The Measure Concentration Problem for Convex Bodies

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Motivated by the measure concentration results for cone volume measures and dual curvature measures, we study the general problem of measure concentration. Let $\mu(K,\cdot)$ be a general geometric measure of a convex body K in $\mathbb{R}^n$, which is Borel over the unit sphere. The i-dimensional subspace concentration of $\mu(K,\cdot)$ refers to the maximal concentration of the measure over all i-dimensional subspaces, defined as

$$c_i(K,\mu) = \sup_{\xi \in G_{n,i}} \mu(K, \xi \cap \mathbb{S}^{n-1}),$$

where $i = 1, 2, ..., n-1$ and $G_{n,i}$ is the Grassmann manifold of i-dimensional subspaces in $\mathbb{R}^n$. The measure concentration problem is to find the maximum concentration constant,

$$c_i(\mu) = \frac{1}{|\mu|} \sup_K c_i(K,\mu),$$

over a class of convex bodies. We show that for origin symmetric convex bodies the L^p area measure $\mu = S_{i,p}(K,\cdot)$, for $i = 1, 2, ...n-1$, and $p \neq 0$, then $c_i(\mu) = 1$, so the L^p area measure can become completely concentrated on a lower dimensional subspace.