Equidistant sets and their approximation

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The equidistant set of two sets K and L (called focal sets) in the Euclidean space $\mathbb{R}^n$, denoted $\{K = L\}$, is the set of all points that have the same distance from both K and L, measured with the infimum metric. Apart from the elementary cases (segment and angle bisectors), the most fundamental examples are the classical conics and convex polytopes.

A particularly interesting class of equidistant sets is when the focal sets are finite, allowing exact (computer-assisted) calculations. This case is closely related to the concept of Voronoi diagrams, and we present some examples how results for the latter can be translated to results for the former. For more general sets, the continuity theorem of Ponce and Santibanez is a fundamental tool, allowing the approximation of equidistant sets (or their focal sets) in the Hausdorff metric. Using this, we can show that any convex body is an equidistant set (approximating it by convex polytopes), and we can also apply this process for computer-based applications (replacing the original focal sets with finite ones). An important real-life example is the determination of territorial sea borders between nations.

We also discuss a recently posed open problem, namely, what are the closed sets that can be realized as the equidistant sets of two *connected* disjoint closed sets, and some other important classes of equidistant sets: *equidistant functions* whose graphs are the equidistant sets of the graph of another function and the x axis.

References

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