

OPTIMALITY CONDITIONS FOR CONVEX CONTAINMENT PROBLEMS UNDER AFFINITY

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Historically, problems involving the Banach–Mazur distance and related quantities have often been handled by finding a best possible inner or outer approximation (usually with respect to the volume) with the hope that the other side behaves well enough. Prominent results in this direction include the characterizations of the John and Loewner ellipsoids and their extensions to general convex bodies. While these results have led to several tight bounds and are influential even in other branches of mathematics, they do not actually consider the problem of finding optimal simultaneous inner and outer approximations under affine transformations. We address this gap by establishing necessary optimality conditions for such simultaneous approximations. In the case of approximation by ellipsoids, these conditions fully characterize the optimal solutions. We also discuss applications of the new conditions, including proofs of the upper bounds on Banach–Mazur distance to the ball that allow easier analysis of the extreme cases.