Poisson hulls and Efron type formulae

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(Joint work with Günter Last)

We introduce a hull operator on Poisson point processes, the easiest example being the convex hull of the support of a point process in Euclidean space. Assuming that the intensity measure of the process is known on the set generated by the hull operator, we discuss estimation of an expected linear statistic built on the Poisson process. In special cases, our general scheme yields an estimator of the volume of a convex body or an estimator of an integral of a Hölder function. We show that the estimation error is given by the Kabanov–Skorohod integral with respect to the underlying Poisson process.

A crucial ingredient of our approach is a spatial strong Markov property of the underlying Poisson process with respect to the hull. Using this spatial Markov property, we derive several distributional identities. In application to the convex hull of a finite Poisson process, our results generalise classical (and also more recent) identities connecting the number of vertices and the volume of the convex hull, namely, Efron's identity. Our results apply to general Poisson hulls (e.g., to ball hulls) and predominantly even to general stopping sets.