Characterization of Cone-Volumes via Pyramids

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Motivated by the logarithmic Minkowski problem for polytopes, we study the cone-volume set $C_{\mathrm{cv}}(U)$, consisting of cone-volume vectors of polytopes $P(U,b) = \{x \in \mathbb{R}^n : U^T x \leqslant b\}$, where $b \geqslant 0$ is a non-negative right-hand side and $\mathrm{vol}(P(U,b)) = 1$. Therefore, the set $C_{\mathrm{cv}}(U)$ is contained in the simplex $\mathrm{conv}(\mathbf{e}_1, \dots, \mathbf{e}_{|U|})$.

We investigate geometric conditions on U under which the standard basis vector $\mathbf{e}_i$ is a cone-volume vector, or, more generally, when a sequence of cone-volume vectors converges to $\mathbf{e}_i$. The outer unit normal vectors $u_i \in U$, corresponding to the vectors $\mathbf{e}_i$, can be characterized by subsets $S \subseteq U$ with the property $\mathrm{pos}(S) = \mathrm{lin}(S)$, which further correspond to pyramid structures with outer unit normals contained in U. To analyze the limit process, we establish a new inequality for cone-volumes.

The characterization of outer unit normal vectors $u_i \in U$ with this property relies on the structure of positive bases, offering new insights into cone-volume configurations.