Expectation of weighted intrinsic volumes of random polytopes

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(Joint work with Ferenc Fodor)

Let K be a convex body in $\mathbb{R}^d$, let $j \in \{1, \ldots, d-1\}$ and let ϱ be a positive and continuous probability density function with respect to the d-dimensional Hausdorff measure on K. Denote by $K_{(n)}$ the convex hull of n points chosen randomly and independently from K according to the probability distribution determined by ϱ.

For the case when $\varrho \equiv 1/V(K)$ and ∂K is C^2_+, M. Reitzner proved an asymptotic formula (and also I. Bárány if ∂K is C^3_+) for the expectation of the difference of the jth intrinsic volumes of K and $K_{(n)}$, as $n \to \infty$. K. J. Böröczky, L. M. Hoffmann and D. Hug extended this result to the case when $\varrho \equiv 1/V(K)$ and the only condition on K is that a ball rolls freely in K. K. J. Böröczky, F. Fodor, M. Reitzner and V. Vígh also showed that under the same assumptions, for the mean width, the existence of a rolling ball inside K is a necessary condition.

K. J. Böröczky, F. Fodor, and D. Hug proved an asymptotic formula for the weighted volume approximation of K under no smoothness assumptions on ∂K. We study the expectation of weighted intrinsic volumes for random polytopes generated by non-uniform probability distributions in convex bodies with very mild smoothness conditions. We assume no smoothness assumption on ∂K if $d/2 < j \leqslant d-1$, otherwise we assume that a ball rolls freely inside K.