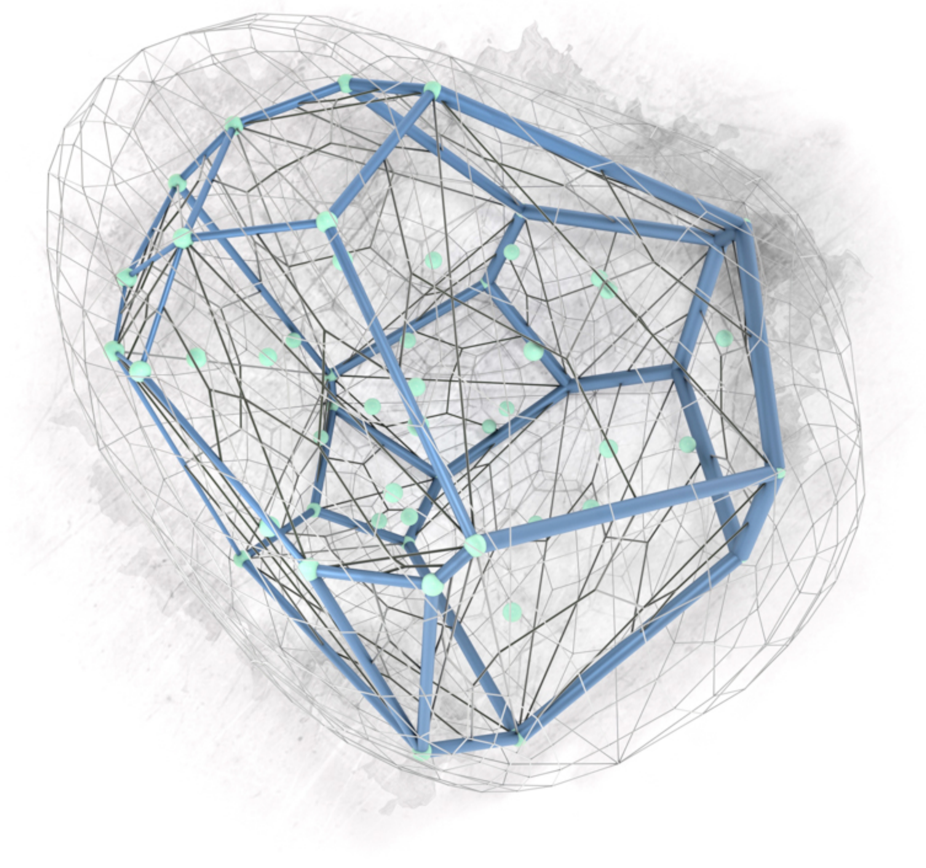
2025 Szeged Workshop on Convexity

29-31 May, 2025

University of Szeged, Hungary



Book of Abstracts

Programme

May 29, Thursday

8:00 – 8:25	*Registration*
8:25 – 8:30	*Opening words*
8:30 – 9:10	**Elisabeth M. Werner**, *Floating bodies for ball-convex bodies*
9:15 – 9:35	**Ansgar Freyer**, *Equivariant valuations of lattice polygons*
9:40 – 10:00	**Jonas Knoerr**, *A Paley-Wiener-Schwartz theorem for valuations on convex functions*
10:00 – 10:30	*Coffee break*
10:30 – 11:10	**Carsten Schütt**, *Expected extremal area of facets of random polytopes*
11:15 – 11:35	**Florian Besau**, *Random polytopes in convex bodies: a bridge between extremal containers*
11:40 – 12:00	**Mohamed Abdeldjalil Mouamine**, *Vector-valued valuations on convex functions*
12:00 – 14:00	*Lunch break*
14:00 – 14:40	**Károly J. Böröczky**, *Extremality of the mean width and the ℓ-norm*
14:45 – 15:05	**Márton Naszódi**, *John ellipsoids of revolution*
15:10 – 15:30	**Florian Grundbacher**, *Optimality conditions for convex containment problems under affinity*
15:30 – 16:00	*Coffee break*
16:00 – 16:20	**Christian Kipp**, *Shadow systems, decomposability and isotropic constants*
16:25 – 16:45	**Anna Zamojska-Dzienio**, *Partitions of unity and barycentric algebras*
16:50 – 17:10	**Sylvia Silberger**, *Maximal sublattices and Frattini sublattices of convex geometries with* $\mathrm{cdim} = 2$
17:15 – 18:30	*Poster session*
	Alexandra Bakó-Szabó, *Pairs of random points from a shell*
	Barnabás Gárgyán, *Extremality of diagonal sections of the cube*
	Balázs Grünfelder, *Variances of non-euclidean random polytopes*

Márk Oláh, *Equidistant sets and their approximation*

Dániel István Papvári, *Expectation of weighted intrinsic volumes of random polytopes*

Ádám Sagmeister, *Circle packings of the hyperbolic plane*

Shanshan Wang, *Normed variants of a theorem of Macbeath*

May 30, Friday

8:30 – 9:10	**Andrea Colesanti**, *The mixed Christoffel problem*
9:15 – 9:35	**Georg C. Hofstätter**, *Mixed volumes and mixed area measures of bodies of revolution*
9:40 – 10:00	**Lily Liu**, *The measure concentration problem for convex bodies*
10:00 – 10:30	*Coffee break*
10:30 – 11:10	**Ilya Molchanov**, *Poisson hulls and Efron type formulae*
11:15 – 11:35	**Tom Baumbach**, *Characterization of cone-volumes via pyramids*
11:40 – 12:00	**István Talata**, *On the homothetic kissing numbers of a tetrahedron*
12:00 – 14:00	*Lunch break*
14:00 – 14:50	*Public lecture*
	María A. Hernández Cifre, *Mean type successive radii of convex bodies*
15:00 – 15:20	**Mei Han**, *Packing minima of convex bodies*
15:25 – 15:45	**Zsolt Lángi**, *On the approximation of convex bodies by monostable polyhedra*
15:45 – 16:15	*Coffee break*
16:15 – 16:35	**Jesús Yepes Nicolás**, *On (complemented) Brunn-Minkowski type inequalities for measures*
16:40 – 17:00	**Nico Lombardi**, L_p *Brunn-Minkowski type inequalities under projection constraints*
17:05 – 17:25	**Lidia Gordo Malagón**, *On* L_p *Brunn-Minkowski type inequalities for a general class of functionals*
18:30 – 21:00	*Reception*

May 31, Saturday

8:30 – 9:10	**Alina Stancu**, *On the planar homothethy conjecture*
9:15 – 9:35	**Fabian Mussnig**, *A Pólya-Szegő type inequality for convex functions*
9:40 – 10:00	**Jacopo Ulivelli**, *Between Brascamp-Lieb and Colesanti inequalities*
10:00 – 10:30	*Coffee break*
10:30 – 11:10	**Jan Kotrbatý**, *A generalization of Godbersen's conjecture*
11:15 – 11:35	**Thomas Jahn**, *Minkowski chirality*
11:40 – 12:00	**Mia Runge**, *(r,D,R)-Blaschke-Santaló diagrams for three-dimensional convex bodies*

Abstracts

Public lecture

Mean type successive radii of convex bodies

María A. Hernández Cifre

Universidad de Murcia

(Joint work with Lidia Gordo Malagón)

For a convex body K of the n-dimensional Euclidean space, we define the successive outer and inner radii, denoted respectively by $\mathrm{R}_i(K)$ and $\mathrm{r}_i(K)$, $i = 1, \dots, n$, in the following way: $\mathrm{R}_i(K)$ is the smallest radius of a solid cylinder with i-dimensional spherical cross section containing K, whereas $\mathrm{r}_i(K)$ is the radius of the greatest i-dimensional ball contained in K. These measures generalize the well-known functionals diameter, minimal width, circumradius and inradius of K, and can be also defined via the circunradius/inradius of suitable projections/sections of the convex body K.

Our aim in this talk is to present the known results as well as some extensions and recent developments about them. Furthermore, the relation between the different successive radii and other relevant geometric measures, like the volume or the Gaussian measure, will be also pointed out.

Invited talks

Extremality of the mean width and the ℓ-norm

Károly J. Böröczky

HUN-REN Alfréd Rényi Institute of Mathematics

(Joint work with Ferenc Fodor and Daniel Hug)

Barthe, Schechtman and Schmuckenschlager proved that the cube maximizes the mean width of symmetric convex bodies whose John ellipsoid (maximal volume ellipsoid contained in the body) is the centered Euclidean unit ball, and the regular crosspolytope minimizes the mean width of symmetric convex bodies whose Lowner ellipsoid is the centered Euclidean unit

ball. In addition, the extremality of the regular simplex without the symmetry assumption has been verified. In the talk, we discuss close to be optimal stronger stability versions of these results, together with their counterparts about the ℓ-norm based on Gaussian integrals. We also consider related stability results for the mean width and the ℓ-norm of the convex hull of the support of (even) isotropic measures on the unit sphere.

The mixed Christoffel problem

Andrea Colesanti

University of Florence

(Joint work with Matteo Focardi, Pengfei Guan and Paolo Salani)

We will start by briefly reviewing the notions of mixed volumes and mixed area measures of convex bodies. Then we will describe two important problems related to area measures: the Minkowski and the Christoffel problems (along with their intermediate version, the Christoffel-Minkowski problem). We will then pass to a similar problem concerning mixed area measures, that we call *mixed Christoffel problem*, and present some new existence results.

A generalization of Godbersen's conjecture

Jan Kotrbatý

Charles University Prague

The long-standing Godbersen's conjecture asserts that the Rogers–Shephard inequality for the volume of the difference body is refined by an inequality for the mixed volume of a convex body and its reflection in the origin. The conjecture is known in several special cases, notably for anti-blocking convex bodies. In this talk, we will propose a generalization of Godbersen's conjecture that refines Schneider's generalization of the Rogers–Shephard inequality to higher-order difference bodies and we will show it is true for anti-blocking convex bodies. We will also present an equivalent formulation of our conjecture in terms of the Alesker product of smooth, translation invariant valuations.

Poisson hulls and Efron type formulae

Ilya Molchanov

University of Bern

(Joint work with Günter Last)

We introduce a hull operator on Poisson point processes, the easiest example being the convex hull of the support of a point process in Euclidean space. Assuming that the intensity measure of the process is known on the set generated by the hull operator, we discuss estimation of an expected linear statistic built on the Poisson process. In special cases, our general scheme yields an estimator of the volume of a convex body or an estimator of an integral of a Hölder function. We show that the estimation error is given by the Kabanov–Skorohod integral with respect to the underlying Poisson process.

A crucial ingredient of our approach is a spatial strong Markov property of the underlying Poisson process with respect to the hull. Using this spatial Markov property, we derive several distributional identities. In application to the convex hull of a finite Poisson process, our results generalise classical (and also more recent) identities connecting the number of vertices and the volume of the convex hull, namely, Efron's identity. Our results apply to general Poisson hulls (e.g., to ball hulls) and predominantly even to general stopping sets.

Expected extremal area of facets of random polytopes

Carsten Schütt

Christian-Albrechts Universität, Kiel

(Joint work with Brett Leroux, Luis Rademacher and Elisabeth Werner)

We study extremal properties of spherical random polytopes, i.e. the convex hull of random points chosen from the unit Euclidean sphere in $\mathbb{R}^n$. The extremal properties of interest are the expected values of the maximum and minimum surface area among facets. We determine the asymptotic growth in every fixed dimension, up to absolute constants.

On the planar homothety conjecture

Alina Stancu

Concordia University

(Joint work with M. Alfonseca, F. Nazarov, D. Ryabogin, and V. Yaskin)

We will present a positive partial result on the planar homothety conjecture stating that the curve of flotation is homothetic to the boundary of the convex body K for some arbitrary density of the body in $(0, |K|/2)$ if and only if the convex body is an ellipse.

Floating bodies for ball-convex bodies

Elisabeth M. Werner

Case Western Reserve University

(Joint work with C. Schütt and D. Yalikun)

We consider floating bodies in the class of ball-convex bodies. A right derivative of volume of these floating bodies leads to a surface area measure for ball-convex bodies which we call relative affine surface area. We show that this quantity is a rigid motion invariant, upper semi continuous valuation.

Contributed talks

Characterization of Cone-Volumes via Pyramids

Tom Baumbach

TU Berlin

Motivated by the logarithmic Minkowski problem for polytopes, we study the cone-volume set $C_{\text{cv}}(U)$, consisting of cone-volume vectors of polytopes $P(U, b) = \{x \in \mathbb{R}^n : U^T x \leq b\}$, where $b \geq 0$ is a non-negative right-hand side and $\text{vol}(P(U, b)) = 1$. Therefore, the set $C_{\text{cv}}(U)$ is contained in the simplex $\text{conv}(\mathbf{e}_1, \ldots, \mathbf{e}_{|U|})$.

We investigate geometric conditions on U under which the standard basis vector $\mathbf{e}_i$ is a cone-volume vector, or, more generally, when a sequence of cone-volume vectors converges to $\mathbf{e}_i$. The outer unit normal vectors $u_i \in U$, corresponding to the vectors $\mathbf{e}_i$, can be characterized by subsets $S \subseteq U$ with the property $\text{pos}(S) = \text{lin}(S)$, which further correspond to pyramid structures with outer unit normals contained in U. To analyze the limit process, we establish a new inequality for cone-volumes.

The characterization of outer unit normal vectors $u_i \in U$ with this property relies on the structure of positive bases, offering new insights into cone-volume configurations.

Random Polytopes in Convex Bodies: A Bridge Between Extremal Containers

Florian Besau

Friedrich Schiller University Jena

(Joint work with A. Gusakova and C. Thäle)

Random polytopes arise as the convex hull of a finite number of independent points drawn uniformly from a convex body in $\mathbb{R}^d$. Classical theory typically contrasts polytopal and smooth convex containers, which often represent extremal cases–for instance, in the asymptotic behavior of the expected volume as the number of points tends to infinity.

In this talk, that is based on recent joint work [arXiv:2411.19163], I will introduce and analyze random polytopes generated within convex bodies formed as products of lower-dimensional balls. These constructions naturally interpolate between the cube–a polytopal container realized as a product of one-dimensional balls–and the Euclidean ball, the prototypical smooth container. We will allow the component balls to vary in dimension and consider not only the uniform distribution, but also certain non-uniform distributions which naturally arise.

We establish precise asymptotic rates for the expected face numbers and expected volume of these random polytopes. Our approach yields new geometric insights and offers a unified framework that bridges the classical extremal cases of convex containers.

Equivariant valuations of lattice polygons

Ansgar Freyer

FU Berlin

(Joint work with Monika Ludwig and Martin Rubey)

The classical Betke-Kneser Theorem characterizes the coefficients of the Ehrhart polynomial as a basis of the vector space of valuations that are invariant under lattice-preserving affine transformations. In analogy to recent developments in the Euclidean theory of valuations, it is tempting to ask for classification results for valuations on lattice polytopes that exhibit an equivariant behavior with respect to lattice-preserving affine maps.

A natural source of such valuations are the so-called Ehrhart tensor coefficients introduced by Böröczky and Ludwig, which generalize the classical Ehrhart coefficients to tensors of rank higher than zero. However, in contrast to the invariant case, these valuations do not constitute a basis of all equivariant valuations. An example of a valuation on lattice polygons that cannot be obtained as a linear combination of Ehrhart tensor coefficients was given by Ludwig and Silverstein.

In the talk we provide the full classification of equivariant valuations in the planar case. To this end, we will develop an extension argument to unbounded polyhedra along with some basic invariant theory.

On L_p Brunn-Minkowski type inequalities for a general class of functionals

Lidia Gordo Malagón

Universidad de Murcia

(Joint work with J. Yepes Nicolás)

The L_p version (for $p \geqslant 1$) of the Brunn-Minkowski inequality proven by Firey in the 60's for two convex bodies containing the origin $K, L \subset \mathbb{R}^n$, and recently extended to non-empty compact sets by Lutwak, Yang and Zhang, asserts that the volume is a (p/n)-concave functional, namely,

$$\mathrm{vol}\big((1-\lambda)\cdot K +_p \lambda\cdot L\big)^{p/n} \geqslant (1-\lambda)\mathrm{vol}(K)^{p/n} + \lambda \mathrm{vol}(L)^{p/n}$$

for all $\lambda \in (0,1)$.

In this talk, we will collect different Brunn-Minkowski type inequalities for a general class of functionals, defined on a certain family of subsets, when dealing with the p-sum of the sets involved.

As a particular case of our general approach, we derive new L_p Brunn-Minkowski type inequalities for both the Wills functional and the standard Gaussian measure. Furthermore, we will provide other remarkable examples of functionals satisfying L_p Brunn-Minkowski type inequalities, such as different absolutely continuous measures with radially decreasing densities.

Optimality Conditions for Convex Containment Problems Under Affinity

Florian Grundbacher

Technical University of Munich

(Joint work with Tomasz Kobos)

Historically, problems involving the Banach–Mazur distance and related quantities have often been handled by finding a best possible inner or outer approximation (usually with respect to the volume) with the hope that the other side behaves well enough. Prominent results in this direction include the characterizations of the John and Loewner ellipsoids and their extensions to general convex bodies. While these results have led to several tight bounds and are influential even in other branches of mathematics, they do not actually consider the problem of finding optimal simultaneous inner and outer approximations under affine transformations. We address this gap by establishing necessary optimality conditions for such simultaneous approximations. In the case of approximation by ellipsoids, these conditions fully characterize the optimal solutions. We also discuss applications of the new conditions, including proofs of the upper bounds on Banach–Mazur distance to the ball that allow easier analysis of the extreme cases.

Packing minima of convex bodies

Mei Han

TU Berlin

(Joint work with Martin Henk and Fei Xue)

In 2021, Henk, Schymura and Xue introduced packing minima, associated with a convex body and a lattice, as packing counterparts to the covering minima of Kannan and Lovász. Motivated by conjectures on inequalities for the successive minima, we introduce a refined definition of packing minima for centered convex bodies in this talk. For these packing minima, we present novel volume inequalities.

Mixed volumes and mixed area measures of bodies of revolution

Georg C. Hofstätter

TU Wien

(Joint work with L. Brauner and O. Ortega-Moreno)

Sometimes, two mixed volume functionals with different reference bodies are in fact equal. For reference bodies that are rotation symmetric around an axis, we determine when this is the case, using mixed spherical projections and tools from valuation theory. As an application, for convex bodies of revolution, we provide a complete solution to Christoffel–Minkowski type problems of intermediate degrees.

Minkowski chirality

Thomas Jahn

Friedrich Schiller University Jena

(Joint work with A. Caragea, K. von Dichter, K.K. Gottwald, F. Grundbacher, M. Runge

We introduce Minkowski chirality, a novel quantity that maps convex bodies to real numbers, quantifying their asymmetry with respect to reflections across subspaces of a given dimension in an optimal containment fashion. Specifically, we define the Minkowski chirality $\alpha_j(K)$ of the convex body $K \subset \mathbb{R}^n$ as the infimum of the circumradii of K with respect to its images under reflections across j-dimensional subspaces of $\mathbb{R}^n$. This extension builds upon the well-known concept of Minkowski asymmetry, which pertains to 0-dimensional subspaces.

In this talk, our focus will be on studying the range of the Minkowski chirality α_1 for two specific types of convex bodies: parallelograms and triangles. We will analyze the optimal subspaces in both cases and identify the convex bodies that exhibit the maximum and minimum values of α_1 within these families.

Shadow systems, decomposability and isotropic constants

Christian Kipp
TU Berlin

Intuitively speaking, a local maximizer of the isotropic constant has to be sufficiently 'rigid' to rule out modifications that provably increase the isotropic constant. In this talk, we discuss necessary conditions for local maximizers in terms of two notions of decomposability: (1) RS-decomposability (due to Campi, Colesanti and Gronchi) and (2) Minkowski decomposability of the polar body. Starting with a computation of the second derivative of the isotropic constant, we discuss how a dimension argument can be used out to rule out that certain convex bodies can be local maximizers of the isotropic constant. Using this approach, we recover the result that a local maximizer has to be RS-indecomposable (due to Campi, Colesanti and Gronchi) and extend it to a slightly larger class of shadow systems. With regard to the second notion of decomposability, the approach can be used to show that the polar body of a local maximizer of the isotropic constant can only have few Minkowski summands; more precisely, its dimension of decomposability is at most $\frac{1}{2}(n^2 + 3n)$.

A Paley–Wiener–Schwartz Theorem for valuations on convex functions

Jonas Knoerr
TU Wien

Much of the immense progress in Geometric Valuation Theory over the last 30 years is based on Alesker's solution to McMullen's Conjecture, which characterizes continuous translation invariant valuations on convex bodies as uniform limits of linear combinations of mixed volumes. In fact, Alesker established a much stronger result, called the Irreducibility Theorem, which provides a representation theoretic description of the space of these valuations. One of key consequences of this description was the introduction of the notion of *smooth valuations*, which enjoy very strong regularity properties and admit a variety of integral and differential geometric descriptions.
In this talk I will present a different approach to regularity results of this kind for dually epi-translation invariant valuations on convex functions. In particular, we will discuss how one can directly obtain suitable integral representations of these functionals from a Paley–Wiener–Schwartz-type regularity characterization of certain distributions associated to these valuations in terms of the decaying properties of their Fourier-Laplace transform.

On the approximation of convex bodies by monostable polyhedra

Zsolt Lángi

Budapest University of Technology and Rényi Institute of Mathematics

(Joint work with Csaba D. Tóth)

A convex polyhedron in $\mathbb{R}^3$ is called *monostable* if it can be balanced on a horizontal plane only on one of its faces. These objects were introduced by Conway at the end of the 1960s, and were described by Shephard in 1968 as 'a remarkable class of convex polyhedra' whose properties 'it would probably be very rewarding and interesting to make a study of'. In 1969 there problems were proposed by Conway regarding monostable polyhedra, which since then were re-stated in some open problem books on geometry. Two of these problems were solved by the presenter in a recent paper. In this talk we sketch the solution of the third problem, asking the following: Which convex bodies can be approximated arbitrarily well, measured in Hausdorff distance, by monostable polyhedra?

The Measure Concentration Problem for Convex Bodies

Lily Liu

New York University

Motivated by the measure concentration results for cone volume measures and dual curvature measures, we study the general problem of measure concentration. Let $\mu(K, \cdot)$ be a general geometric measure of a convex body K in $\mathbb{R}^n$, which is Borel over the unit sphere. The i-dimensional subspace concentration of $\mu(K, \cdot)$ refers to the maximal concentration of the measure over all i-dimensional subspaces, defined as

$$c_i(K, \mu) = \sup_{\xi \in G_{n,i}} \mu(K, \xi \cap \mathbb{S}^{n-1}),$$

where $i = 1, 2, ..., n-1$ and $G_{n,i}$ is the Grassmann manifold of i-dimensional subspaces in $\mathbb{R}^n$. The measure concentration problem is to find the maximum concentration constant,

$$c_i(\mu) = \frac{1}{|\mu|} \sup_K c_i(K, \mu),$$

over a class of convex bodies. We show that for origin symmetric convex bodies the L^p area measure $\mu = S_{i,p}(K, \cdot)$, for $i = 1, 2, ...n-1$, and $p \neq 0$, then $c_i(\mu) = 1$, so the L^p area measure can become completely concentrated on a lower dimensional subspace.

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L_p Brunn-Minkowski type inequalities under projection constraints

Nico Lombardi

University of Florence

(Joint work with A. Colesanti, E. Saorín Gómez, and J. Yepes Nicolás)

In the classical Brunn-Minkowski theory, if the convex bodies K, L are assumed to share a common projection onto a hyperplane, namely $P_{u^\perp}(K) = P_{u^\perp}(L)$, for some direction u, then the Brunn-Minkowski inequality admits the following improvement:

$$\mathrm{vol}_n\big((1-\lambda)K + \lambda L\big) \geq (1-\lambda)\mathrm{vol}_n(K) + \lambda \mathrm{vol}_n(L),$$

for every $\lambda \in [0, 1]$. For $p \geq 1$, the L_p Brunn-Minkowski inequality establishes

$$\mathrm{vol}_n\big((1-\lambda)K + \lambda L\big)^{p/n} \geq (1-\lambda)\mathrm{vol}_n(K)^{p/n} + \lambda \mathrm{vol}_n(L)^{p/n},$$

for every $\lambda \in [0, 1]$ and convex bodies K and L containing the origin.

In this talk, we will discuss possible improvements for the L_p Brunn-Minkowski inequality when the convex bodies share a common projection, and some recent results in this direction.

Vector-valued valuations on convex functions

Mohamed Abdeldjalil Mouamine

TU Wien

(Joint work with Fabian Mussnig)

Following the work of A.Colesanti, M.Ludwig, and F.Mussnig, who established a functional version of Hadwiger's theorem which characterizes functional intrinsic volumes on the space of convex super-coercive functions, we explore a novel family of valuations. Specifically, we characterize continuous, $\mathbb{R}^n$-valued, epi-translation invariant and rotation equivariant valuations and show that they can be constructed using Hessian measures. These valuations are derived from a Steiner formula applied to a functional version of the moment vector. We also establish integral geometric formulas for these valuations.

A Pólya–Szegő-type inequality for convex functions

Fabian Mussnig

TU Wien

(Joint work with Jacopo Ulivelli)

The classical Pólya–Szegő inequality can be seen as a far-reaching functional analog of the Euclidean isoperimetric inequality. Several generalizations are known, among which we want to highlight a recent version by Bianchi, Cianchi, and Gronchi [1] that considers simultaneous symmetrization of both the Sobolev function and the Young function.

We present a new Pólya–Szegő-type inequality for convex functions which coincides with the inequality of Bianchi, Cianchi, and Gronchi in special cases. Our approach builds upon a previous strategy of Klartag [2] and connects mixed volumes of higher-dimensional convex bodies with mixed Monge–Ampère measures of convex functions.

References

[1] G. Bianchi, A. Cianchi, and P. Gronchi, *Anisotropic symmetrization, convex bodies, and isoperimetric inequalities.* Adv. Math. **462** (2025), Art. 110085.

[2] B. Klartag, *Marginals of geometric inequalities.* In *Geometric Aspects of Functional Analysis: Israel Seminar 2004–2005*, pp. 133–166, Springer, Berlin, 2007.

John ellipsoids of revolution

Márton Naszódi

Alfréd Rényi Institute of Mathematics and ELTE

(Joint work with Grigory Ivanov, Zsolt Lángi and Ádám Sagmeister)

The largest volume ellipsoid E contained in a convex body K in d-dimensional space is a central object in convexity. Fritz John gave conditions in terms of the contact points of K and E guaranteeing that E is of largest volume. In this ongoing work, we study the problem of finding the largest volume ellipsoid of revolution in K.

(r, D, R)-Blaschke-Santaló diagrams for three-dimensional convex bodies

Mia Runge

Technical University Munich

We present complete (r, D, R)-Blaschke-Santaló diagrams for the 1- and 2-norm in 3-space. For each case, we prove one new inequality and show that it completes the description of the diagram. Furthermore, we give a complete description of the values the inradius, circumradius and diameter of a three-dimensional body can attain, when we consider generalized normed spaces.

Maximal Sublattices and Frattini Sublattices of Convex Geometries with $cdim = 2$

Sylvia Silberger

Hofstra University

(Joint work with Kira Adaricheva, Anna Zamojska-Dzienio, Adam Mata)

A proper sublattice $\mathcal{M}$ of a lattice $\mathcal{L}$ is called *maximal* if it is the case that for any other proper sublattice $\mathcal{S}$, $\mathcal{M} \subseteq \mathcal{S}$ implies $\mathcal{M} = \mathcal{S}$. The *Frattini sublattice* of a lattice $\mathcal{L}$ is the intersection of all of its maximal sublattices.

In 1973, two papers, on by C. C. Chen, K. M. Koh, and S. K. Tan and the other by I. Rival both showed maximal sublattices of distributive lattices were always intervals. The former also included results descibing the Frattini sublattice of distributive lattices and more progress on that front was made by M.E. Adams, P. Dwinger, and J.Schmid in 1996. Shortly thereafter, in 1997, M. E. Adams, R Freese, J.B. Nation, and J. Schmid showed the complements of maximal sublattices of lattices in the larger class of bounded lattices were always intervals. In the same paper they also showed that the every bounded lattice is a Frattini sublattice of another bounded lattice. In this paper, Adams, Freese, Nation and Schmid posed the conjecture that spurred our research: complements of maximal sublattices of the even broader class of semi-distributive lattices are also intervals. Semidistributive lattices are lattices that are both $\vee-$semidistributive ($SD_\vee$) and $\wedge-$semidistributed ($SD_\wedge$) and in our explorations on this matter, we arrived at the conjecture that the complements of maximal sublattices of $SD_\vee$ lattices are unions of intervals with the same minimal element and dually, the complements of maximal sublattices of $SD_\wedge$ lattices are unions of intervals with the same maximal element. As part of our work on this conjecture we focused on a sub-class of $SD_\vee$ lattices called *convex geometries*.

In this talk we describe all of the possible complements of maximal sublattices of finite convex geometries with $cdim = 2$ and give a partial description of the possible Frattini sublattices of finite convex geometries with $cdim = 2$.

On the homothetic kissing numbers of a tetrahedron

István Talata

Budapest University of Economics and Business

Let K be a d-dimensional convex body, and let c be a real number, $c \neq 0$. Then, for every $v \in \mathbb{R}^d$, $cK + v = \{c \cdot k + v \mid k \in K\}$ is a homothetic copy of K whose ratio of homothety is c. The homothetic kissing number $H(K, c)$ of K with ratio of homothety c is the maximum number of mutually nonoverlapping homothetic copies of K with ratio of homothety c that can be arranged so that all touch K. This notion is a generalization of the translative kissing number of a convex body.

Let $T \subseteq \mathbb{R}^3$ be a tetrahedron. We give upper and lower bounds for $H(T, c)$ for various values of c.

Between Brascamp–Lieb and Colesanti inequalities

Jacopo Ulivelli

TU Wien

It is a well-known fact that both the Brunn–Minkowski and the Prékopa–Leinlder inequalities have equivalent local versions. Namely, Minkowski's second inequality and the Brascamp–Lieb inequality. The former can be restated as a Poincaré-type inequality due to Colesanti. In this talk, we present with similar methods a new inequality (again equivalent to the Prékopa–Leinlder inequality) which encoroporates both the Brascamp–Lieb and the Colesanti inequality. Interestingly, a new term appears which encodes the interaction between the two inequalities.

ON (COMPLEMENTED) BRUNN-MINKOWSKI TYPE INEQUALITIES FOR MEASURES

Jesús Yepes Nicolás
Universidad de Murcia
(Joint work with A. Zvavitch)

In this talk, we will discuss various functional and geometric forms of Brunn-Minkowski type inequalities, in both its classical form and its complemented version. We will study these inequalities in the setting of different absolutely continuous measures on $\mathbb{R}^n$ with radially decreasing densities, by paying special attention to the cases of the volume (the n-dimensional Lebesgue measure) and the standard Gaussian measure. Furthermore, we will show some other related inequalities when involving different operations between subsets of $\mathbb{R}^n$.

PARTITIONS OF UNITY AND BARYCENTRIC ALGEBRAS

Anna Zamojska-Dzienio
Warsaw University of Technology

Barycentric coordinates provide solutions to the problem of expressing an element of a compact convex set as a convex combination of a finite number of extreme points of the set. A barycentric coordinate system is usually characterized by two properties: the "partition of unity" property (understood as a partition of the constant function $\mathbb{1}_\Pi$ where the output value is 1) and the "linear precision" property (a partition of the identity function 1_Π on Π). Barycentric coordinates have been studied widely within the geometric literature, typically in response to the demands of interpolation, numerical analysis and computer graphics.

In [1, 2] the authors brought an algebraic perspective to the problem. They developed a general framework for the study of barycentric coordinate systems on a given convex polytope, founded on the theory of *barycentric algebras* introduced in the nineteen-fifties independently by M.H. Stone and H. Kneser for the axiomatization of real convex sets.

In this talk we focus on the discussion of relations between different subclasses of partitions of unity giving them an algebraic interpretation [3].

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Poster presentations

Pairs of random points from a shell

Alexandra Bakó-Szabó
University of Szeged
(Joint work with Ferenc Fodor)

We study the distribution of pairs of uniform and other random points from a spherical shell of two concentric balls. We determine the density function of their distance using characteristic functions and integrals of Bessel functions.

Extremality of diagonal sections of the cube

Barnabás Gárgyán
Warwick University & University of Szeged
(Joint work with Gergely Ambrus)

In this talk, we study the volume of diagonal central hyperplane sections of the cube. We demonstrate that, apart from the minimal, maximal, and the main diagonal sections, no subdiagonal central sections of the cube have extremal volume. Our approach relies on estimates on the rate of decay of the Laplace-Pólya integral, which are established by purely combinatorial means based on recursive arguments.

Variances of non-euclidean random polytopes

Balázs Grünfelder
University of Szeged
(Joint work with Ferenc Fodor)

We prove asymptotic upper bounds on the variances of the volume and vertex number of non-euclidean random polytopes, such as spherical random polytopes in spherical convex bodies, and hyperbolic random polytopes in convex bodies in hyperbolic space. We show two ways to prove variance upper bounds: applying the gnomonic projection, one can deduce these results from the weighted random models in the Euclidean space; and a direct proof via a non-euclidean version of the economical cap theorem.

Equidistant sets and their approximation

Márk Oláh

University of Debrecen

(Joint work with Ábris Nagy, Myroslav Stoika and Csaba Vincze)

The equidistant set of two sets K and L (called focal sets) in the Euclidean space $\mathbb{R}^n$, denoted $\{K = L\}$, is the set of all points that have the same distance from both K and L, measured with the infimum metric. Apart from the elementary cases (segment and angle bisectors), the most fundamental examples are the classical conics and convex polytopes.

A particularly interesting class of equidistant sets is when the focal sets are finite, allowing exact (computer-assisted) calculations. This case is closely related to the concept of Voronoi diagrams, and we present some examples how results for the latter can be translated to results for the former. For more general sets, the continuity theorem of Ponce and Santibanez is a fundamental tool, allowing the approximation of equidistant sets (or their focal sets) in the Hausdorff metric. Using this, we can show that any convex body is an equidistant set (approximating it by convex polytopes), and we can also apply this process for computer-based applications (replacing the original focal sets with finite ones). An important real-life example is the determination of territorial sea borders between nations.

We also discuss a recently posed open problem, namely, what are the closed sets that can be realized as the equidistant sets of two *connected* disjoint closed sets, and some other important classes of equidistant sets: *equidistant functions* whose graphs are the equidistant sets of the graph of another function and the x axis.

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Expectation of weighted intrinsic volumes of random polytopes

Dániel István Papvári

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(Joint work with Ferenc Fodor)

Let K be a convex body in $\mathbb{R}^d$, let $j \in \{1, \ldots, d-1\}$ and let ϱ be a positive and continuous probability density function with respect to the d-dimensional Hausdorff measure on K. Denote by $K_{(n)}$ the convex hull of n points chosen randomly and independently from K according to the probability distribution determined by ϱ.

For the case when $\varrho \equiv 1/V(K)$ and ∂K is C^2_+, M. Reitzner proved an asymptotic formula (and also I. Bárány if ∂K is C^3_+) for the expectation of the difference of the jth intrinsic volumes of K and $K_{(n)}$, as $n \to \infty$. K. J. Böröczky, L. M. Hoffmann and D. Hug extended this result to the case when $\varrho \equiv 1/V(K)$ and the only condition on K is that a ball rolls freely in K. K. J. Böröczky, F. Fodor, M. Reitzner and V. Vígh also showed that under the same assumptions, for the mean width, the existence of a rolling ball inside K is a necessary condition.

K. J. Böröczky, F. Fodor, and D. Hug proved an asymptotic formula for the weighted volume approximation of K under no smoothness assumptions on ∂K. We study the expectation of weighted intrinsic volumes for random polytopes generated by non-uniform probability distributions in convex bodies with very mild smoothness conditions. We assume no smoothness assumption on ∂K if $d/2 < j \leq d-1$, otherwise we assume that a ball rolls freely inside K.

Circle packings of the hyperbolic plane

Ádám Sagmeister

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(Joint work with Konrad Swanepoel)

Problems related to circle packings are central in discrete geometry. Here, given $n \in \mathbb{N}$, we want to find the maximum number of pairs of touching circles in a packing of n congruent circles of the hyperbolic plane. It is known, that on the Euclidean plane, the extremum comes from a spiral construction of the tiling of the plane with regular triangles. Here we give both lower and upper bounds in the hyperbolic plane. In particular, we prove that if the radius of the circles is not too small, the number of touching pairs is less than the one coming from the order 7 triangular tiling.

Normed variants of a theorem of Macbeath

Shanshan Wang

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(Joint work with Zsolt Lángi)

A theorem of Sas states that in plane among convex bodies of unit area, ellipses are hardest to approximate in terms of area difference with an inscribed convex polygon with a fixed number of vertices. He also showed that the only extremizers of this problem are the ellipses. Macbeath, in a classical theorem, generalized this result for higher dimensions and proved that the volume difference between a convex body and a largest volume inscribed convex polytope with a fixed number of vertices is maximal, relative to the volume of the body, for Euclidean balls. Our goal in this talk to investigate normed variants of this problem, and find largest volume polytopes with a fixed number of vertices and insribed in the unit ball of the norm, have the largest/smallest Busemann volume, Holmes-Thompson volume, Gromov's mass and mass* as defined by the norm.

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