $T$ belonging to $\mathcal{P} \cap (\overline{\Omega}_2 \setminus \Omega_1)$, which in turn yields a pair $(u, v)$ satisfying (1), (2) for the chosen value of $\lambda$. The proof is complete. □

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