Random Polytopes in Convex Bodies: A Bridge Between Extremal Containers

Florian Besau

FSU Jena

(Joint work with A. Gusakova and C. Thäle)

Random polytopes arise as the convex hull of a finite number of independent points drawn uniformly from a convex body in $\mathbb{R}^d$. Classical theory typically contrasts polytopal and smooth convex containers, which often represent extremal cases–for instance, in the asymptotic behavior of the expected volume as the number of points tends to infinity.

In this talk, that is based on recent joint work [arXiv:2411.19163], I will introduce and analyze random polytopes generated within convex bodies formed as products of lower-dimensional balls. These constructions naturally interpolate between the cube–a polytopal container realized as a product of one-dimensional balls–and the Euclidean ball, the prototypical smooth container. We will allow the component balls to vary in dimension and consider not only the uniform distribution, but also certain non-uniform distributions which naturally arise.

We establish precise asymptotic rates for the expected face numbers and expected volume of these random polytopes. Our approach yields new geometric insights and offers a unified framework that bridges the classical extremal cases of convex containers.