L_p Brunn-Minkowski type inequalities under projection constraints

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In the classical Brunn-Minkowski theory, if the convex bodies K, L are assumed to share a common projection onto a hyperplane, namely $P_{u^{\perp}}(K) = P_{u^{\perp}}(L)$, for some direction u, then the Brunn-Minkowski inequality admits the following improvement:

$$\operatorname{vol}_n((1-\lambda)K + \lambda L) \ge (1-\lambda)\operatorname{vol}_n(K) + \lambda \operatorname{vol}_n(L),$$

for every $\lambda \in [0, 1]$. For $p \ge 1$, the L_p Brunn-Minkowski inequality establishes

$$\operatorname{vol}_n((1-\lambda)K+\lambda L)^{p/n} \ge (1-\lambda)\operatorname{vol}_n(K)^{p/n} + \lambda \operatorname{vol}_n(L)^{p/n},$$

for every $\lambda \in [0, 1]$ and convex bodies K and L containing the origin.

In this talk, we will discuss possible improvements for the L_p Brunn-Minkowski inequality when the convex bodies share a common projection, and some recent results in this direction.